

## **Finding Behaviour Patterns along Highways by Image Processing**

**Gyula Max**

Department of Automatization and Applied Informatics  
Budapest University of Technology and Economics  
H-1521 Budapest, Po. Box. 91, Hungary  
Tel: +36 1 463 2870, fax: +36 1 463 2871, e-mail: max@aut.bme.hu

*Abstract: We spend more and more time on travelling. Travelling by metro or tram gives us almost certain time to get home. And what about is travelling by car? Many persons travel hours to go working or to get home. In rush time the main roads and highways are full of cars. Can we simulate drivers' behaviour and their decision making process? This paper tries to find patterns in drivers' behaviour and simulate these processes.*

*Keywords: traffic simulation, image processing*

### **1 Introduction**

Travelling is our fun. Our hobby takes much time. Every day we spend at least one or two hours in travelling. Get on metro or tram if you want to get there in time. If you go by car it is not sure you reach your target in time. The more cars are in the street the surer is to be late. We try to summarize the reasons of your not getting there. The purpose of this paper is to study the results of two simulation methods along highways. Using the same initial conditions and the same probability variables during the simulations are there any differences between results of the two simulation methods? A number of approaches have recently been developed to simulate moving objects in traffic [1-12]. Recent studies have dealt with the dynamical phase transitions of pedestrians [1] or moving objects [2,3]. One-dimensional models have been proposed for the traffic flow on a highway [4,5] and two-dimensional models have been presented to describe the traffic flow in a city [6,7]. When the car density increases, the dynamical jamming transition occurs from the freely moving traffic to the jammed traffic. This paper shows two fundamentally difference models based on computer simulation. The first model is an adaptive, decentralized, parameterless model (ADP) with transition probabilities, while the second one is two-lane Lighthill-Whitham-Richards model based on discretized differential equations.

## 2 Simulation Model and Traffic Rules

Our basic traffic system contains two models. Both models are based on four-way traffic model with large number of small mobile objects as in Fig. 1. In both cases movements are represented in a two-dimensional square lattice considering traffic rules during the simulation. The rules are simply. Four types of objects exist. These objects move to left, right, up, and down in a square lattice. Each object can move to any of its four neighbouring lattice points. Even though each object has an intended travel direction it can move forwards, move sideways, move backwards, or stay without moving. One lattice point can not be occupied by two or more objects. If this happened an accident is occurred. Initially, the objects are placed randomly on the lattice with density  $p$  where  $0 \leq p \leq 1$ . The left, right, up, and down objects coloured by green, red, blue and black respectively.

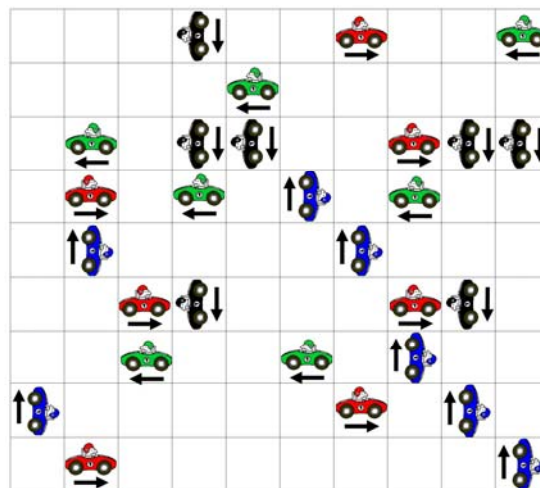


Figure 1  
Objects moving left, right, up and down on square lattice

In each time step objects can move left, right, up, and down in a two-dimensional square. Using their previous parameters these rules help us to follow drifts of objects describing new positions, speeds of the objects. The object density remains constant and equals to the initial density value, since no the objects are added or removed after the initialization. The objects are described by their density, positions, speeds, flows and drifts. Definitions of these elements are the following:

**Density** is the number of vehicles present on a given length of highway. Normally, density is reported in terms of vehicles per mile or vehicles per kilometre. High densities indicate that individual vehicles are very close together, while low densities imply greater distances between vehicles.

**Position** shows exact place of the vehicle. It can be measured along a line or given by their coordinates. Generally, it refers to two-dimensional surface.

**Speed** of a vehicle is defined as the distance it travels per unit of time. Most of the time, each vehicle on the highway will have a speed that is somewhat different from those around it. In quantifying the traffic flow, the average speed of the traffic is the significant variable. The average speed, called the space mean speed, can be found by averaging the individual speeds of all of the vehicles in the study area. The average speed is used as velocity in this paper.

**Flow** is one of the most common traffic parameters. Flow is the rate at which vehicles pass a given point on the roadway, and is normally given in terms of vehicles per hour. Also the free flow speed is interesting. This is the mean speed that vehicles will travel on a roadway when the density of vehicles is low. Under low-density conditions, drivers no longer worry about other vehicles. They subsequently proceed at speeds that are controlled by the performance of their vehicles, the conditions of the roadway, and the posted speed limit.

The main direction of the movement is the drift. It shows the greatest possibility of the movement. On the highway one drift exists. This drift is from left to right in Europe. Of course, vehicles can turn to left or right. Possibility of turning is much less than possibility of go ahead.

During the simulation a continuous, random, sequential procedure is used to update objects. At each time step, objects are randomly chosen one after the other  $n$  times and updated, where  $n$  is the number of objects in the lattice. Using this procedure each object is updated once at each time step.

In our model two lines segment of a highway is represented. According to the basic four ways model objects can move to right, up, and down on the highway. The intended travel directions of the objects are from left to right as it is shown by Fig. 2. Objects can move from left to right during their travel, or they can move sideways, or stay without moving in jam. Moving back is not forbidden.

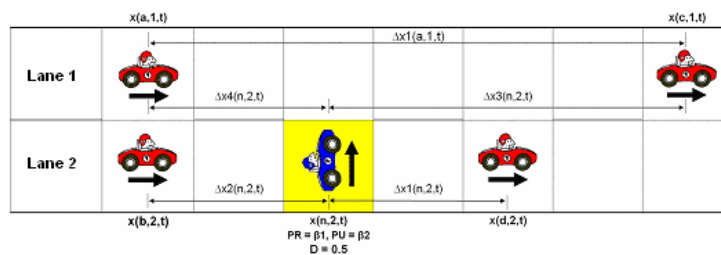


Figure 2  
 Movements on two lane highway

## 2.1 ADP Model

Our main target is to try to avoid congested regions and move objects towards less congested regions. Since we explain moving objects in the highway the traffic rules are described here for right-moving objects. The traffic rules for other objects are similar. The right object will be in one of the 16 configurations shown in Fig. 3. The grey square in the middle denotes the right object under consideration. The neighbouring circles denote neighbouring lattice points that are occupied by other objects. The object would move from left to right. The arrows denote possible directions in which the right object can move.

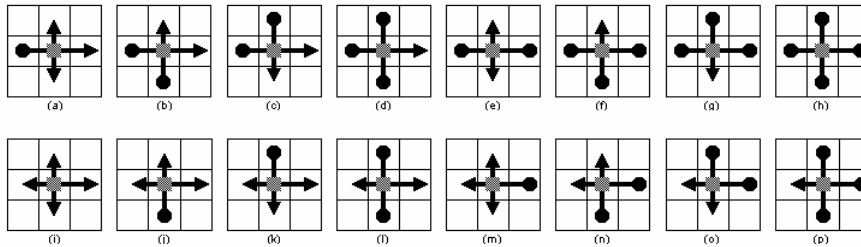


Figure 3  
 Sixteen possible configurations of a right moving object

In the real world objects move back very rarely. The upper line in Fig. 3[(a)-(h)] shows movements without step back. Sometimes back step is very useful, if the environment is known. Our task is to find next position in the lattice. Let's surround each object by four 3x3 square regions as it is shown in Fig. 4. The objects look at the congestion levels in these four surrounding boards, and move the object to the less congested region as it is shown in Fig. 5.

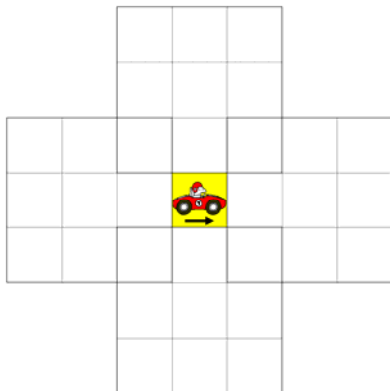


Figure 4

3x3 regions in left, right, up, and down directions surrounding an object

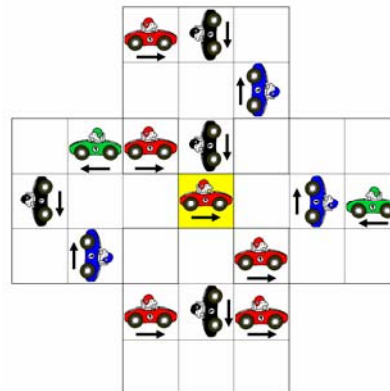


Figure 5

The object is surrounded by other objects in

The transition probability of the right object moving in right (forward), up (left turn), down (right turn) and left (back step) direction is denoted by PR, PU, PD, and PL respectively. There are 16 configurations of the transition probabilities of the right object. Table 1 shows values of probabilities for each configuration. In Table 1, NR, NU and ND denote the number of unoccupied lattice points in the 3x3 region in right, up, down and left direction respectively as well as  $N=NR+NU+ND+1$  denotes unoccupied lattice points without step back. In the above rules, the probability of the right object staying at the current lattice point without moving is  $PS = 1 - (PR+PU+PD+PL)$ . In this case the object can move but it is staying in its current position. According to the real world, in case of continuous flow the probability of not moving equals to zero.

Table 1  
Probability values of movements

No	PR	PU	PD	PL
a	NR/N	NU/N	ND/N	0
b	$NR/(NR+NU+1)$	$NU/(NR+NU+1)$	0	0
c	$NR/(NR+ND+1)$	0	$ND/(NR+ND+1)$	0
d	1-PS	0	0	0
e	0	$NU/(ND+NU+1)$	$ND/(ND+NU+1)$	0
f	0	1-PS	0	0
g	0	0	1-PS	0
h	0	0	0	0
i	$(1-PB) \cdot NR/N$	$(1-PB) \cdot NU/N$	$(1-PB) \cdot ND/N$	PB
j	$\frac{(1-PB) \cdot NR}{(NR+NU+1)}$	$\frac{(1-PB) \cdot NU}{(NR+NU+1)}$	0	PB
k	$\frac{(1-PB) \cdot NR}{(NR+ND+1)}$	0	$\frac{(1-PB) \cdot NR}{(NR+ND+1)}$	PB
l	$1 - PB - PS$	0	0	PB
m	0	$\frac{(1-PB) \cdot NU}{(ND+NU+1)}$	$\frac{(1-PB) \cdot ND}{(ND+NU+1)}$	PB
n	0	$1 - PB - PS$	0	PB
o	0	0	$1 - PB - PS$	PB
p	0	0	0	1

In order to evaluate Table 1, the probability of back step (Pback or PL) must be calculated. The idea behind these traffic rules is that the right object tries to avoid congested regions and move towards less congested regions and back step is not supported. In other words, the probability of the right object moving in a given direction is proportional to the number of unoccupied lattice points in the 3x3

region in that direction. In the rules (a)–(h), the left direction (step back) is occupied and the right object moves in available right, up and down direction with probability proportional to NR, NU and ND. In the rules (i)–(p), the left direction is unoccupied and the right object moves left with probability Pback and moves in available right, up and down direction with probability proportional to NR, NU and ND. The probability Pback of the right object (forward) moving left (back) is proportional to NL/K<sup>2</sup>, where NL is the number of unoccupied lattice points in left region and K is the size of the surrounded regions as well as 1–(NR+NU+ND+1)/(3·K<sup>2</sup>+1) shows the rate of occupied lattice points in right, up and down regions (1).

Pback = PB = P(not to moving right, up and down) times P(moving left) i.e.

$$P_{back} = \left(1 - \frac{NR + NU + ND + 1}{3K^2 + 1}\right) \cdot \frac{NL}{K^2} \quad (1)$$

Fig. 5 shows an example where an object in the middle is surrounded by other objects. The object in the middle is moving right. This is case (i) with NR = 6, NU = 4, ND = 5 and NL = 5. The transition probabilities of the middle object are PR = 1152/4032, PU = 768/4032, PD = 960/4032, Pback = PL = 960/4032 and PS = 192/4032 or in case of (b) with NR = 6, NU = 4 the probabilities are PR = 6/11, PU = 4/11, and PS = 1/11.

This calculation does not take into consideration the intended travel direction of objects. Even though, the transition probability depends on drift. Take the intended travel direction into consideration in case of (b) as PR<sub>drift</sub>=D+(1-D)/2 and PU<sub>drift</sub>=(1-D)/2 as well as in case of (c) as PR<sub>drift</sub>=D+(1-D)/2 and PD<sub>drift</sub>=(1-D)/2, where D indicates the strength of the drift. If no choice, like in case (f) P<sub>drift</sub> = 1. In case of continuous flow PS<sub>drift</sub> = 0.

Taking into consideration the drifts figures of Table 1., equations of (b) change as follow:

$$PR = \frac{NR}{NR + NU + 1} \rightarrow PR = \frac{NR * PR_{drift}}{NR * PR_{drift} + NU * PU_{drift} + 1 * PS_{drift}} \quad (2)$$

$$PU = \frac{NU}{NR + NU + 1} \rightarrow PU = \frac{NU * PU_{drift}}{NR * PR_{drift} + NU * PU_{drift} + 1 * PS_{drift}} \quad (3)$$

Using the new formulas values change. New value of PR is 18/20 and PU is 1/20. Value of PS is also changed into 1/20.

It is important to realize the adaptive, decentralized, and parameterless nature of the traffic rules. The transition probabilities PL, PR, PU, and PD of a mobile object at a given time depend on congestion level in the surrounding regions at that time. The transition probabilities of objects change with time depending on

traffic conditions. Each object makes its own decision about its movement in decentralized manner depending only on local congestion in the surrounding regions. There is no centralized control system that controls the movement of all objects. The traffic rules do not contain any parameters. There is no need to choose specific parameter values. This adaptive, decentralized, and parameterless traffic model is different from earlier traffic models [1,2] where the transition probabilities PL, PR, PU, and PD are predetermined and fixed for all objects at all time based on drift parameter regardless of congestion.

## 2.2 LWR Model

Some LWR models based of the two-lane were proposed in [8–11]. They considered the momentum equation and then developed a continuum modelling approach for the two-lane traffic. In [12] a two-lane car-following model with lateral distance is presented and it showed that the stability of traffic flow can be improved and the kink waves may appear in the unstable regime. However, the lane changing was not allowed in their work and some complex phenomena cannot be revealed. In this paper, we consider the effect of lateral distance and the resultant lane changing in the two-lane traffic.

Consider a highway with two traffic lanes ( $L = 1, 2$ ), as shown in Fig. 2. We label vehicles from 1 to  $m$  by  $(1, \dots, n-1, n, n+1, \dots, m)$ . These vehicles are distributed in lanes in position  $X(x(1,L,t), \dots, x(n-1,L,t), x(n,L,t), x(n+1,L,t), \dots, x(m,L,t))$  at time  $t$  and are allowed to change lane if necessary. In reality, when a vehicle changes lane, its velocity should be decomposed into two components: the longitudinal and transverse velocities. To simplify the analysis, we assume that a lane-changing vehicle moves instantaneously into the target lane and is at the same longitudinal location as that before changing lane.

We first discuss the car-following rules for the forward movement in the longitudinal direction, which can be expressed for the selected vehicle as

$$\frac{dX(n, L, t + \tau_L)}{dt} = V(\Delta x1(n, L, t), \Delta x3(n, L, t), \Delta v1(n, L, t)) \quad (4)$$

where  $\Delta x1(n, L, t)$  and  $\Delta v1(n, L, t)$  are the headway and relative velocity between two vehicles  $n$  and  $n+1$  in the same lane  $l$  at time  $t$ , respectively,  $\Delta x3(n, L, t)$  is the lateral distance (i.e., the distance between the vehicle in the selected lane and the closest vehicle in the target lane in front of this vehicle) at time  $t$ .  $\tau_L$  is the delay time within which the vehicle velocity reaches its optimal velocity in the target lane. This value is different in the lanes.

Expanding the left-hand side of Eq. (4) in Taylor's series and omitting the high-order terms, we have

$$\frac{dx(n, L, t + \tau_L)}{dt} = \frac{dx(n, L, t)}{dt} + \tau \frac{d^2x(n, L, t)}{dt^2} \quad (5)$$

We define the optimal velocity in lane L as a linear combination of the headway-induced component and the velocity-induced component in the following form

$$V_L(\Delta x1(n, L, t), \Delta x3(n, L, t), \Delta v1(n, L, t)) = V_L(\Delta x1(n, L, t), \Delta x3(n, L, t)) + \lambda_L \cdot \Delta v1(n, L, t) \quad (6)$$

where  $\lambda_L \in (0, 1)$  is the sensitivity factor for the relative velocity and independent of time, velocity and position. We assume that the physical condition for driving in lane 1 (the far-side lane) is better than that in lane 2 (the curb-side lane), and thus  $\lambda_1 > \lambda_2$ . Substituting Eqs. (5), (6) into Eq. (4), we have

$$\frac{d^2 X(n, L, t)}{dt^2} = \alpha_L (V_L(\Delta x1(n, L, t), \Delta x3(n, L, t)) - \frac{dx(n, L, t)}{dt} + \lambda_L \cdot \Delta v1(n, L, t)) \quad (7)$$

where  $\alpha_L = 1/\tau_L$  represents the driver's sensitivity with respect to the difference between the optimal and current velocities in lane L, and  $\alpha_L \cdot \lambda_L$  is the sensitivity coefficient in response to the stimulus  $\Delta v(n, L, t)$ . We define the weighted headway as

$$\bar{\Delta x}(n, L, t) = \beta_1 \cdot \Delta x1(n, L, t) + \beta_2 \cdot \Delta x3(n, L, t) \quad (8)$$

where  $\beta_1$  and  $\beta_2$ , with  $\beta_1 + \beta_2 = 1$  and  $\beta_1 \geq \beta_2$ , are the weights for  $\Delta x1(n, L, t)$  and  $\Delta x3(n, L, t)$ , respectively. So, the velocity  $V_L(\Delta x1(n, L, t), \Delta x3(n, L, t))$  in Eq. (7) takes the following form

$$V_L(\Delta x1(n, L, t), \Delta x3(n, L, t)) = V_L(\bar{\Delta x}(n, L, t)) = \frac{1}{2} v_{Lmax} \cdot (\tanh(\bar{\Delta x}(n, L, t) - h_{3L}) + \tanh(h_{3L})) \quad (9)$$

where  $h_{3L}$  and  $v_{Lmax}$  are the safety distance and maximum speed in target lane, respectively. In the studies [13-14], the effect of  $\Delta x3(n, L, t)$  was not modelled (i.e.,  $\beta_2 = 0$ ). In [15] both  $\Delta x1(n, L, t)$  and  $\Delta x3(n, L, t)$  were taken account in lane changing behaviour (i.e.,  $\beta_2 \neq 0$ ).

Having solved and discretized Eq. 7 with help of Eqs. (8) and (9) we have the following formula:

$$X(n, L, t + 2\tau) = X(n, L, t + \tau) + \tau \cdot \frac{1}{2} v_{Lmax} \cdot (\tanh(\beta_1 \cdot \Delta x1(n, L, t) + \beta_2 \cdot \Delta x3(n, L, t) - h_{3L}) + \tanh(h_{3L})) + \lambda_L \cdot \tau \cdot (X(n, L, t + \tau) - X(n, L, t)) \quad (10)$$



### 3 Simulation

In the simulation program 1500 meters length of a highway segment of two lines were represented. The traffic rules are the following: in one of the two lines the  $i^{\text{th}}$  object ( $O_i$ ) starts in the highway at the time  $t_{i0}$  its speed is  $v_{i0}$  at position of  $X(i,L,t_{i0})=0$ , where  $v_{i0} \leq v_{1\text{max}} = v_{2\text{max}} = 180$  km/h. Without step back the possible drifts of  $O_i$  right moving object are (b), (c), (d), (e), (f), (g), (h) from Fig. 3. In other words, objects can move ahead or turn to left or right, can change lines, and can wait if they can not move. Line changing rules were declared as follows:

$$\Delta x1(n, L, t) < 2 \cdot h_{3L} \quad (11)$$

$$\Delta x3(n, L, t) > \Delta x1(n, L, t) \quad (12)$$

$$\Delta x4(n, L, t) > h_{4L} \quad (13)$$

Equations (11) and (12) represent the incentive criterion for the lane-changing decision. Equation (13) is the safety criterion, where  $\Delta x4(n, L, t)$  is the distance between the vehicle  $n$  in lane  $L$  and the closest vehicle in the target lane behind this vehicle at time  $t$  (see Fig. 2), and  $h_{4L}$  is the corresponding safety distance. According to this lane-changing rule, the reaction time required by a lane-changing decision can be neglected.

Since effects of  $\beta1$  and  $\beta2$  are similar to effects of probability variables of ADP model these figures connect our models. In each step new values of PR and PU or PD are calculated. Using (8),  $\beta1 + \beta2 = 1$  and  $\beta1 \geq \beta2$  formulas  $\beta1$  is substituted with PR and  $\beta2$  is substituted with PU or PD according to the lane changing. These rules can be independently used in modelling the lane-changing behaviour and we have a common area in our models. In the LWR simulation two sets of initial values are necessary beside of the common starting values. So first the ADP model is calculated and the results are transferred as second half of the starting values of the LWR model.

This computer program uses sequential update rule. Initially, the mobile objects characterized by their timestamps and speeds are numbered randomly from 1 to  $n$  where  $n$  is the total number of objects on the lattice. At each time step, each object is updated once in the sequential order from 1 to  $n$ . As we have seen the transition probabilities depend on the drift of objects. The transition probabilities of right moving objects from left to right are higher than probabilities of turning.

At each time step  $T$ , each object decides where it wants to move based on neighbour configuration and transition probabilities as it was described above. The conflicts may arise because more than one object may want to move to the same lattice point at the same time step. If the conflict of movement can be resolved than newer figure is calculated. If not, an accident happens, the speeds of objects become to zero blocked one line of the highway.

During examination, the velocity of the system is defined as number of objects that move from one lattice point to another lattice point at time step  $t$  divided by the total number of objects in the system. Throughout the paper, the simulations are run for 20 000 time steps. The average velocity ( $v$ ) of the system is defined as the average of  $v(t)$  over  $10\,000 \leq t \leq 20\,000$  time steps averaged over 7 different random initial conditions. The symbols  $N_{\max}$ ;  $p$ ; and  $D$  denote the system size, object density, and drift of objects, respectively.

We want to examine the following:

- similarity between ADP and LWR models,
- changes of flow characteristic of continuous and interrupted flow,
- effects of special events (i.e. accident).

## 4 Results

Our main goal is to find a good traffic model for simulation. A model is good if it can follow moving objects with relatively small errors for a long time or can predict their movements in the observed region. This paper showed two, basically difference, two-dimensional highway traffic models. It introduces us to basic techniques and elements of the traffic description. In the ADP model the transition probabilities of moving objects were corrected (see Table 1) as compared with [2]. This paper combined the effects of transition probabilities and drifts. New equations were made to calculate common effects (2), (3). The paper shows that (2) and (3) describe traffic model in proper perspective similar to [2]. Drift forces moving objects to continuous flow.

In both models in case of large systems besides increasing object density average velocity decreasing significantly that is higher density forced moving objects to get slower. Accidents made total jam in case of large systems. This paper studied flow and congestion of moving objects on two-line highway using a simple two-dimensional traffic system. The equations of the models in this paper show common effects of transition probabilities and drifts of moving objects. The objects try to avoid congestion among themselves using simple congestion-avoiding traffic rules. As it was seen there were strong relation between traffic relative distances and velocity though, the exact relationship between these parameters were not known yet. During the computer simulation our traffic rules tried to avoid accidents. It was successful only at low density. Unfortunately, our models can not follow each other's movements. The numbers of the similar steps were very small. During the simulation neither starting states in the 10000<sup>th</sup> steps nor in the final steps were the same. Each simulation gives different results. It means that these simulation can not be used for predictions. Though we found

many matching patterns during the simulation as it can be seen in Table 2. A pattern was matched if the positions and the velocities of the moving objects were the same. Increasing density shows strong correlation to matching patterns.

Table 2  
Results of the simulations

Density	Final state	Patterns matching	Similar steps	Accidents
0,1	0	23700	3,194	0
0,2	0	35736	2,892	0
0,3	0	113014	3,987	0
0,4	0	152994	3,132	79
0,5	0	148642	3,365	153
0,6	0	120616	3,169	297
0,7	0	219324	2,835	461
0,8	0	167494	3,394	553
0,9	0	372440	3,426	617

Finding matching patterns showed interesting possibilities in traffic observation. When matching pattern is found in each model next few steps are the same. This amount is not huge, but it can be used for prediction. During evaluation of traffic situations our computers are not fast enough. Maximum 6-8 frames are evaluated in each second because of demand of the high processor time. We would have more evaluation time if we knew next few positions of our moving objects. These models help us to increase evaluation period because next one or two positions and velocities can be predicted without image processing. The evaluation time increases with image processing time of one frame minus prediction time of the model. We can say that using one of these prediction models the evaluation period of one frame can be increased from 150 ms to 300 ms at higher speed level in the highways or to 400 ms at smaller velocity in case of urban traffic.

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