Computational Intelligence assisted Theory Building in Economics

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Abstract: In the area of contemporary complex economic systems there arises several difficulties in theory building because of high level requirements for mathematical knowledge's and skill for using mathematical tools. In this situation the scholars can addressee great expectation to the product of computational intelligence. Fortunately the devices and tools usable for this purpose isn't put very high requirements to potential computational intelligence product propose. It is interesting and surprising yet, that also relatively simple devices and tools can effectively serve for problem solving in meant area of science. On the other hand the realisation of more advanced theory building is putting extraordinary requirements on tools prepared by computational intelligence. Our purpose in this paper is not so ambitious. We focus our attention to more simple tasks, and so we will show only a few examples demonstrating this conclusion for early stages of theory building. But such preliminary projects are necessary for more advanced theory building too. Among others there are such subjects those in common economic textbook are prearranged as easy for understanding, but in reality the opposite is true. One from the set of such subjects is broadly familiar problem of monopoly. Intimately spoken new ICT products and services coupling with them entering to market perturbed the former structural stability of that one. So in one hand the manufacturer is immediately becoming real monopolist searching for price and product amount on the market, i.e., he must challenge against uncertainty of demand curve (he must looking for shape and location of that one). On the other hand, the demand side, that is, consumers must react to new situation not only in the market but they are forced them to little by little changing their former economic, social, cultural etc. behaviour. So the new product entering on market is at least two valued entity perturbed the structural stability of market from both sides. But the principal impact is on whole global knowledge society progression. The complex behaviour of that one masterly helps to solve overtly progression in computational intelligence.

Keywords: complex economic systems, economic dynamics, economic model building, new product monopoly, perturbation, qualitative economics, structural stability, topological equivalence

Introduction

Theory building (TB) in economics so as in other social sciences is first of all a process of creativity and imagination realised by authentic scholar. The result of such activities of human mind is becomes of some germs from one side for perturbation impacts to former theory if such exists, and from other side for creation of very new theory if such one nil. So the theory building is actually the composed activity of human mind united destruction and creation. This integration of two polar opposite activities we are using for founding our conclusion that theory building has a nature of creative perturbation arising in former structural stability of economic theory, that is bit by bit vanishing the former structural stability and equivalently, that is gradual creating of, and/or step by step moving to, new structural stability. Using topological approach the former theory after upper descript activities is forced to such change that the successive form of theory looks as if former one lost his topological equivalence. On the other sides of view, because the theory building consist several further activities the job is more complex than it looks in first site. Actually, among other distinctive features, the theory is some kind of story. So the theory building is also special story telling which is parallel with other activities in this process. This complex activity is also on learning and understanding by scrawling and/or featuring and design models and algorithms assisted by CI. The novelties, in decrypted complex activities as theory building is, connected with computational intelligence (CI) assistance are lying in its potential character that may be very intelligent, ready and efficient collaborator in qualitative economic analysis and in theory building. The theory builder may acting in perpetual very vital dialog with virtual co-worker and interposes it as interface into gap between own mind and the objective economic reality. We are put emphasis on our belief that this connection between authentic human intelligence (which is some function upon brain) and authentic computational intelligence (which is some function upon computer) isn't the same as connection brain-computer. Apart from that we have to say that so human intelligence as computational intelligence are disposing with imputed intelligence of third subjects products and using them not only in own but naturally also in mutual cooperative cognition process. On the side of virtual partner of theory builder such third persons are CI professionals.

Actually, the live interaction processes among three entities displayed in Fig. 1 is much more complex, but that is another story than one we interesting in this paper. For better understanding our own problem it is suitable to bring some familiar economic examples, because our focus is on economic theory building. Before it is fitting to note that for scholar in meant process there is much to be gained from questioning the assumptions of a former well established theory. Those gains may become the starting point for preliminary destruction of former theory. For example we use known fact that traditional economic theory (in majority named as neoclassical economy) is based on few key assumptions such as perfect

information of agents, stable equilibrium (and/or proclivity to homeostasis) of the firms and of whole economy, perfect competitions, etc. So the theory builder can successfully benefit greatly from relaxing these assumptions. The strong destructing perturbation in former theory, for example is releasing the out of this world assumption on stable equilibrium not only in economic systems (that is in constructions of mind), but first of all in objective economic reality. Because the reverse is right: the real economy being in motions far from equilibrium and only time to time is approaching the near neighbour loci, or area close to equilibrium: real economy demonstrates some pattern of discrete form of cyclical behaviour. Not at all, some of cyclical behaviour shows complex crowd modes of behaviour. But this change from equilibrium to non-equilibrium assumptions isn't fallen from heaven. It is result of former theoretical investigation of that matter.

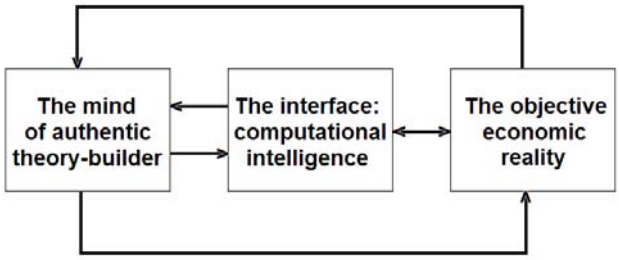

Figure 1
The simple schema of location and function of computational intelligence

It must be draw attention to the fact that theory building in economics also requires careful reflection on the importance and uniqueness of the phenomenon at hand, the questions explored, and the context of the research. Because theories is ought to serve as signposts that tell us what is important, why it is important, what determines this importance, and what outcomes should be expected. We are familiar also with assertion that theories in several cases is guide the subjects looking for information and knowledge through what was found and why it enriches or even challenges his/her understanding. Theoretically grounded studies pay particular attention to the context of their research and account for its complexity, uniqueness and richness. Such requirements are also very natural screened-off area of theory building. All of said before is several times more confused in the condition of evolving global knowledge society. On the other hand novelties arriving in the same content of that one not surprisingly becoming strong tools helping scholars in solving emerging problems connected with theory building upon new evolving phenomena. Namely, it is to be said that in economics, i. e. in early qualitative analysis and quick to fasten on their result theory building, the major role may play, and/or is really played by computational intelligence at present. Naturally it is clear today, that after first mass produced IBM PC entered to the life of scientist and/or researchers the ICT devices begin to play ceaseless increasing role in scientific activities and in theory building too.

These are the reasons that in narrower community such processes get the name computer-assisted theory building. But in this name is some way and in contents too misunderstanding. From the one hand it is true, that for researcher the PC and/or notebook are their every day assistant at present. But on the other hand greater truth is that real assistant is born on the base created by computational intelligence. This is the reason why we preferred the term computational intelligence assisted theory building. By the way, it is clear that the difference for example between every day using of PC and in opposite using PC and/or in excellence cases also multi-parallel platforms for building virtual partners for dialog like complex virtual laboratories are is colossal.

In the area of contemporary complex economic systems there arises several difficulties in theory building because of high level requirements for mathematical knowledge's and skill for using mathematical tools. In this situation the scholars can addressee great expectation to product of computational intelligence. Fortunately the devices and tools usable for this purpose isn't put very high requirements to potential computational intelligence product propose in preliminary early steps in the process of theory building. It is interesting and surprising yet, that also relatively simple devices and tools can effectively serve for problem solving in meant area of science. On the other hand the realisation of more advanced theory building is putting extraordinary requirements on tools prepared by computational intelligence. Our purpose in this paper is not so ambitious. We are focusing our attention to more simple tasks, and so we will show only a few examples demonstrating this conclusion for early stages of theory building. Among others there are such subjects those in common economic textbook are prearranged as easy for understanding, but in reality the opposite is true. One from the set of such subjects is broadly familiar problem of monopoly. Intimately spoken new ICT products and services coupling with them entering to market perturbed the former structural stability of that one. So in one hand the manufacturer is immediately becoming real monopolist searching for price and product amount on the market, i.e., he must challenge against uncertainty of demand curve (he must looking for shape and location of that one). On the other hand, the demand side, that is, consumers must react to new situation not only in the market but they are forced them to step by step changing their former economic, social, cultural etc. behaviour, shortly, he must change his former life style. So the new product entering on market is at least two valued entity perturbed the structural stability of market from both sides. But the principal impact is on whole global knowledge society progression. The complex behaviour of that one masterly helps to solve overtly progression in computational intelligence. Similar economic market issues are the problems of duopoly and oligopoly, and on the opposite side of market there is the entity of market type: perfect competition. For the purpose of this paper the obscurity of cyclical economic growth is interested too. But because of lack of place we can't take it to this paper.

1 Popular Economic Theories Innovated (Rebuilt and Rearranged) by the Aid of Simple CI Products

For easy ingoing to the area under discussion of this paper we begin with rearranging well-known, but a little more complex economic theories as are frequent with assistance of simple software available for wide community of economist's. Among other the offer in this field is wide enough, for example products of distributed CI, such as products in MAS's, ANN's, ES's, EP's, GA', Petri net's, and others and/or their mixtures, but we are using probably the best way to do it for the purpose of this paper by devices, iDmc, STELLA and partially by Excel. In this way we wont to avoid great palette of problems attached with modelling and simulation of social system, because with the alive (perpetually living) economy we may to act and work as with complex social system. In short, to act with complex social system, modelling it, and simulating with them is difficult. As interested scholars known, it is difficult, basically, for

a) Two creator mental reasons: syntactic and semantic complexity, and

b) Two pragmatic reasons: the need of managing clusters of interconnected models and necessitates constraining model possibilities. So we are purposefully resigned to this job in this paper.

For, it is not need to emphasize that theory building is a process of deep creativity and imagination. It demands careful reflection on the importance and uniqueness of the phenomenon at hand, the questions explored, and the context of the research. Theories serve as signposts that tell us what is important, why it is important, what determines this importance, and what outcomes should be expected. Theories also guide the reader through what was found and why it enriches or even challenges our understanding. Theoretically grounded studies pay particular attention to the context of their research and account for its complexity, uniqueness and richness. These studies also offer compelling arguments, provide a fair test of these arguments, and use findings to refine and enrich the theory they have invoked.

1.1 Economic Theories: iDmc and Excel Assisted Building the Cases of Modified Monopoly Model

Conventional economic theory is arranged so, that deals only with two basic market form: perfect competitive market and market of unitary product, in other word monopoly. From ordinary textbooks of economics occasionally come up deception that monopolist is in advantageous position because he can deliberately choose the level of price and amount of his product for yielding uppermost profit. For realisation of this pleasant goal he must know at least the entirely demand curve established in the market. But this is very unrealistic assumption indeed in

very general monopolistic situation. Much harder is that problem with new product monopoly because the demand curve is beginning to emerge as far as new product entering to market. But the monopolist decision making must be done before this, with fairly long advance.

1.1.1 Model of Monopolist's Searching with Linear Demand Function

For first demonstration we are using subsequent simple task appropriate for description monopolist possible behaviour by entering future market with new ICT product. Supposing that monopolist just knows a few point on the linear demand function, recently visited in its more or less erratic search of maximum profit. It is however unrealistic assumption but it is convenient for demonstration the problems. Namely the model is based on subsequent formal statements:

Demand curve function is: $P_D = 5 - 2Q$. Revenue curve function is: $R = 5Q - 2Q^2$. Cost curve function is: $C = 2Q - Q^2 + 0.7Q^3$. Profit function is: $\Pi(Q) = 3Q - Q^2 - 0.7Q^3$, because in economics is: $\Pi(Q) = R - C = 5Q - 2Q^2 - 2Q + Q^2 - 0.7Q^3 = 3Q - Q^2 - 0.7Q^3$. And after derivation we are: $\Pi'(Q) = 3 - 2Q - 2.1Q^2$, and for simulation we shall dealing with quadratic equation: *3-2x-2.1x2=0,* which after solution given plot of quadratic parabola,

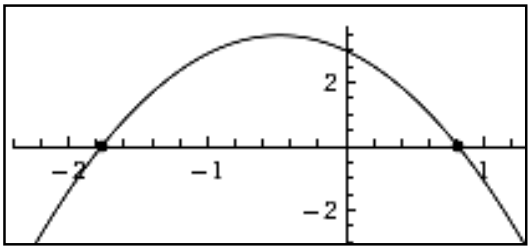

Figure 2
The quadratic parabola with two worked out roots

and their two roots are: x_1 = *-1.7627862939148853,* x_2= *0.8104053415339328.* Marginal revenue function is: $MR = 5 - 4Q$. Marginal cost function is: $MC = 2 - 2Q + 2.1Q^2$, (*x=y=0.7378364083397065*).

1.1.2 Virtual Laboratory in iDMC

For simple demonstration of differences between trajectory output from experiment and from basin of attraction one, we are building laboratory in "Trajectory" and in "Basin of attraction" routine of iDmc.

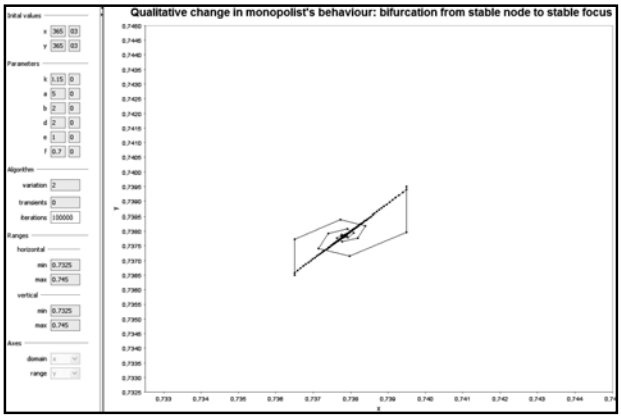

Figure 3
Bifurcation of the unstable fixed point from reaching stable node, to reaching stable focus because the longitude of searching step was raised

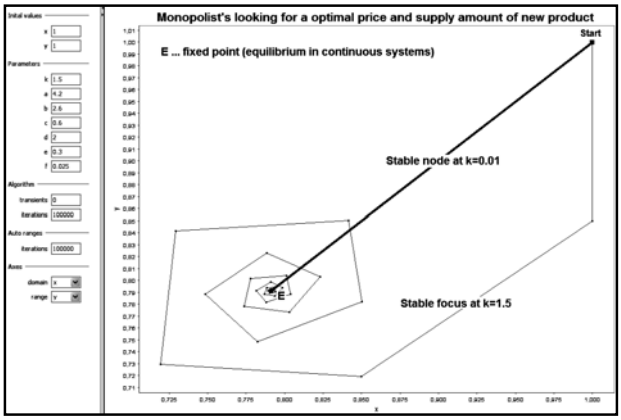

Figure 4
The trajectories tracking to stable equilibrium (to fixed point)

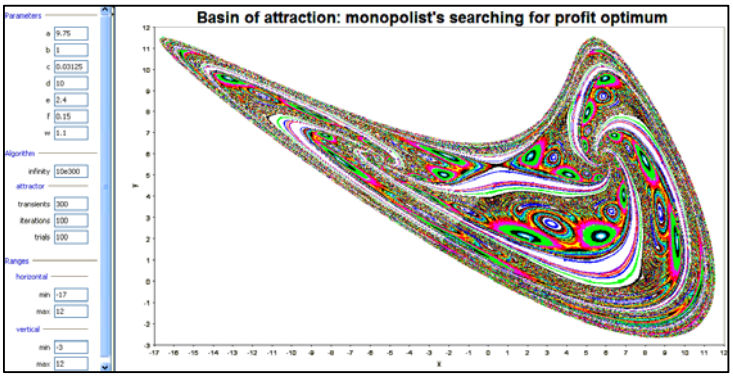

Figure 5
The sensitivity of the model to starting point values

1.2 Model with Cubic Demand Function Built on Ideas Proposed by T. Puu

1.2.1 Basics for Model of Monopolist's Searching with Cubic Demand Function

For the searching process T. Puu defined map in implicit form as

$$T:\begin{cases} x' = f(x,y) \\ y' = g(x,y) \end{cases}, \tag{1}$$

which he explicitly itemized as

$$\begin{matrix} f(x,y) = y, \\ a \\ g(x,y) = y + \sigma\left((a-e) - (b-f)(x+y) + (m-g)(x^2+2xy+y^2) - d(x^3+x^2y+xy^2+y^3)\right) \end{matrix} \tag{2}$$

and again in map form is look as

$$T:\begin{cases} x' = y \\ y' = y + \sigma\left((a-e) - (b-f)(x+y) + (m-g)(x^2+2xy+y^2) - d(x^3+x^2y+xy^2+y^3)\right) \end{cases} \tag{3}$$

The original Puu's design of searching algorithm for relations (1)-(3) is to estimate difference of marginal costs MC and marginal revenue MR (see curves in Fig. 6) from the two last known (numerical values) points of the profit function, and, as Puu articulates: "in the vein of Newton, to use a given step length to move in the direction of increasing profits". He denotes the next last and last ascertained values of profits by x and y respectively, and the step length by σ and he get the next value as

$$y + \sigma \frac{\Pi(y) - \Pi(x)}{y - x} \tag{4}$$

and for avoiding possible instability the denominator, he factored out the quotient

$$\frac{\Pi(y) - \Pi(x)}{y - x} = a - e) - (b - f)(x + y) + (m - g)(x^2 + 2xy + y^2) - d(x^3 + x^2 y + xy^2 + y^3) \tag{5}$$

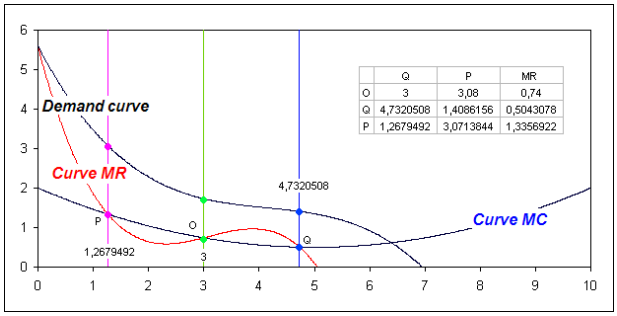

Figure 6

Three basic curves used for building laboratory for monopoly model with cubic demand

On the above ideas and relations we are building virtual laboratory for proposed model with cubic demand function of monopolist's searching.

1.2.2 Experimentation with Settings Appropriates for Demonstration the Task

For better understanding the characteristic features of complex dynamical system as that one is, we bring a few snapshots resulting from experiments realised in iDmc laboratory, only for visual imprint, without verbal comments. In our opinion however such snapshot can offer better and more material for imagination than long verbal text. By the way, it is also the great virtue of building theories assisted by CI. Namely such joint activities realised between authentic live human imagination and complex processes running in authentic virtual laboratory can percolate to the deeper stratum of brain structures.

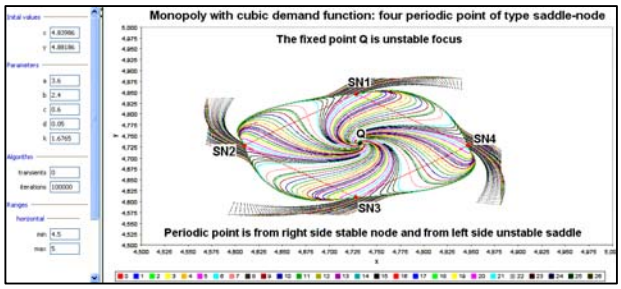

Figure 7
By trajectories visualised qualitative nature of four periodic points

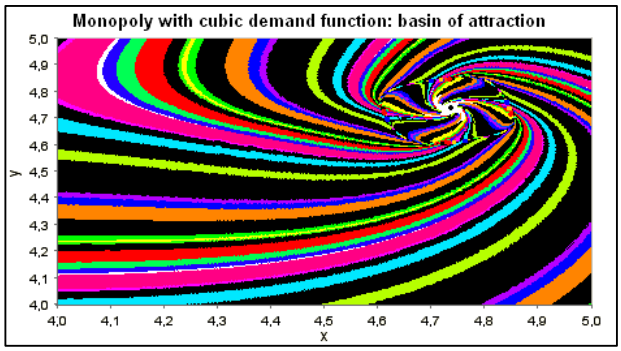

Figure 8
The same task as in Fig. 7 realised in routine “Basin of attraction”

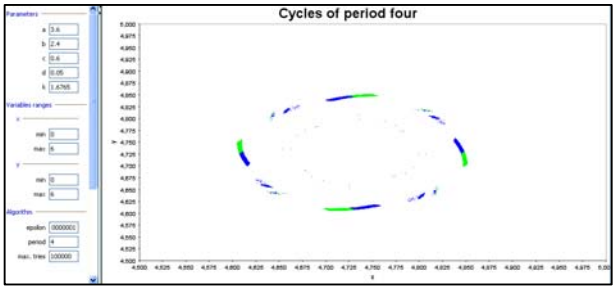

Figure 9
Experiment is realised in algorithm “Cycles”

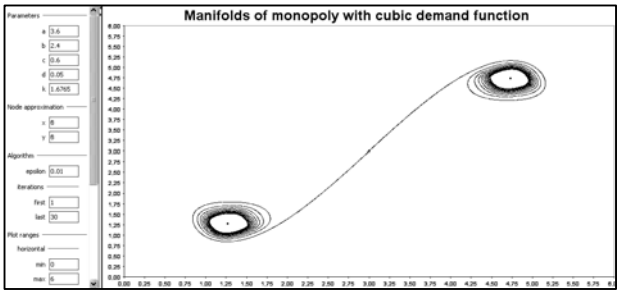

Figure 10
Right and left manifold and two unstable fixed point and unstable central fixed point

1.2.3 Simple Experiments in Excel Usable in Early Phase of Theory Building

With simple experiments realised in Excel built laboratory we can also reach some interesting looks on the on the complex dynamical system. A few snapshots show examples how we can use experiment result in theory building.

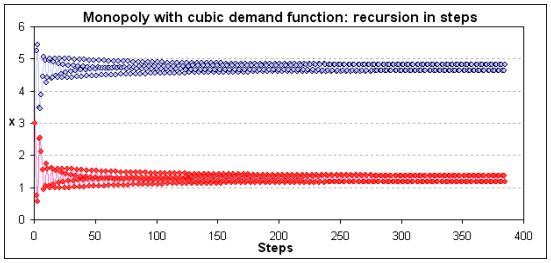

Figure 11
Experiment in Excel with same values as in iDMC

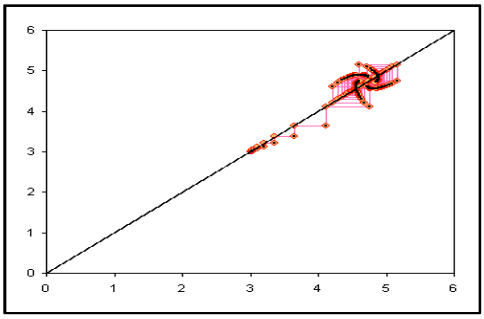

Figure 12
Cobweb animation in Excel: approaching four saddle-node periodic points on cycle

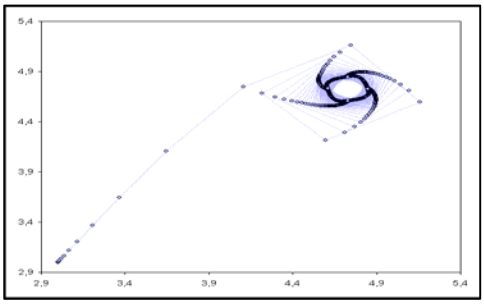

Figure 13
Evolution to saddle-node points via focus trajectory with four branches

1.3 Economic Theories: STELLA Assisted Building and Telling Story in Case of Modified Cournot Duopoly

The duopoly model of A. A. Cournot is other very suitable case for demonstration CI assisted economic theory building. More we can to say: it is perfectly fitting for opening phase of CI assisted building economic theory. By the way, from my opinion Cournot was the first mathematician and economist who applied mathematics to the field of economics, not necessarily to produce numerical precision in a predictive fashion, but rather to provide clearer formulation of economic relationships. That is the approach I called qualitative computational economics, because parallel with mathematics the computational intelligence have to dominate in economic theory nowadays.

1.3.1 Building Model and Virtual Laboratory

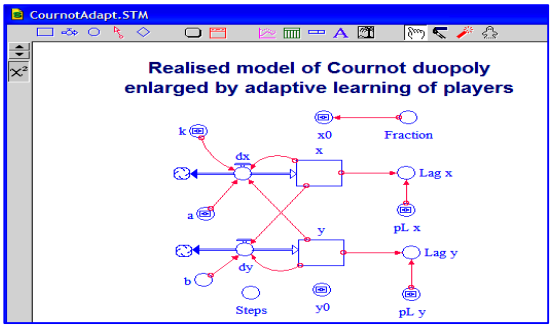

Figure 14
Basic scheme of Cournot duopoly enlarged model realised in map/model level of STELLA 8.1

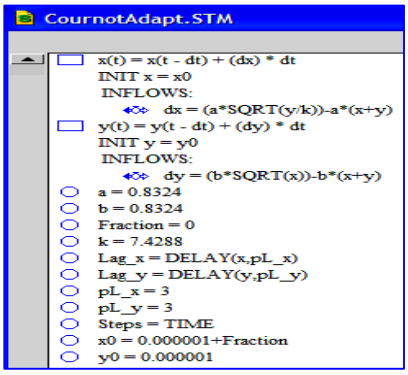

Figure 15

Principles of basic scheme (the fillings of building boxes)

1.3.2 Building Stories on Duopoly Theories

For building entire theory for communication in wider community of scholars it is important to convert tentative theory into for readable for others. In such activities CI can efficiently helps. One from simple approach to this task is to build story telling in programmed form by, for example STELLA software. Only for visual imprint we show two snapshots made in this devices, when the story run by hand pushing mode via spacebar.

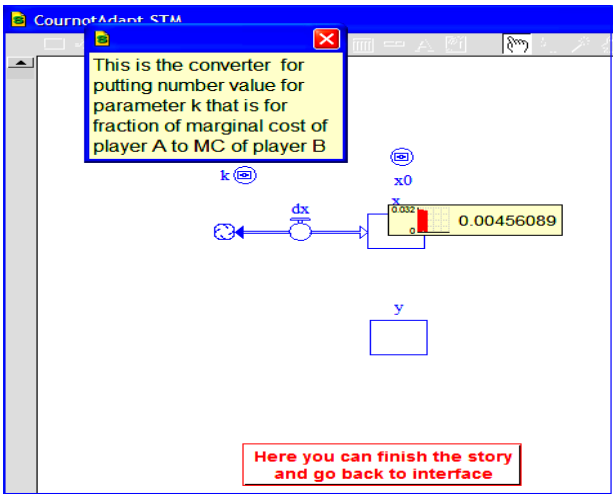

Figure 16

Realising story telling on model building (the progress of story watching is in early phase)

Somebody interested in the subject theory by working in story telling showed with us on snapshot from STELLA virtual laboratory (see Fig. 17) can gain more knowledge's and information's than if he is reading only the conventional form of theory, for example in paper book and/or in monographs. For example in virtual case he can parallel by reading verbal text and by gazing at static pictures and/or graphs, diagrams, etc. he can practising virtual laboratory experiments by his own ideas and purposes.

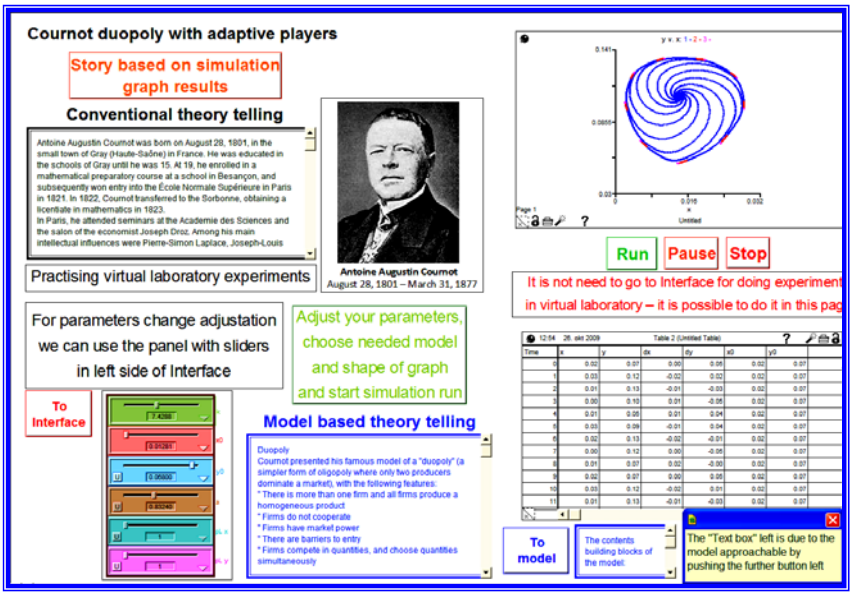

Figure 17
Realising story telling on contemporary duopoly theory (the progress of story watching is at half)

1.3.3 Preparatory Processes for Building Contemporary Duopoly Theory Assisted by Computational Intelligence (Using iDmc and STELLA)

In the same purpose as before we bring several result of experimentation in virtual laboratories built in STELLA and iDmc.

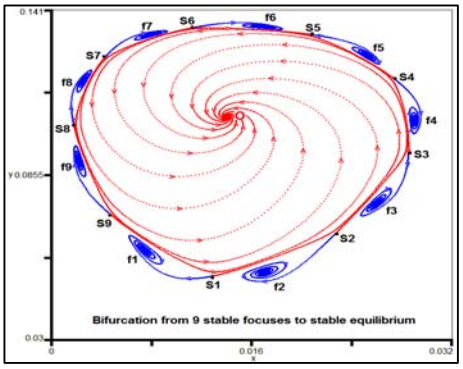

Figure 18
Demonstration of high sensibility of system on starting point of variables x_{on}, y_{on}

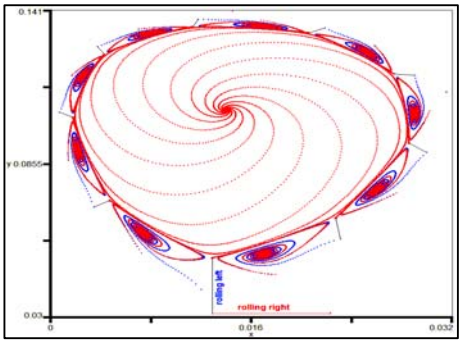

Figure 19
Exhibition of model high sensitivity to starting loci: the trajectories tracking to stable equilibrium (to fixed point) and to complex invariant closed curve **Γ** from three starting loci and to 9 stable focuses

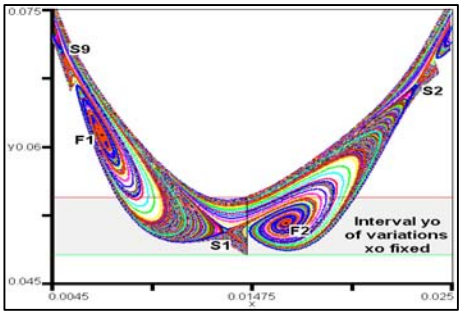

Figure 20
The trajectories tracking to stable equilibrium (to fixed point) and to complex invariant closed curve **Γ**

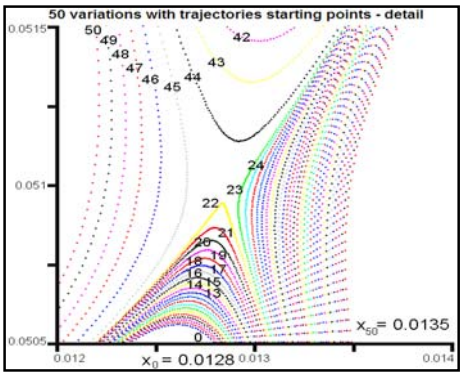

Figure 21
The visualisation of sensitivity of model to starting value of trajectory simulation

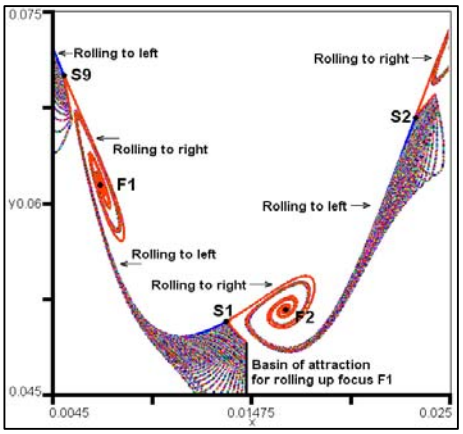

Figure 22
Visualisation of rolling direction change impacted by bifurcation

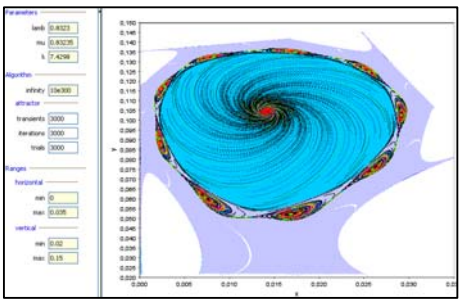

Figure 23
The snapshot of basin shows qualitative shapes depending from variation in starting points

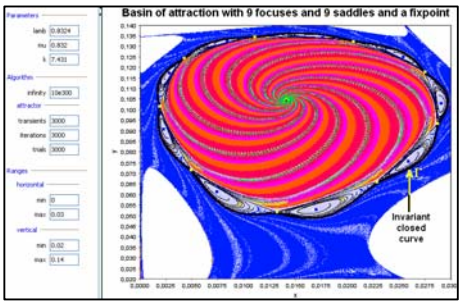

Figure 24
The change of qualitative shapes in basin caused by change of longevity of searching parameter

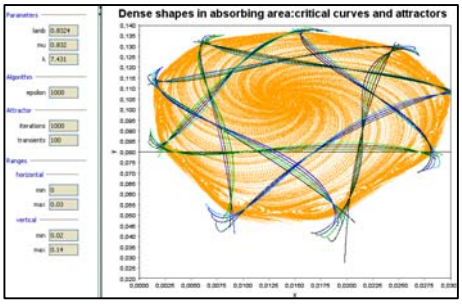

Figure 25
Exhibition the similar situation achieved in routine "Absorbing area": there are the attractor overlapped by several critical lines LC

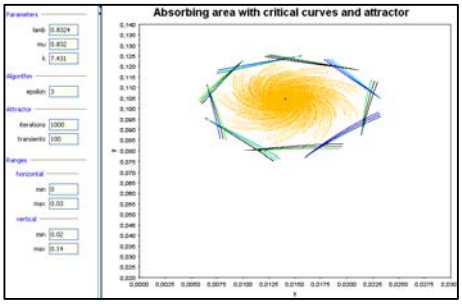

Figure 26
Other dense shapes in absorbing area

Conclusions

With progressing of knowledge based society and their economy there is organically emerging a wide cluster of new problems having to challenge by economic science in this time. Because of such processes are extraordinary complex for scholars investigating new emerging entities and having to build theories on achieved results coming up very difficult tasks. Complications in socio-economic evolution are emerging first of all as a result of wide frontal entering of products and services due to ICT and connected with them other High-Tech products and processing devices. On the other hand, fortunately, the same devices are effectively helping overcome several tasks born from those difficulties. Actually, from the sight of new theory building the most promising helps can the scholars expect from progress in computational intelligence. The author in this paper focuses attention to this area of using CI result and product for gaining more effective and effortlessness methods for building theories on complex economic phenomena.

In this paper the author understand and formulate the computational intelligence study of economies as complex dynamic discrete systems. That is he approach economic processes by constructively simulation them. This distinct him from reduction bearing approaches using in main stream economics analysing economic systems. With this qualitative distinction he, together with other scholars creates new ways for understanding of economics systems in the phase of global knowledge based society. The author is stresss in whole paper this qualitative character, because it is right that the dominant part of the complete economic theory is qualitative in his character. So he warns from misunderstanding this one in comparison with more pragmatically oriented econometrical studies in which CI helping as very strong performer.

With efforts realised in this paper the author would like to advocate for wider using of advanced products of computational intelligence for investigation in complex qualitative economic phenomena and for building virtual co-developers living in appropriate software useful to personas moving in environment of economic sciences. The author is access this character, because it is right that the dominant part of the complete economic theory is qualitative in his character. He also proposes the idea of integrative approaches aided by product of advanced CI in this paper. Namely because only integrative methods aided by sophisticated CI products is able completely deal with new, unprecedented process emerging in contemporary global knowledge society.

Emergence of very strange behaviour by executing experiments in the virtual laboratory simulation of dynamical behaviours of economic systems is good starting point for imagination upon complexities in real economic life. For example this approach among others shows interesting story that is connected with nature of subcritical Neimark-Sacker bifurcation, the case that is very important for understanding hidden complexities not only in natural systems but in

socio/economic evolution too. In such virtual experiments that is to say, the emergence and existence of a repelling invariant closed curve which bounds the basin of attraction of the stable fixed point implies for economic scholars fundamentally important disposition the very process. Those is lying in fact that small perturbations of the system have no effects on its dynamical behaviour, while large enough disturb may drift the system to another attractor. In this sense the system after small perturbation keeping on structurally stable state forever. By other word the system after such perturbation is keeping on topologically equivalent state and/or dynamical regime. In such situation upon the whole system the basic fixed point governs. We are exhibited above that those qualitative regimes realizing such motion is either stable node, and/or stable focus depending on the longevity size of probing step parameter. Based on such approaches shoved in upper part of paper it is possible coming to the end that situation, first of all for conventional (main stream) economist's buildings their theories, is not as optimistic as it like in first sight. Unfortunately, the size of disturbations, even sometimes there are in clusters, emerging in real economies are outlying from one that we may called "small" as we do above. In such situation our attractive fixed point losing his attractive force and delivering it to closed invariant curve. But descript situation is a little more complex. For example, if the shock is so large enough that may drift the system to another attractor the scholar have to reveal hidden coherences, such as requirements of the coexistence of the fixed point with a different attracting set are. So the deceptions which are nourishing in main stream economic about sustain economic equilibrium and about sinusoidal shape of business cycle are easy to reveal by proposed methods assisted by CI.

Coming to end, it is obvious that somebody from CI community can say that we are offered relatively very simple bundle of devices for demonstration cases in this paper. But we did so for ease entry to the process of using cheap devices available for theorizing purposes in economics at least for preliminary phase of investigation and theory building, that is, this paper is not directed to scholars in clean CI theory. On the other hand we are convinced that the magnification of new task arriving with evolving global knowledge based economy may be interesting also for somebody of meant scholars, from the point of view of their professional interests.

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