Applicability of Fuzzy Flip-Flops in the Implementation of Neural Networks

Rita Lovassy[1], László T. Kóczy[2], László Gál[3]

[1] Inst. of Information Technology, Mechanical and Electrical Eng., Széchenyi István University, Győr, Hungary, on leave from Inst. of Microelectronics and Technology, Kandó Kálmán Faculty of Electrical Engineering, Budapest Tech, Hungary
E-mail: lovassy.rita@kvk.bmf.hu

[2] Inst. of Information Technology, Mechanical and Electrical Eng., Széchenyi István University, Győr, Hungary and Dept. of Telecommunication and Media Informatics, Budapest University of Technology and Economics, Hungary
E-mail: koczy@sze.hu, koczy@tmit.bme.hu

[3] Inst. of Information Technology, Mechanical and Electrical Eng., Széchenyi István University, Győr, Hungary, on leave from Dept. of Technology, Informatics and Economy, University of West Hungary
E-mail: gallaci@ttmk.nyme.hu

Abstract: The concept of various type fuzzy flip-flops (F^3) has already been proposed. We have done some investigations on a large scope of F^3s based on different t-norms and conorms. Also we have shown that a few F^3 types are suitable for realizing neurons in multilayer perceptrons. The aim of this paper is to present a comparison of the performance of several type neural networks based on fuzzy J-K and also fuzzy D flip-flops (the latter derived from the former type). The behavior of algebraic, Yager, Dombi and Hamacher type fuzzy flip-flop neural networks are presented. The best fitting t-norm and corresponding fuzzy flip-flop type will be presented in terms of function approximation capability.

Keywords: t-norm, t-conorm, fuzzy J-K flip-flop, fuzzy D flip-flop, Fuzzy Flip-Flop Neural Network (FNN)

1 Introduction

Neural networks and fuzzy set theory has been the object of intense study and application, especially in the last decade. There are several manners to combine neural networks and fuzzy logic, which may differ essentially according to approaches and the tasks. With the use of fuzzy logic techniques, neural computing can be integrated in symbolic reasoning to solve complex real world problems. In fact, artificial neural networks, expert systems, and fuzzy logic systems share common features and techniques in the context of approximate reasoning. This paper investigates the choice of function approximator for a neural network based fuzzy flip-flops. A fuzzy flip-flop network is proposed, in which an artificial neural network-like approach is designed to construct the knowledge base of an expert system. The approximation of a mathematical function (using examples in the form of input-output pairs) is a central issue in subjects as diverse as pattern recognition, control theory and statistics. We present some investigations on the usefulness of several logical connectives, followed by a purposeful fuzzy sequential system design in order to construct a network performing good learning and approximation.

The paper is structured into five sections. After the introduction, in Section 2, we present the concept of a single fuzzy J-K flip-flop, using the fundamental equation as it was proposed in [10].

In Section 3, a comparative study of several types of fuzzy J-K flip-flop with feedback (case of $K=1-Q$), fuzzy D flip-flop (case of $K=1-J$) and a different interpretation to define fuzzy D flip-flop [1] based on various norms has been presented.

Section 4 is devoted to the investigation of the F^3 based neurons and the Multilayer Perceptrons (MLP) [8] constructed from them. We show that the proposed the Fuzzy Flip-Flop Neural Network (FNN) architecture it can be use for approximating various test functions. Comparison between different types of FNNs and the ideal *tansig* (hyperbolic tangent sigmoid transfer function) characteristics NN are presented in Section 5.

2 The Concept of Fuzzy J-K Flip-Flop

The fuzzy J-K flip-flop is an extended form of binary J-K flip-flop. In this approach the truth table for the J-K flip-flop is fuzzified, where the binary NOT, AND and OR operations are substituted by their fuzzy counterparts, i.e. fuzzy negation, t-norm, and co-norm respectively. The next state $Q(t+1)$ of a J-K flip-flop is characterized as a function of both the present state $Q(t)$ and the two present inputs $J(t)$ and $K(t)$. For simplicity (t) is omitted in the next. The so called fundamental equation of J-K type fuzzy flip-flop [10] is

$$Q(t+1) = (J \vee \neg K) \wedge (J \vee Q) \wedge (\neg K \vee \neg Q) \tag{1}$$

where $\neg, \wedge, \vee$ denote fuzzy operations (e.g. $\neg K = 1 - K$). As a matter of course, it is possible to substitute the standard operations by any other reasonable fuzzy operation triplet (e.g. De-Morgan triplet), thus obtaining a multitude of various fuzzy flip-flop (F^3) pairs.

In [7] we studied the behavior of F^3 based on various fuzzy operations.

In the next Section we will give an overview of the different type J-K F^3s, based on familiar norms well known from the literature, namely the algebraic, Yager, Dombi and Hamacher, using the standard complementation in every case. After introducing their characteristic equations we will illustrate their behavior by the graphs belonging to the next states of fuzzy flip-flops for typical values of *Q, J* and *K*.

3 J-K F^3s Based on Various Fuzzy Connectives

The algebraic, Yager, Dombi and Hamacher t-norms, combined with the standard negation, was analyzed to investigate, weather and to what degree they present more or less sigmoidal (S-shaped) *J-Q(t+1)* characteristics in particular cases, when *K=1-Q*, *K=1-J*, with fixed value of *Q*. Algebraic t-norm presents non-sigmoidal behavior, with piecewise linear characteristics and several breakpoints, but having the advantage of the hardware implementation of F^3. Circuits based on algebraic norms are presented earlier in [9]. The implementation was done by using fuzzy gate circuits.

3.1 Fuzzy J-K Flip-Flops Based on Some Classes of t-Norms

Using the algebraic norms and the standard negation

$$i_A(a,b) = ab \tag{2}$$

$$u_A(a,b) = a + b - ab \tag{3}$$

$$c(a) = 1 - a \tag{4}$$

The fundamental equation of the algebraic type fuzzy flip-flop [9] can be rewritten in the form

$$Q(t+1) = J + Q - JQ - KQ \tag{5}$$

Yager, in [11], proposed an infinite family of possible fuzzy operation pairs. The intersection of two fuzzy sets *a* and *b* applying the Yager t-norm has the expression

$$i_w(a,b) = 1 - \min\left[1, ((1-a)^w + (1-b)^w)^{1/w}\right] \quad \text{for } a,b \in [0,1] \tag{6}$$

where values of parameter w lie within the open interval $(0,\infty)$. By the way for w = 1 it gives the Łukasiewicz t-norm. For simplicity we use the following denotation $i_w(a,b) = a \ i_w \ b$.

The dual expression of t-conorm is defined by

$$u_w(a,b) = \min\left[1,(a^w + b^w)^{1/w}\right] \tag{7}$$

for w as before. Similarly to (7) $u_w(a,b) = a \ u_w \ b$.

Using such as triplet, the maxterm form in the unified equation (1) can be rewritten as

$$Q(t+1) = \left(J \ u_w \left(1-K\right)\right) \ i_w \ \left(J \ u_w \ Q\right) \ i_w \ \left(\left(1-K\right) \ u_w \left(1-Q\right)\right) \tag{8}$$

Several values of parameter w in the Yager-norm were considered, in an effort to tune the *J-Q(t+1)* characteristics of the corresponding F^3.

The pair of Dombi-class operators (similarly, a De-Morgan triplet) are defined as follows [6]:

$$i_\alpha(a,b) = a \ i_\alpha \ b = \frac{1}{1+\left[\left(1/a-1\right)^\alpha + \left(1/b-1\right)^\alpha\right]^{1/\alpha}} \quad \text{and}$$

$$u_\alpha(a,b) = a \ u_\alpha \ b = \frac{1}{1+\left[\left(1/a-1\right)^{-\alpha} + \left(1/b-1\right)^{-\alpha}\right]^{-1/\alpha}} \tag{9}$$

The unified equation of the next state can be expressed as

$$Q(t+1) = \left(J \ u_\alpha \left(1-K\right)\right) \ i_\alpha \left(J \ u_\alpha \ Q\right) \ i_\alpha \ \left(\left(1-K\right) \ u_\alpha \left(1-Q\right)\right) \tag{10}$$

Parameter α lies within the open interval $(0,\infty)$. If J = 0, K = 0 or Q = 0 (9) results in division by 0 the expressions are extended to their respective limit values. Both the Yager and the Dombi operators are classic (monotonic, commutative, associative and limit preserving) t-norms and co-norms.

Hamacher t-norms are the following [6] and the unified equation (1) can be rewritten as

$$i_H(a,b) = \frac{ab}{v+(1-v)(a+b-ab)}, \text{ and} \qquad u_H(a,b) = \frac{a+b-(2-v)ab}{1-(1-v)ab} \tag{11}$$

for $v \in (0,\infty)$

$$Q(t+1) = \left(J \ u_H \left(1-K\right)\right) \ i_H \left(J \ u_H \ Q\right) \ i_H \ \left(\left(1-K\right) \ u_H \left(1-Q\right)\right) \tag{12}$$

3.2 Fuzzy J-K flip-flop, *K=1-Q* (fuzzy J-K flip-flop with feedback)

The next figures depict the behavior by the graphs belonging to the next states of different type fuzzy J-K flip-flops for various typical values of *Q, J* and *K,* in the particular case, when *K=1-Q*. Figure 1 bring example for non-signoidal F^3s (algebraic types). Using the parameterized families of Yager, Dombi, Hamacher norms for typical parameter values, we obtained more or less S-shaped *J-Q(t+1)* characteristics. The sections of the 3D surface are approximately sigmoidal as it is shown in Figures 2-4.

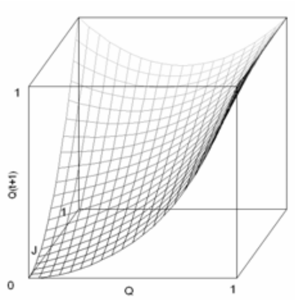

Figure 1
Algebraic type J-K F^3

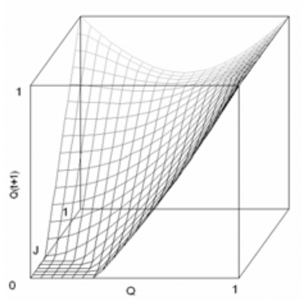

Figure 2
Yager type J-K F^3 w=2

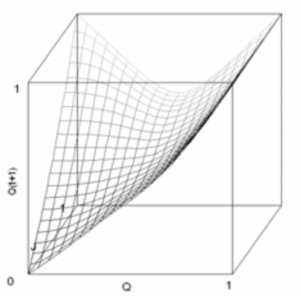

Figure 3
Dombi type J-K F^3 α=2

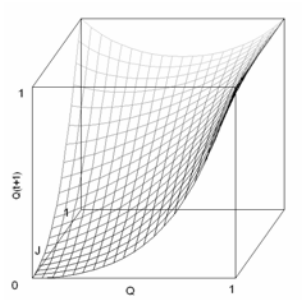

Figure 4
Hamacher type J-K F^3 ν=10

3.3 Fuzzy J-K flip-flop, *K=1-J* (fuzzy D flip-flop)

Connecting the inputs of the fuzzy J-K flip-flop in a particular way, namely, by applying an inverter in the connection of the input J to K, case of *K=1-J*, a fuzzy D flip-flop is obtained. Substituting $\neg K = J$ in equation (1) and let *D=J,* the fundamental equation of fuzzy D flip-flop will be

$$Q(t+1) = (D \vee D) \wedge (D \vee Q) \wedge (D \vee \neg Q) \tag{13}$$

Figures 5-8 show the behavior of the fuzzy D flip-flop introduced above, substituting to equation (13) the algebraic, Yager, Dombi and Hamacher norms. For a well selected parameter (i.e. w=2, α=2, ν=10) and Q values, the *J-Q(t+1)* characteristics present nice quasi sigmoidal behavior. As an alternative approach, Choi and Tipnis [1] proposed an equation which exhibits the characteristics of a fuzzy D flip-flop, as follows

$$Q(t+1) = (D) \wedge (D \vee Q) \wedge (\neg Q \vee D) \tag{14}$$

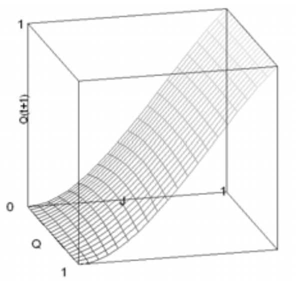

Figure 5
Algebraic type D F^3

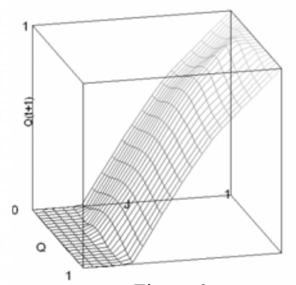

Figure 6
Yager type D F^3

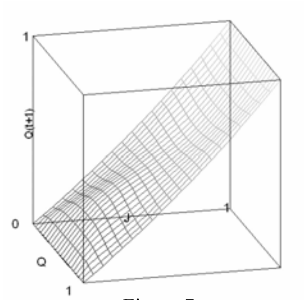

Figure 7
Dombi type D F^3

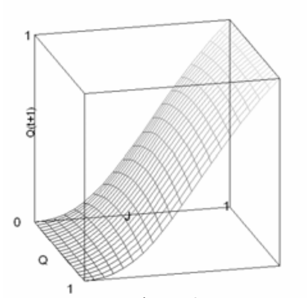

Figure 8
Hamacher type D F^3

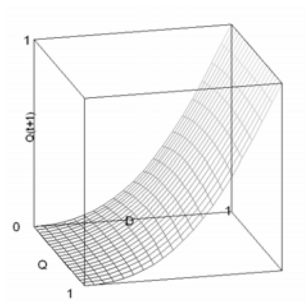

Figure 9
Hamacher type Choi D F^3

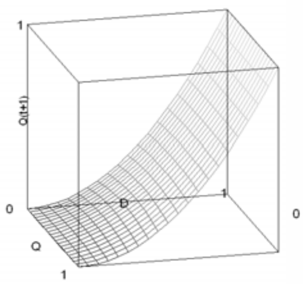

Figure 10
Algebraic type Choi D F^3

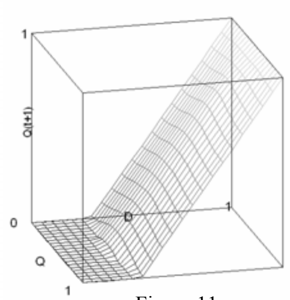

Figure 11
Yager type Choi D F^3

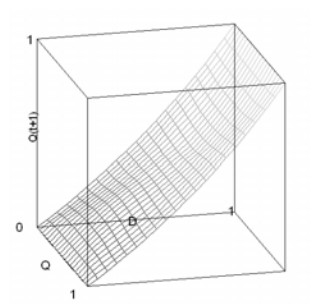

Figure 12
Dombi type Choi D F^3

We will refer to this new type of fuzzy D flip-flop as Choi type fuzzy D flip-flop (because of the first author B. Choi). Comparing the characteristical equation of the fuzzy D flip-flop (13), with expression (14), there is an essential difference between the two fuzzy flip-flops. Substituting *D=J=1-K*, the two formulas differ in the first member. $D = D \vee D$ holds only in the exceptional case, when the t-conorm is idempotent. Idempotence for T and S means that [2]

$T(x,x) = x$ and $S(x,x) = x$ for all $x \in [0,1]$;

It can be proved [3] that t-norm T is idempotent iff T = min, and t-conorm S is idempotent iff S = max. For example, using the algebraic norm

$$u_A(a,a) = a + a - a \cdot a = 2 \cdot a - a^2 = a \tag{15}$$

is true only in the borderline cases, i.e. when $a = 0$, or $a = 1$. It is surprising how much the satisfaction of idempotence influences the behavior of the fuzzy D flip-flops. Although, the *J-Q(t+1)* Choi fuzzy D flip-flop characteristics for Hamacher, algebraic, Yager and Dombi norms (Figures 9-12) also present approximately sigmoidal behavior. Comparing Figures 5-8 and 9-12 belonging to the two types of fuzzy D flip-flop with the same norms, it can be seen that, for the same value of Q, the curvature differs, which fact leads to a rather different behavior in the applications.

4 The Fuzzy Flip-Flop-based Neurons

Next, a fuzzy network is proposed, in which an artificial neural network-like approach is designed to construct the knowledge base of an expert system.

We study the effect of applying some well know t-norms in the investigation of the F^3 based neurons and the MLPs constructed from them. An interesting aspect of these F^3s is that they have a certain convergent behavior when their input J is excited repeatedly. This convergent behavior guarantees the learning property of the networks constructed this way.

In our approach the weighted input values are connected to input J of the fuzzy flip-flop based on a pair of fuzzy t-norm and t-conorm, having quasi-sigmoidal transfer characteristics. The output signal is then computed as the weighted sum of the input signals, transformed by the transfer function [4].

In this concept, *K=1-Q* (feedback J-K F^3), or *K=1-J* (D F^3) is proposed. When input K of the F^3 is connected with output $\overline{Q}$, or when input K is connected with J, an elementary fuzzy sequential unit with just one input is obtained. Now J can be considered as an equivalent of the traditional input of the neuron. The behavior of Choi type fuzzy D flip-flop was also evaluated for comparison.

From the neural networks perspective (regarding to the ability to use the learning and adaptation mechanisms used with classic neuron models), suitable t-norms may be deployable for defining fuzzy neurons.

4.1 Fuzzy Flip-Flop Network

A very commonly used architecture of neural network is the multilayer feed forward network, which allows signals to flow from the input units to the output units, in a forward direction. In general, two trainable layer networks with sigmoid transfer functions in the hidden layer and linear transfer functions in the output layer are universal approximators [5].

The model for the neural system now proposed is based on two hidden layers constituted from fuzzy flip-flop neurons. Networks now proposed are sensitive to the number of neurons in their hidden layers. Too few neurons can lead to underfitting, too many neurons can cause similarly undesired overfitting. The functions to be approximated are represented by a set of input/output pairs. All the input and output signals are distributed in the unit interval. During network training, the weights and thresholds are first initialized to small, random values.

5 Function Approximation by Multilayer Networks

5.1 Single Sine Wave (Various Norms)

A fuzzy flip-flop based neural network, with a transfer function using algebraic, Yager, Dombi and Hamacher operators in the hidden layers furthermore a linear transfer function in the output layer, was used to approximate a single period of the sine wave. The number of neurons was chosen after experimenting with different size hidden layers. Smaller neuron numbers in the hidden layer result in worse approximation properties, while increasing the neuron number results in better performance, but longer simulation time. The training was performed for different size hidden layers and finally a 1-4-4-1 FNN was proposed as good and fast enough.

Different random initial weights were used and the network was trained with Levenberg-Marquardt algorithm with 100 maximum numbers of epochs as more or less sufficient.

In our present experiments we forced Q = 0.32, because this value ensured rather good learning abilities. We suppose however that flexible Q values might lead to even better learning and approximation properties in the future.

The expression of the function to be approximated was:

$$y = \sin(c_1 * x)/2+0.5, \qquad (16)$$

where the input vector x generated a sinusoidal output y. The value of constant c_1 was chosen 0.07, to keep the wavelet in the unit interval. The parameter of Dombi, Yager and Hamacher operators were fixed α=2, w=2 and ν=10, which values provided good learning and convergent properties. Figure 13 presents the graphs of the simulations in case of fuzzy J-K flip-flop with feedback based neural network. It can be observed that the algebraic F^3 provides a fuzzy neuron with rather bad learning ability. Figures 15 and 17 compare the behavior of fuzzy D flip-flop and Choi type fuzzy D flip-flop based NNs. Table 1 summarizes the 100 runs average approximation goodness, by indicating the Mean Squared Error (MSE) of the training values for each of the ideal *tansig*, algebraic, Yager, Dombi and Hamacher types of FNNs. Comparing the minimum and median (median value of the array) values, the Yager and Dombi types FNNs performed best using fuzzy J-K flip-flop with feedback, thus they can be considered as rather good function approximators. It is interesting that according to the numerical illustrations the average of 100 runs mean squared errors in case of fuzzy D flip-flop and Choi type fuzzy D flip-flop type NNs, the best results after the idealistic *tansig* function is given by the Hamacher and Yager F^3, which is followed by the Dombi and finally the algebraic one. The Hamacher and Yager types FNNs have excellent approximation properties. It is surprising how much the satisfaction of idempotence influences the behavior of the fuzzy D flip-flop based NN. Comparing Figures 15 and 17 belonging to the two types of fuzzy D flip-flop with the same norms, it can be seen that, for the same value of Q, the value of the MSE differs, which fact leads to a rather different behavior in the applications.

5.2 Two Superimposed Sine Waves with Different Period Lengths (Various Norms)

When instead of a single sine wave a more complex wave form was used, in order to obtain the same results we increased the neuron numbers in the hidden layers to 8 neurons in each. We proposed a 1-8-8-1 F^3 based neural network to approximate a combination of two sine wave forms with different period lengths described with the equation

$$y = \sin(c_1 * x) * \sin(c_2 * x)/2+0.5. \qquad (17)$$

The values of constants c_1 and c_2 were selected to produce a frequency proportion of the two components 1:0.35. Same as in subsection 5.1 we compared the network function approximation capability in the above mentioned cases as is shown in Figures 14, 16 and 18.

It is interesting that according to the numerical illustrations, the average of 100 runs mean squared error of training and validation values (Table 2), the sequence is again the same as it was in the case of the single sine wave.

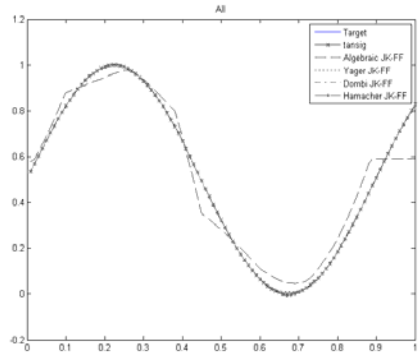

Figure 13
Simulation result fuzzy J-K FNN

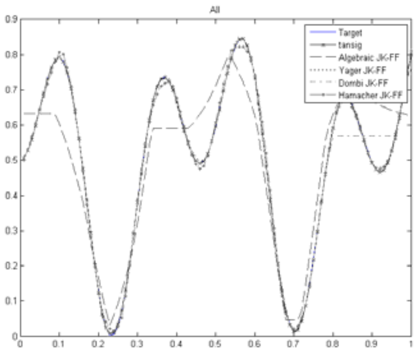

Figure 14
Simulation result fuzzy J-K FNN

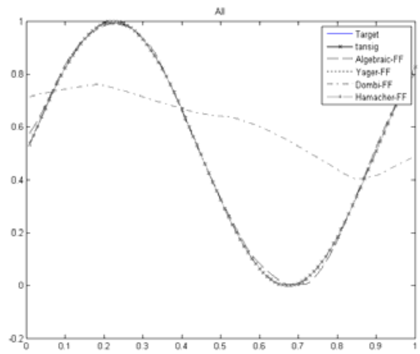

Figure 15
Simulation result fuzzy D FNN

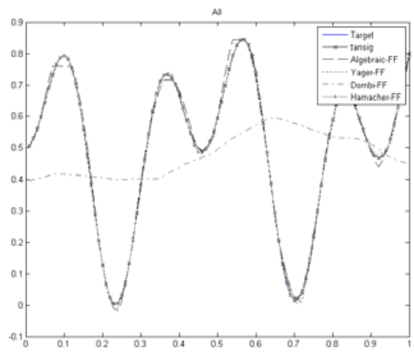

Figure 16
Simulation result fuzzy D FNN

TABLE 1
SINGLE SINE WAVE

F^3 Neuron Type	JK-FF		D-FF		CHOID-FF	
	Minimum	Median	Minimum	Median	Minimum	Median
tansig	$2.72x10^{-11}$	$2.54x10^{-8}$	$6.64x10^{-14}$	$3.31x10^{-8}$	$2.63x10^{-11}$	$4.29x10^{-8}$
Algebraic	$3.38x10^{-3}$	$6.38x10^{-2}$	$1.17x10^{-4}$	$1.30x10^{-2}$	$7.63x10^{-3}$	$5.92x10^{-2}$
Yager	$8.43x10^{-7}$	$3.57x10^{-2}$	$2.15x10^{-6}$	$6.51x10^{-3}$	$7.61x10^{-5}$	$2.59x10^{-2}$
Dombi	$3.53x10^{-8}$	$4.89x10^{-2}$	$7.20x10^{-2}$	$2.17x10^{-1}$	$9.30x10^{-2}$	$1.90x10^{-1}$
Hamacher	$7.64x10^{-6}$	$1.21x10^{-2}$	$2.02x10^{-7}$	$9.31x10^{-3}$	$1.79x10^{-4}$	$3.42x10^{-2}$

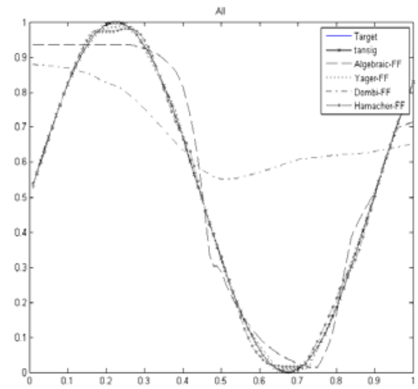

Figure 17
Simulation result fuzzy ChoiD FNN

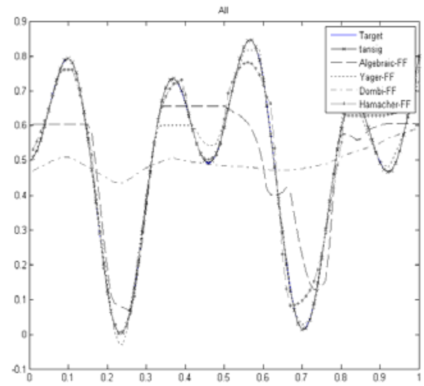

Figure 18
Simulation result fuzzy ChoiD FNN

TABLE 2
TWO SINE WAVES

F^3 Neuron Type	JK-FF		D-FF		CHOID-FF	
	Minimum	Median	Minimum	Median	Minimum	Median
tansig	2.96×10^{-8}	1.61×10^{-6}	3.13×10^{-8}	9.05×10^{-7}	1.88×10^{-8}	1.53×10^{-6}
Algebraic	2.17×10^{-2}	4.73×10^{-2}	1.56×10^{-4}	2.01×10^{-2}	1.50×10^{-2}	4.52×10^{-2}
Yager	8.04×10^{-6}	2.12×10^{-2}	2.78×10^{-6}	4.37×10^{-3}	2.51×10^{-4}	1.27×10^{-2}
Dombi	9.13×10^{-5}	3.19×10^{-2}	4.88×10^{-2}	1.50×10^{-1}	5.12×10^{-2}	1.40×10^{-1}
Hamacher	3.50×10^{-5}	2.25×10^{-2}	3.56×10^{-6}	3.20×10^{-3}	9.67×10^{-4}	2.62×10^{-2}

Conclusions

In this paper, we proposed the use of fuzzy flip-flop based neural network (FNN) for performing function approximation based on a combination of test functions. We compared the performance three different types of FNNs. Obviously, the performance of FNNs depends from the choice of different fuzzy flip-flop types. The results were promising in the sens that the proposed fuzzy D flip-flop based NN using Yager and Hamacher norms was found to be superior to the other approaches in approximating of test functions. In the future we plan to do simulations with a wide range of different functions and patterns to confirm our hypothesis. It may be worth while comparing a multitude of Yager, Hamacher and Dombi type F^3s when parameters are assuming their whole range.

Acknowledgement

Research supported by the National Scientific Research Fund grant # T048832, further by Széchenyi István University Main Research Direction Grant and Budapest Tech grants.

References

[1] B. Choi, K. Tipnis, "New Components for Building Fuzzy Logic Circuits", Proc. of the 4th Int. Conf. on Fuzzy Systems and Knowledge Discovery, Vol. 2, pp. 586-590, 2007

[2] E. Czogała, J. Leski, "Fuzzy and Neuro-fuzzy Intelligent Systems", Physica-Verlag, Springer Verlag, 2000

[3] J. C. Fodor, M. Rubens, "Fuzzy Preference Modelling and Multicriteria Decision Support", Kluwer Academic Pub., 1994

[4] M. Hagan, H. Demuth, "Neural Networks for Control", Invited Tutorial, American Control Conference, San Diego, pp. 1642-1656, 1999

[5] K. M. Hornik, M. Stinchcombe, H. White, "Multilayer Feedfordward Nerworks are Universal Approximators", Neural Networks, Vol. 2, No. 5, pp. 359-366, 1989

[6] G. J. Klir, B. Yuan, "Fuzzy Sets and Fuzzy Logic: Theory and Applications", Prentice Hall, Upper Saddle River, NJ, pp. 50-88, 1995

[7] R. Lovassy, L. T. Kóczy, "Comparison of Elementary Fuzzy Sequential Digital Units Based on Various Popular T-norms and Co-norms", Proc. 3rd Romanian-Hungarian Joint Symposium on Applied Computational Intelligence, Timişoara, 2006, pp. 164-181

[8] R. Lovassy, L. T. Kóczy, L. Gál, "Multilayer Perceptron Implemented by Fuzzy Flip-Flops", IEEE World Congress on Computational Intelligence, WCCI 2008, Hong Kong, pp. 1683-1688

[9] K. Ozawa, K. Hirota, L. T. Kóczy, K. Omori, "Algebraic Fuzzy Flip-Flop Circuits", Fuzzy Sets and Systems 39/2, North Holland, 1991, pp. 215-226

[10] K. Ozawa, K. Hirota, L. T. Kóczy, "Fuzzy Flip-Flop", In: M. J. Patyra, D. M. Mlynek, eds., Fuzzy Logic. Implementation and Applications, Wiley, Chichester, 1996, pp. 197-236

[11] R. R. Yager, "On a General Class of Fuzzy Connectives", Fuzzy Sets and Systems, Vol. 4, 1980, pp. 235-242