Mamdani-type Inference in Fuzzy Signature Based Rule Bases

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Abstract: The concept of fuzzy signatures might be useful when modeling complex, well structured problems, where one or several components of the structure are determined at a higher level by a sub-tree of other components. The data set belonging to the problem has an arbitrary structure, from which the structure of the data may slightly differ. An aggregation operator is given for each node, for the purpose of modifying the structure, so that data with missing components can be evaluated. Deducing a conclusion from an observation having such a structure is a key issue. In this paper fuzzy signature based rule bases will be introduced, then the generalisation of the well known Mamdani method for signature based rules will be shown step-by-step. Finally, an example of inference on fuzzy signatures will be discussed.

Keywords: fuzzy signatures, inference on fuzzy signatures

1 Fuzzy Signatures

In 1967 Goguen introduced L-fuzzy sets [1] as the generalisation of the original concept of fuzzy sets introduced by Zadeh [2] in 1965. L-fuzzy membership grades are elements of an arbitrary lattice L:

$$A : x \to L, \quad \forall x \in X \tag{1}$$

In 1980 Kóczy introduced vector valued fuzzy sets [3], which are special L-fuzzy sets, where L is the lattice of n-dimensional fuzzy vectors, $L = [0,1]^n$ in (1).

Vector valued fuzzy sets assign to each element of X a set of quantitative features rather than a single degree, this way providing additional information about the specific element.

Fuzzy signatures [4] are a generalised form of vector valued fuzzy sets, where each vector component is possibly another nested vector. This generalisation can be continued recursively to any finite depth, thus forming a signature with depth *m*.

$$A_S : x \to [a_i]_{i=1}^{k}, a_i = \begin{cases} [0,1] \\ [a_{ij}]_{j=1}^{k_i} \end{cases}, a_{ij} = \begin{cases} [0,1] \\ [a_{ijl}]_{l=1}^{k} \end{cases}, \quad \forall x \in X \tag{2}$$

The structure of fuzzy signatures can be represented both in vector form and also in a tree structure. Let us consider a simple example.

The basic structural vector is: $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, where the sub-trees are $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}$; $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix}$; $x_{22} = \begin{bmatrix} x_{221} \\ x_{222} \\ x_{223} \end{bmatrix}$ and $x_3 = \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix}$

The vector form of the structure is: $x = \begin{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \\ \begin{bmatrix} x_{21} \\ \begin{bmatrix} x_{221} \\ x_{222} \\ x_{223} \end{bmatrix} \\ x_{23} \end{bmatrix} \\ \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} \\ x_4 \end{bmatrix}$

The tree structure of the same signature can be seen in Figure 1.

Fuzzy signatures can be considered as special, multidimensional fuzzy data, where some of the components are interrelated in the sense that a sub-group of variables determines a feature on a higher level. This way, complex and interdependent data components can be described and evaluated in a compact way. The big advantage of fuzzy signatures is, that they can deal with situations, where some of the data components are not known.

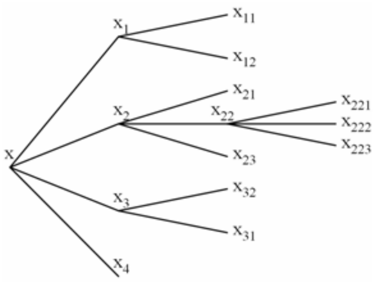

Figure 1
The tree structure of an example fuzzy signature

1.1 Fuzzy Signature Sets

The structure of fuzzy signature sets is similar to that of variable fuzzy signatures. The only difference is that instead of storing fuzzy variables on the leaves of the structure, a membership function is present on each leaf (see Figure 2).

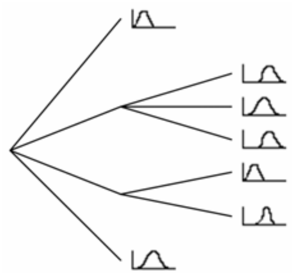

Figure 2
The tree structure of an example fuzzy signature set

1.2 Aggregation on Fuzzy Signatures

As the structure of fuzzy signatures may vary from observation to observation, some kind of structure modifying operation is essential, so that these differently structured signatures may be compared.

Aggregation operations result in a single fuzzy value from a set of other fuzzy values and have to satisfy a set of axioms. Maximum, minimum and arithmetic mean are the most common operators. It is possible to transform fuzzy signature structures in the following way: the fuzzy value of a parent node can be obtained by aggregating the values of its children (or in more general form, the value of its sub-tree) with a suitable aggregation operator, thus reducing the depth of the structure.

Finding the relevant aggregation operator for each node in the structure is a very important problem of fuzzy signatures, because the capability of modifying the structure is a key issue when comparing different signatures. Additional expert knowledge can be taken into account by introducing weights (from the [0,1] interval) for every node in the structure.

The most general form of aggregation operators is the Weighted Relevance Aggregation Operator (WRAO) introduced in [6]. The values and weights belonging to each child l in the sub-tree are denoted by x_l and w_l respectively. The definition of the WRAO is as follows:

$$@(x_1, x_2, \dots, x_n; w_1, w_2, \dots, w_n) = \left\{ \frac{1}{n} \sum_{i=1}^{n} (w_l x_l)^p \right\}^{\frac{1}{p}} \tag{3}$$

where p is the aggregation factor of the above function. $(p \in \Re,\ p \neq 0)$

Some of the well-known aggregation operators are special cases of WRAO depending on the value of p.

$p \to -\infty$, WRAO $\to$ minimum

$p = -1$, WRAO $=$ harmonic mean

$p \to 0$, WRAO $\to$ geometric mean

$p = 1$, WRAO $=$ arithmetic mean

$p \to \infty$, WRAO $\to$ maximum

2 Fuzzy Signatures in Rule Bases

The general form of fuzzy rule bases is as follows:

R_1 : If x is A_1 then y is B_1

R_2 : If x is A_2 then y is B_2

…

R_r : If x is A_r then y is B_r

where A_i are multidimensional fuzzy sets and B_i are fuzzy sets.

The above rule base can be extended to function on fuzzy signatures by the generalisation of the rules.

In the antecedent part of the rules, A_i can either be fuzzy signature sets or simply fuzzy variable signatures, where the fundamental structures of the signatures are similar and the corresponding aggregation operators are uniform for each rule. For fuzzy signature sets, the domain of the fuzzy sets on the leaves of the structure is [0,1]. The consequents of the rules remain fuzzy sets.

Observation A' is either a fuzzy signature singleton or a fuzzy signature set (over the domain [0,1]). The structure of the observation does not necessarily correspond exactly to the fundamental structure of the rules, but often it can be obtained by removing some sub-trees or leaves. This indicates that some information might be missing from the observation.

Signature A' is obtained from the original fuzzy singleton or fuzzy set observations by normalising the domains on each leaf of the signature.

2.1 Mamdani Inference in Fuzzy Signature Based Rule Bases

A slightly modified version of the algorithm introduced by Mamdani in 1975 [7] is proposed here, which will work on rule bases with fuzzy signatures, as well.

2.1.1 Calculating the Degree of Matching for Every Rule

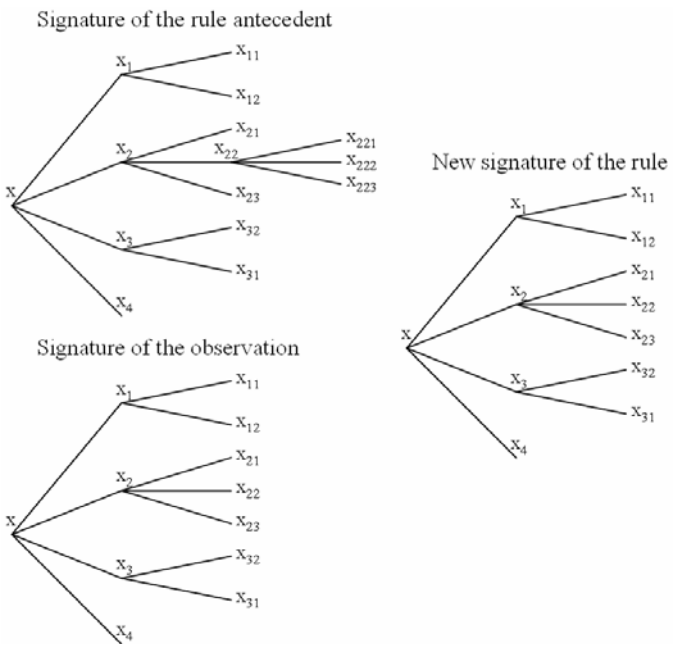

Figure 3

A whole sub-tree is missing from the observation

1 Finding the common structure:

The signatures A_i and A' have to be reduced to their maximal common sub-tree.

a) If a whole sub-tree is missing in the structure of the observation compared to the structure in the rule antecedent, then that sub-tree is reduced to its root by aggregation. The structures can be seen in Figure 3.

In the rule's new structure the fuzzy set (or variable) at the leaf marked x_{22} is calculated by aggregating the fuzzy sets (or variables) that originally where on the leaves x_{221}, x_{222} and x_{223}:

$$x_{22} = @_{22}(x_{221}, x_{222}, x_{223}) \tag{4}$$

b) In the case that only a few children of a node are missing from the observed tree structure, then the maximal common sub-tree of the rule's and the observation's signature has to be determined. At this point, the signature of the antecedent and the signature of the observation both have to be reduced to the common structure using the aggregation operators defined for the rule base.

The first step in reducing the antecedent of the rule and the observation to the common structure shown in Figure 4 is reducing the sub-tree belonging to x_{22} as shown in equation (4).

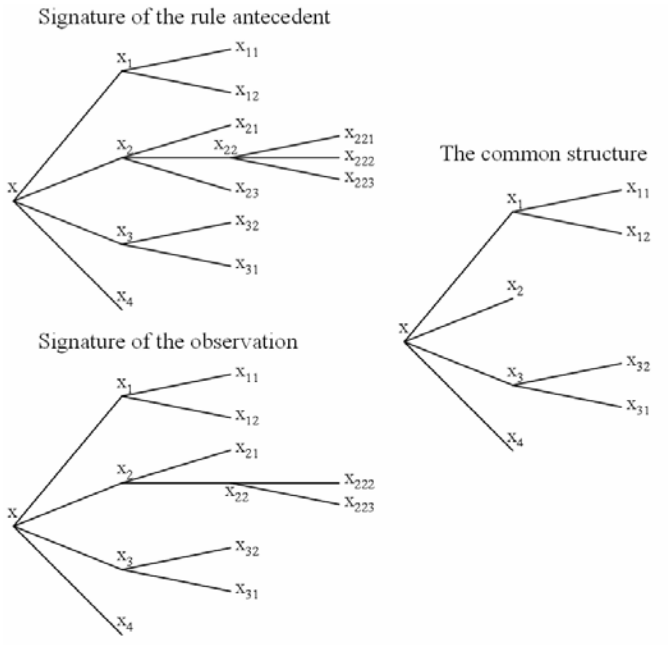

Figure 4
Two values (from nodes x_{23} *and* x_{221}) are misssing from the observation

The fuzzy set (or fuzzy value) calculated for the node x_{22} is then taken into account at the next level, when x_2 is calculated:

$x_2 = @_2(x_{21}, x_{22}, x_{23})$ in the case of the rule antecedent, and

$x_2 = @_2(x_{21}, x_{22})$ in the case of the observation.

In the aggregation, missing data is treated as though its weight in the WRAO was 0.

Let $A_i^{(r)}$ and $A'^{(r)}$ be signatures of the rule antecedent and the observation respectively, after the reduction of the signatures to their common structure.

2 Constructing the signature representing the degree of matching:

For each leaf l of the signature structure, the degree of matching between the fuzzy set (or value) found on the equivalent leaf of the rule antecedent and the observation has to be calculated by applying the formula used in the Mamdani method:

$$M_i(l) = \max(\min(A'^{(r)}(l), A_i^{(r)}(l))) \tag{5}$$

where $S(l)$ denotes the fuzzy set (or value) found on leaf l of the signature S. M_i is the signature representing the degree of matching between rule i and the observation.

3 Calculating the degree of matching for rule i:

The degree of matching for rule i is obained by reducing the previously computed fuzzy signature M_i to its root using the aggregation operators belonging to the rule base. The result of the aggregation is a fuzzy value representing the degree of matching of rule i and the observation, which is denoted by w_i.

The above process is repeated for all rules i ($i=1,\ldots,r$), so that all the degrees of matching ($w_1,\ldots,w_r$) are produced.

2.1.2 Inference Engine

From this point onwards, inference for fuzzy signature based rule bases does not differ from inference on original, fuzzy set rule bases.

First, the truncated output fuzzy set B_i^* is calculated for each rule i as:

$$\mu_{B_i^*}(y) = \min(w_i, \mu_{B_i}(y)) \tag{6}$$

The conclusion fuzzy set B' is calculated from the truncated fuzzy sets as:

$$\mu_{B'}(y) = \max_{i=1}^{r}(\mu_{B_i^*}(y)) \tag{7}$$

2.1.3 Defuzzification

To calculate a crisp result from the obtained fuzzy set B' representing the observation, one of many defuzzification methods known from literature can be used.

3 Example of Fuzzy Signature Based Inference

Let the antecedents of the rules in the rule base be fuzzy signature sets, where the membership functions of the fuzzy sets on the leaves of the structure are chosen from sets A, B, C, D ,E (of Figure 5).

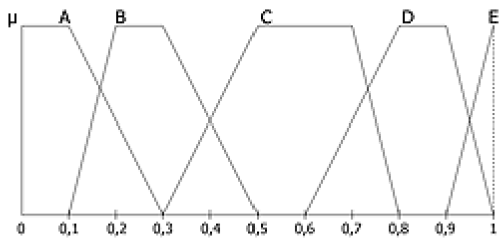

Figure 5
Example fuzzy sets for use in rule antecedents

The membership function of the consequent fuzzy set of each rule is either *F* or *G* (of Figure 6).

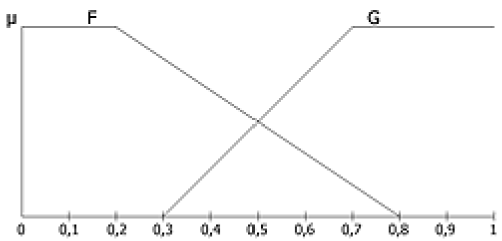

Figure 6
Example fuzzy sets for use in rule consequents

3.1 The Structure

The fundamental structure of the fuzzy signatures involved in the example is the structure seen in Figure 1.

3.2 The Aggregation Operators

Let us define the aggregation operators $@_i$ which generate the fuzzy set (or value) assigned to nodes x_i from the fuzzy sets (or values) assigned to their immediate children (x_{i1} and x_{i2} (and x_{i3} if needed)). Recursively, the fuzzy set (or value) on node x_{22} can be aggregated from the values of its children: x_{221}, x_{222} and x_{223}.

The value of the root x also has to be obtained (at least for aggregating the signature representing the degree of matching), so an aggregation operator @ is needed at this level, that generates the value of x from the value of its children: x_1, x_2, x_3 and x_4.

The formulae:

$$\begin{aligned} x &= @(x_1, x_2, x_3.x_4) \\ x_1 &= @_1(x_{11}, x_{12}) \\ x_2 &= @_2(x_{21}, x_{22}, x_{23}) \\ x_{22} &= @_{22}(x_{221}, x_{222}, x_{223}) \\ x_3 &= @_3(x_{31}, x_{32}) \end{aligned} \tag{8}$$

In the example the following operators are used: @ , $@_1$ and $@_2$ are the arithmetic mean, $@_{22}$ the minimum and $@_3$ the maximum. These are all special cases of WRAO, where all the weights are 1.

3.3 Aggregation on Fuzzy Sets

When aggregating fuzzy sets, the membership values for each element x of X (in this case x ∈ [0,1]) are calculated for all the fuzzy sets which are subject to the aggregation. The original aggregation operator is then used on these membership values, to obtain the aggregated value belonging to x. The membership function of the aggregated fuzzy set is:

$$\begin{aligned} & Ag = @(A_1, A_2, \ldots, A_k) \\ & \forall x \in [0,1] \quad \mu_{Ag}(x) = @\{\mu_{A_1}(x), \mu_{A_2}(x), \ldots, \mu_{A_k}(x)\} \end{aligned} \tag{9}$$

In the example below let the fuzzy sets A_1 and A_2 have membership functions C and D seen in Figure 5. The aggregation shown in Figure 7 is the arithmetic mean operator. The resulting fuzzy set (denoted by H) is marked with a broken line.

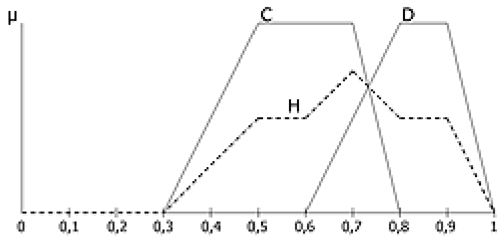

Figure 7
Aggregation of fuzzy sets *C* and *D* with the arithmetic mean operator

3.4 The Rules

The rule base contains rules with differently structured fuzzy signatures. Let the vector form of rule *j* be the following:

R_j : If *x* is $[[D \quad C] \quad [C \quad A \quad B] \quad [E \quad B] \quad D]^T$ then *y* is F

3.5 The Inference

The process of inference is shown for the following fuzzy signature observation:

$$x' = [0.4 \quad [0.45 \quad [0.25 \quad 0.65 \quad 0.5] \quad 0.35] \quad [0.95 \quad 0.6] \quad 0.75]^T$$

3.5.1 Calculating the Degree of Matching Between the Rule Antecedent and the Observation

1) Finding the common structure is shown in Figure 8.

The new structure of the fuzzy signature set in the rule is obtained by aggregating fuzzy sets *C* and *D* so that a new fuzzy set is obtained for leaf x_1. The aggregation operator used is that of leaf x_1 (arithmetic mean). This aggregation was shown in Figure 7.

The new fuzzy signature set in the rule is: $[H \quad [C \quad A \quad B] \quad [E \quad B] \quad D]^T$

The new structure of the fuzzy signature in the observation is obtained by aggregating the fuzzy values of the sub-tree belonging to x_{22} (with the aggregation operator $@_{22}$). The new value on node x_{22} is: $\min(0.25, 0.65, 0.5) = 0.25$

The new signature of the observation is:

$$[0.4 \quad [0.45 \quad 0.25 \quad 0.35] \quad [0.95 \quad 0.6] \quad 0.75]^T$$

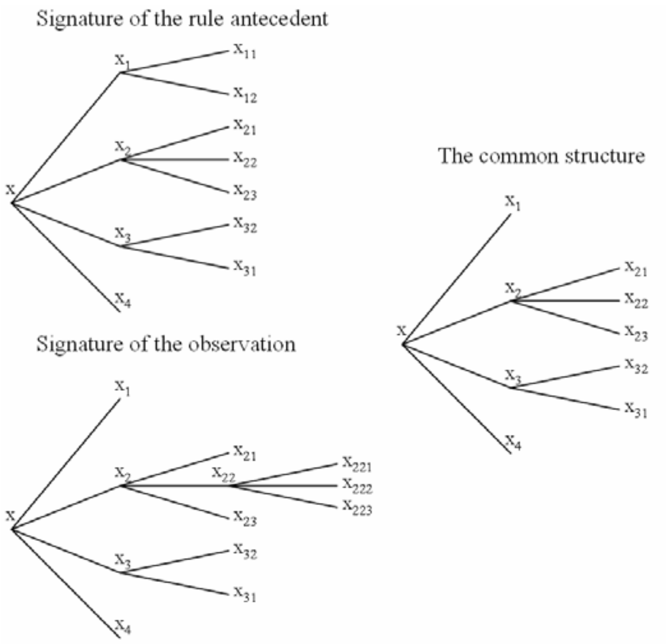

Figure 8
The common structure of the rule and the signature

2) Constructing the signature representing the degree of matching

$$M_j = \begin{bmatrix} \mu_H(0.4) \\ \begin{bmatrix} \mu_C(0.45) \\ \mu_A(0.25) \\ \mu_B(0.35) \end{bmatrix} \\ \begin{bmatrix} \mu_E(0.95) \\ \mu_B(0.6) \end{bmatrix} \\ \mu_D(0.75) \end{bmatrix} = \begin{bmatrix} 0.25 \\ \begin{bmatrix} 0.75 \\ 0.25 \\ 0.75 \end{bmatrix} \\ \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \\ 0.75 \end{bmatrix}$$

3) Calculating the degree of matching for rule *j*.

The signature M_j has to be aggregated to its root. The values assigned to nodes x_i are:

$$x_2 = @_2(x_{21}, x_{22}, x_{23}) = (0.75 + 0.25 + 0.75)/3 = 0.58$$

$$x_3 = @_3(x_{31}, x_{32}) = \max(0.5, 0) = 0.5$$

The degree of matching for rule *j* is:

$$w_j = @(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4)/4 = (0.25 + 0.58 + 0.5 + 0.75)/4 = 0.52$$

3.5.2 Inference Engine

The fuzzy set F (the consequent of rule j) is 'truncated' by w_j.

These steps are performed similarly for all the other rules in the rule base, keeping in mind that the signature structures in the antecedents of the rules may differ from the one shown in this example.

When fuzzy sets B_j^* are calculated for every rule j, the conclusion is obtained in the same way as with classical fuzzy rule bases.

If necessary, defuzzification methods are applied on the concluded fuzzy set in order to obtain a crisp conclusion.

Conclusions

In this paper a brief overview of fuzzy signatures along with their main advantages was given. After this, rule bases containing fuzzy signatures were presented and the idea of Mamdani type systems based on fuzzy signatures were introduced. These could be useful for large data sets, where data can be modeled with fuzzy signatures, because conclusions can be drawn from observed data available in signature form. In the future, various methods for automatically extracting rules from available data to form fuzzy signature based rule bases will be examined.

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References

[1] J. A. Goguen: L-fuzzy Sets, in Journal of Mathematical Analysis and Applications, 1967, Vol. 18, pp. 145-174

[2] L. A. Zadeh: Fuzzy Sets, in Information and Control, Vol. 8, 1965, pp. 338-353

[3] L. T. Kóczy: Vector Valued Fuzzy Sets in Busefal, eté, 1980, pp. 41-57

[4] L. T. Kóczy, T. Vámos, G. Biró: Fuzzy Signatures, in Proceedings of EUROFUSE-SIC'99, Budapest, Hungary, May 25-28, 1999, pp. 210-217

[5] K. W. Wong, A. Chong, T. D. Gedeon, L. T. Kóczy, T. Vámos: Hierarchical Fuzzy Signature Structure for Complex Structured Data, in Proceedings of International Symposium on Computational Intelligence and Intelligent Informatics, Nabeul, Tunisia, May 19-23, 2003, pp. 105-109

[6] B. S. U. Mendis, T. D. Gedeon, J. Botzheim, L. T. Kóczy: Generalised Weighted Relevance Aggregation Operators For Hierarchical Fuzzy Signatures, in Proceedings of International Conference on Computational Intelligence for Modelling Control and Automation, 2006

[7] E. H. Mamdani, S. Assilian: An Expereriment in Linguistic Synthesis With a Fuzzy Logic Controller, in International Journal of Man Machine Studies, Vol. 7, 1975, pp. 1-13