

# Flight Control System of an Experimental Unmanned Quad-Rotor Helicopter

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*Abstract: An autonomous robot with vertical take-off and landing (VTOL) capability could be useful for many applications including search and rescue, exploration in hazardous environments, monitoring, surveillance and investigation or even for intelligence. However design of a flight control system for a small UAV is a challenging task. Unlike in conventional aircrafts the control algorithm has to be implemented in a size limited processing environment and has to cope with smaller mechanical time constants and noisy inertial sensor data. This paper introduces the methodology used in the development of an experimental unmanned quad-rotor helicopter, explores the limits of the linear LQG controller used in the design and for improvement suggests the emerging SDRE nonlinear control design method.*

*Keywords: unmanned aerial vehicles, UAV, VTOL, flight-control system quad-rotor helicopter, embedded control, optimal control, LQR, LQG, state-dependent Riccati equation, SDRE*

## 1 Introduction

Being efficient information collecting gadgets UAVs are getting more and more attention not only in modern warfare but in our everyday life. The main reason is protection of human life. Lost of an unmanned machine shows up only on a damage report. Besides, the absence of onboard human presence permits operational condition and environment beyond the capabilities of piloted airplanes. As a result UAVs can operate even in circumstances endangering life such as high g maneuvers, toxic gases or radioactive environment etc. Another motivating factor is cost. Considering price, maintenance and training of pilots unmanned aerial vehicles are less expensive than piloted ones [1][2].

Aircrafts having vertical take-off and landing capability form a special group among UAVs. These – usually rotary-wing – aircrafts do not require large areas to become airborne therefore they can support mid range monitoring, surveillance, investigation or even intelligence and reconnaissance operations. Recent advances in sensing, data processing and control technology have made it possible to

develop small VTOL aerial robots able to operate in urban areas or even inside buildings. Along with cost and size reduction – that made these systems very attractive for many applications – challenging control and performance problems emerged. The miniaturized inertial sensors are less efficient than the conventional sensors because of noise and drift, and also the small actuators suffer from saturation. Therefore designing flight control system of mini or micro UAVs is still a challenging goal. This complex design task is demonstrated in this material via a control system of an experimental unmanned quad-rotor helicopter [3][4][5].

## 2 The Quad-Rotor UAV

Recently a number of research groups are again investigating the problem of developing a small four-rotor helicopter design. A single rotor helicopter is very dangerous in an indoor or obstacle bounded environment because of the potential for the exposed rotor blades to collide with something and cause the helicopter to crash. Even skilled pilots have trouble navigating them close to the outside of buildings. Four-rotor helicopters are attractive because the rotors are smaller and can be enclosed, making them safer. Also, it may be possible to achieve more stationary hovering with four thrust forces acting at a distance from the centre of gravity than with one force acting through the centre of gravity. A small electrical four-rotor helicopter owing to its mechanical simplicity can be a robust and reliable VTOL UAV construction.

To facilitate the job of the pilot, it is rewarding to automate some functions of the UAV. Manually stabilizing the flight of a four-rotor helicopter by means of controlling the speed of the four rotors is almost impossible. Accordingly the use of an onboard autopilot is necessary to utilize this construction in UAV applications. Besides, the autopilot can provide stable platform for onboard imaging sensors and opportunity to accomplish out of sight operations.

In order to design a stabilizing flight control system an adequate mathematic model has to be determined that mimic the real, physical behavior of the helicopter. This model is derived using equations of rigid body dynamics:

$$\begin{aligned}
 \mathbf{r}(t) &= \mathbf{v}(t) \\
 m\dot{\mathbf{v}}(t) &= \sum_j^k \mathbf{F}_j(t) \\
 \dot{\Phi}(t) &= \Phi(t)\tilde{\omega}_{(k)}(t) \\
 J_{(k)}\dot{\omega}_{(k)}(t) &= -\omega_{(k)}(t) \times J_{(k)}\omega_{(k)}(t) + \sum_k^z M_{k(k)}(t)
 \end{aligned} \tag{1}$$

As shown in Figure 1 Equation 1 describes the linear motion of a rigid body in inertial frame ( $K_0$ ) and the rotational motion in body frame ( $K$ ) where the vectors are  $r(t)$  – position,  $v(t)$  – velocity,  $\omega(t)$  – angular velocity,  $F(t)$  – force,  $M(t)$  – torque, and the matrices are  $\Phi(t)$  – rotation,  $\tilde{\omega}(t)$  – angular velocity,  $J$  – inertia tensor, and  $m$  the mass. To utilize Equation 1 the mass properties, the forces and torques acting on the body have to be determined [6][7][8].

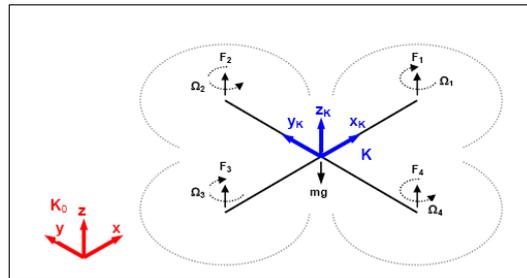


Figure 1

Quad-rotor helicopter with inertial and body frames

The mass of the fuselage or its components can be measured with a precision scale, but the determination of the inertia tensor can be complicated because of the complex structure of the helicopter. Therefore it is beneficial to utilize the capabilities of CAD systems. After designing the helicopter body in a mechanical engineering CAD environment the inertia tensor can be easily approximated with a built in tool as shown in Figure 2 and 3. This tool can compute the mass properties even complicated shapes [9].

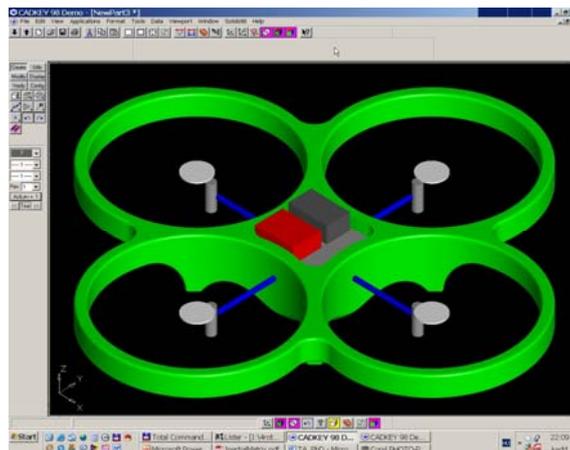


Figure 2

Quad-rotor helicopter in CAD environment

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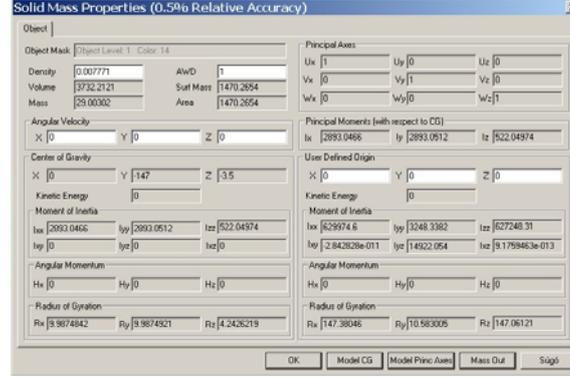


Figure 3

Mass properties of the quad-rotor helicopter

The torque is derived from differential thrust associated with pairs of rotors along with aerodynamic and gyroscopic effects. The thrust and drag generated by a single rotor can be modeled as a quadratic function of the propeller speed. With these assumptions the dynamic model can be stated as

$$\begin{aligned} \dot{\mathbf{i}}(t) &= \mathbf{v}(t) \\ \dot{\mathbf{v}}(t) &= -\mathbf{g} + \frac{1}{m_0} \Phi(t) \cdot \mathbf{F}_{f(K)}(t) \end{aligned} \quad (2)$$

$$\dot{\Phi}(t) = \Phi(t) \tilde{\omega}_{(K)}(t)$$

$$\dot{\omega}_{(K)}(t) = \mathbf{J}_{(K)}^{-1} (-\tilde{\omega}_{(K)}(t) \cdot \mathbf{J}_{(K)} \omega_{(K)}(t) + \mathbf{M}_{aero(K)}(t) - \mathbf{M}_{G(K)}(t))$$

where the lift and the aerodynamic torque is

$$\mathbf{F}_{f(K)} = \begin{bmatrix} 0 \\ 0 \\ C_f (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \end{bmatrix} \quad \mathbf{M}_{aero(K)} = \begin{bmatrix} d \cdot C_f (\Omega_2^2 - \Omega_4^2) \\ d \cdot C_f (\Omega_3^2 - \Omega_1^2) \\ C_f (\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2) \end{bmatrix} \quad (3)$$

and the torque due to gyroscopic effect is

$$\mathbf{M}_{G(K)} = \mathbf{J}_{re} \begin{bmatrix} \omega_y (\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \\ -\omega_x (\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \\ 0 \end{bmatrix}_{(K)} \quad (4)$$

where  $d$ ,  $C_f$ ,  $C_r$ ,  $\mathbf{J}_{re}$  are constants, and  $\Omega_{1,2,3,4}$  are the angular velocities of the rotors.

The electrical motors driving the rotors can be modeled with the well-known DC-motor dynamics [10]:

$$\begin{aligned} \frac{d\omega_m(t)}{dt} J_m &= i(t)K_T - B\omega_m(t) \\ \frac{d i(t)}{dt} L &= u(t) - i(t)R - K_b\omega_m(t) \end{aligned} \quad (5)$$

If the mechanical time constant of the controlled electric motor is much smaller than the mechanical time constant of the controlled helicopter – which is desirable – the angular velocities of the rotors can be treated as inputs to the helicopter instead of the input voltages of the DC-motors. Therefore the complexity of dynamic model of the whole system is reduced. Accordingly the mathematical model of a quad-rotor helicopter – introducing the Euler angles – has the following form [11]:

$$\begin{aligned} \begin{bmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{r}_z \end{bmatrix} &= \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \\ \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} &= \begin{bmatrix} (\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \frac{C_f}{m_0} \\ (\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \frac{C_f}{m_0} \\ -g + (\cos \varphi \cos \theta) (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \frac{C_f}{m_0} \end{bmatrix} \\ \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} \omega_x + \omega_y \sin \varphi \tan \theta + \omega_z \cos \varphi \tan \theta \\ \omega_y \cos \varphi - \omega_z \sin \varphi \\ \omega_y \sin \varphi \sec \theta + \omega_z \cos \varphi \sec \theta \end{bmatrix}_{(K)} \\ \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix}_{(K)} &= \begin{bmatrix} \left( \frac{J_{yy} - J_{zz}}{J_{xx}} \right) \omega_z \omega_y + \left( \frac{d \cdot C_f}{J_{xx}} \right) (\Omega_2^2 - \Omega_4^2) - \left( \frac{J_r}{J_{xx}} \right) \omega_y (\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \\ \left( \frac{J_{zz} - J_{xx}}{J_{yy}} \right) \omega_z \omega_x + \left( \frac{d \cdot C_f}{J_{yy}} \right) (\Omega_3^2 - \Omega_1^2) + \left( \frac{J_r}{J_{yy}} \right) \omega_x (\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \\ \left( \frac{J_{xx} - J_{yy}}{J_{zz}} \right) \omega_x \omega_y + \left( \frac{C_f}{J_{zz}} \right) (\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2) \end{bmatrix}_{(K)} \end{aligned} \quad (6)$$

### 3 Onboard Electronics

The onboard electronics consist of a central processing unit (IMU), an inertial measurement unit, an ultrasonic height sensor, a duplex radio transceiver, an RC receiver, a power stage and a power supply (Figure 4).

To provide sufficient signal processing capability the central processing unit is based on a TMS320F28335 32 bit microcontroller and on a Spartan 3 FPGA device that communicate via a dual port SRAM implemented in the FPGA.

The IMU module consists of 3 accelerometers, 3 gyroscopes and 3 magnetometers with built in sensor fusion algorithm. The ultrasonic height sensor can provide measurements with 1 cm accuracy.

The helicopter can communicate with a ground station via a duplex radio link while the manual control is implemented as a conventional RC receiver-controller combination [11].

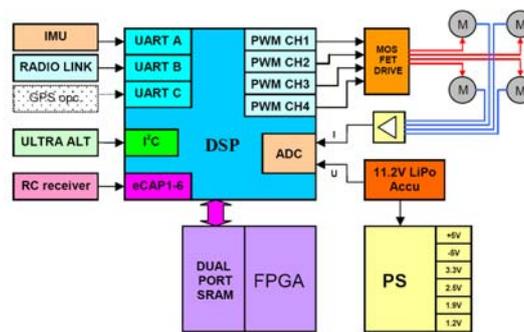


Figure 4

Block diagram of the onboard electronics

### 4 Controller Design

After determining the mathematical model, our task is to design a controller which makes our closed-loop system

- stable,
- follow the reference signal,
- minimize the effect of external disturbances,
- filter the internal disturbances,
- insensitive to parameter uncertainties,
- meet other requirements.

It is usually hard to satisfy all these – often contradictory – requirements since our model is only an approximation of the reality. We are not aware all the effects and cannot measure all the disturbances. Besides to keep the model manageable we have to use simplifications. Therefore only well-founded controller design can guarantee adequate closed-loop system.

Control theory of linear systems has a long and fruitful history. It has elaborated, simple and powerful tools with a mass of successful practical implementations. Therefore it is a common practice in engineering to use linear models and linear control design methods. However, linearization of a highly nonlinear model alone is often not sufficient. The linearized model can only predict the local behavior of the nonlinear system in the neighborhood of an operating point. Besides, there are essentially nonlinear phenomena that cannot be described by linear models. Accordingly we should use the analysis and control theory of nonlinear systems [12][13].

#### 4.1 LQG Controller

The tools of optimal control theory can be effectively used with multi-input multi-output linear systems of the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (7)$$

Control design is called “optimal control” when a specified condition is satisfied. But optimality cannot be interpreted globally; it is just with respect to that certain predefined criterion. Considering optimal Linear Quadratic Regulator (LQR) used here, the overall performance of the resulting closed-loop system is dependent on the suitability of the chosen Q, R weighting matrices in the quadratic criterion:

$$J = \frac{1}{2} \int_{t=0}^{\infty} (x^T Q x + u^T R u) dt \rightarrow \min \quad (8)$$

Hence the choice of an appropriate criterion is crucial in optimal control theory.

The optimal state feedback gain that minimizes the cost functional in Equation 8 can be obtained from the solution of the following equation

$$A^T P + P^T A + Q - P B R^{-1} B^T P = 0 \quad (9)$$

that is known as the algebraic Riccati-equation. The optimal feedback gain is then

$$K_{opt} = R^{-1} B^T P. \quad (10)$$

To implement the closed loop system all the state variables have to be available (Figure 5). However, in practice it is sometimes impossible or impractical to

measure all the state values. Even in the exceptional case that we are able and willing to measure all states we have to deal with measurement noise. In this case we should use state estimator.

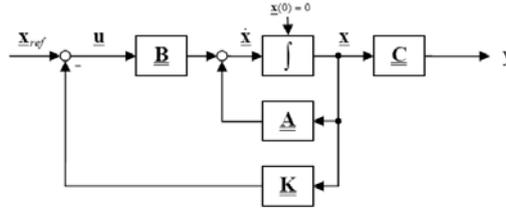


Figure 5  
 Full state-feedback LQR controller

The state estimation starts from the nominal linear system model with uncorrelated, zero mean state disturbance  $w(t)$  and measurement noise  $v(t)$

$$\begin{aligned} \dot{x} &= Ax + Bu + Gw \\ y &= Cx + v \end{aligned} \quad (11)$$

where  $Q_w$  and  $R_v$  are the covariance matrices of the corresponding stochastic signals:

$$\begin{aligned} E\{w(t)\} &= 0 & E\{v(t)\} &= 0 \\ E\{w(t)w^T(t+\tau)\} &= Q_w \delta(\tau) & E\{v(t)v^T(t+\tau)\} &= R_v \delta(\tau) \\ E\{w(t)v^T(t+\tau)\} &= 0 \end{aligned} \quad (12)$$

The block diagram of the well-known Kalman-filter is depicted in Figure 6. The estimation would be best, if the difference  $\tilde{x}(t) = x(t) - \hat{x}(t)$  of the real  $x(t)$  and the estimated  $\hat{x}(t)$  state variables were minimal. Hence, the optimality criterion of a Linear Quadratic Estimator (LQE) can be stated as

$$E\{\tilde{x}(t)\tilde{x}^T(t)\} = \min \quad (13)$$

The optimal error feedback matrix  $L$  can be obtained if we solve the corresponding Riccati-equation:

$$AP + PA^T + GQ_w G^T - PC^T R_v^{-1} CP = 0 \quad (14)$$

From the solution  $P$  the steady state error feedback matrix is

$$L_{opt} = PC^T P^{-1}. \quad (15)$$

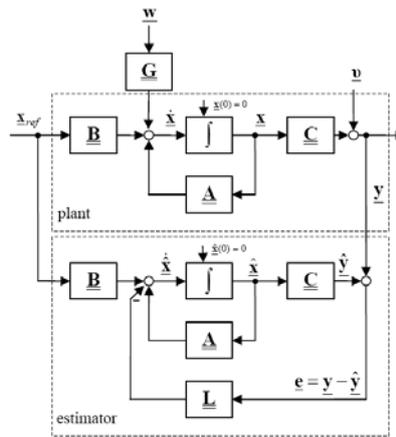


Figure 6  
 Block diagram of the Kalman-filter

It is easy to observe the similarity between the optimal controller and optimal state estimator problem. With the following dual transformations the solution is completely equivalent:

$$A = A^T, \quad Q = Q_w, \quad B = C^T, \quad R = R_0, \quad K = L^T, \quad P = P.$$

The derived controller and estimator above can be put together to form a Linear Quadratic Gaussian (LQG) controller shown in Figure 7.

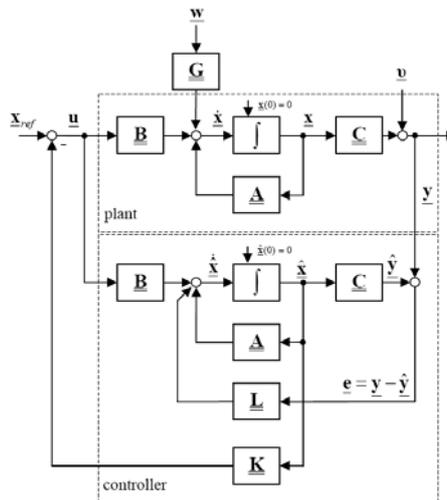


Figure 7  
 Block diagram of the complete LQG controller

The state feedback is accomplished as if the estimated state variables were the real ones. It is the separability principle of the LQG regulator: the LQR and LQE problem can be decoupled [14][15][16].

In order to apply linear control design technique to the quad-rotor helicopter the linear approximation of the nonlinear system (Equation 6) has to be linearized in the vicinity of steady state hovering. In the neighborhood of this operating point the following linear model can be derived:

$$\begin{aligned} \begin{bmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{r}_z \end{bmatrix} &= \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} & \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} &= \begin{bmatrix} \theta(U_1 C_{fn} - 4c_{fn}) \frac{1}{m_0} \\ -\varphi(U_1 C_{fn} - 4c_{fn}) \frac{1}{m_0} \\ -g + (u_1 + u_2 + u_3 + u_4)(C_{fn} - 4c_{fn}) \frac{1}{m_0} \end{bmatrix} \\ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{(K)} & \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix}_{(K)} &= \begin{bmatrix} \left( \frac{d \cdot C_{fn}}{J_{xx}} \right) (u_2 - u_4) \\ \left( \frac{d \cdot C_{fn}}{J_{yy}} \right) (u_3 - u_1) \\ \left( \frac{C_m}{J_{zz}} \right) (u_1 + u_3 - u_2 - u_4) \end{bmatrix}_{(K)} \end{aligned} \quad (16)$$

With this model, and with the appropriate weighting matrices, the optimal state feedback gain can be computed. With the probability properties of the state disturbance and the measurement noise the optimal state estimator gain can also be determined. After extensive simulations and analysis of test flights the given LQG controller implemented in the onboard electronics shown in section 2 can stabilize the near hover flight of experimental quad-rotor helicopter even in the presence of disturbing tosses. However, the system can only recover from roll and pitch angles smaller than 30 degrees due to the linear approximation. On the other hand, the slow horizontal drifting can not be measured with the noisy onboard inertial sensors therefore this effect has to be corrected manually during flight. Without more accurate or additional sensors the drifting can not be reduced to an acceptable level, therefore position regulation is not possible with this electronics [11].

## 4.2 SDRE Controller

Over the past several years, a lot of research effort has concentrated on nonlinear controller synthesis and analysis theory. The inability of linear controller design techniques to handle strongly nonlinear system dynamics has accelerated the development of methods such as feedback linearization, gain-scheduling, recursive backstepping, adaptive control etc. Also, a state-dependent Riccati equation (SDRE) technique has recently proposed for control of nonlinear dynamic systems. This method can be interpreted as a nonlinear counterpart of the LQR design with state dependent system and weighing matrices. Therefore the quadratic optimality criterion to be minimized

$$J = \frac{1}{2} \int_{t_0}^{\infty} (x^T Q(x)x + u^T R(x)u) dt \quad (17)$$

is also dependent on the states. Since the SDRE method approaches the controller design problem by mimicking the LQR formulation for linear systems, the general nonlinear state equation has to be rewritten in a pointwise linear structure called state-dependent coefficient (SDC) form:

$$\dot{x} = f(x, u) \rightarrow \dot{x} = A(x)x + B(x)u \quad (18)$$

This parameterization is possible if and only if  $f(0) = 0$  and  $f(x)$  is continuously differentiable. Additionally the  $A(x)$ ,  $B(x)$  pair has to be pointwise controllable for all  $x$  in the linear sense to ensure that the state-dependent Riccati equation

$$A^T(x)P(x) + P(x)A(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \quad (19)$$

has a solution for every point in the state-space. The state feedback matrix that is also state-dependent here becomes:

$$u = -K(x)x = -R^{-1}(x)B^T(x)P(x)x \quad (20)$$

Unlike LQR design for linear systems the SDRE control can only guarantee local asymptotic stability. Since the explicit closed-loop system equations are usually not known for SDRE controlled systems, the global stability analysis is quite difficult and still an open issue. However, simulation and experiment results show wide range of applicability, good stability performance and due to its LQR nature inherent robustness characteristics of the SDRE method.

For a given nonlinear system there may be several possible SDC forms that result in pointwise stabilizable matrix pair  $A(x)$ ,  $B(x)$ . The choice among them influences the overall performance of the controller in the same way as the choice of the weighing matrices which are also allowed to be function of the states. This flexibility can be utilized in the controller design.

To implement the SDRE controller one has to solve the state-dependent Riccati equation in every sampling interval. The analytical solution is only possible for lower order systems or for systems with special structure. Otherwise numerical algorithms have to be used that demand more computational resources than conventional control algorithms. For example the Schur-method conventionally used to solve algebraic Riccati equations requires approximately  $75n^3$  floating point operations. Hopefully the computational power of modern computers is more than enough to cope with this task although for embedded processing some important issue has to be considered. Since the computational cost is of polynomial growth rate and the choice of the sampling rate is critical, for stability sufficient signal processing capacity has to be available onboard [17][18][19].

### Conclusions

In this paper the onboard electronics and the implemented linear quadratic regulator design of an experimental quad-rotor helicopter is presented. It is concluded that without more precise or additional sensors the horizontal drift in the position can not be reduced. Therefore with inaccurate inertial sensor data position control is not a possibility. Besides to cope with the nonlinear phenomena of the helicopter dynamics a nonlinear controller design approach has to be considered. Theoretical and experimental results suggest that the characteristics of the SDRE method are adequate for a wide range of nonlinear dynamic systems including unmanned aerial vehicles.

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