# Design of Compliant Robotic Micro-Devices

**Štefan Havlík[1], Giuseppe Carbone[2]**

[1] Institute of Informatics, Slovak Academy of Sciences
Severna 5, 974 01 Banská Bystrica, Slovak Republic, email: havlik@savbb.sk
[2] Laboratory of Robotics and Mechatronics, DiMSAT - University of Cassino
Via Di Biasio 43, 03043 Cassino, Italy, email:carbone@unicas.it

***Abstract:*** *Design and modeling compliant robotic mechanisms are discussed in this paper. Such mechanisms are usually constructed form relatively rigid parts connected by flexural segments, or, made as compact elastic structures in cases of micro mechanisms. Problems of description and comparing performance characteristics with the goal to reach maximal positional accuracy are analyzed. The procedure of designing these compliant mechanical structures on example of the robotic micro gripper is shown.*

***Keywords:*** *compliance, stiffness, mechanisms, robot*

## I INTRODUCTION

Compliant mechanical structures and mechanisms represent a broad class of mechanical system where displacements are reached by elastic deformation of their flexural parts / segments. The desired motion is the mechanical response on forces / torques applied on input ports. The meaning and application of compliant mechanisms is getting more and more important especially in small and micro-scale or precise machines where classic constructions from discrete parts are no more realizable. The solution lies in design of compact compliant mechanical structures and using appropriate advanced technology for their manufacturing. Compact designs and MEMS technologies enable to miniaturize dimensions and realize manipulations in micro scale range. Mechanically such mechanisms can be designed as transformers of forces, displacements or force to displacement and vice-versa.

When compare a classic - human scale mechanism with the range of motions within x.$10^{-1}$ (m) it normally exhibits the positioning accuracy x.$10^{-5}$ (m) i.e. the rank of difference is $10^{-4}$. Then, accepting the same ratio for micro-manipulation mechanisms they should reach the nano-scale accuracy. Considering possible sources of errors the total accuracy is influenced by not exactly defined deflections of particular joints / arms. They always exhibit some cross flexural effects that deteriorate final accuracy of the mechanisms. This is one of crucial problems of micro-manipulation and task for design of precise compliant mechanisms.

Naturally, designing complex compliant structures that include elastic and relatively rigid elements suppose using techniques for force and compliance analysis, modeling and simulation of

flexible structures as well. The final design is then always a choice of geometry and parameters that satisfy some optimal / compromise solution. It should be said that when designing a compliant mechanisms much more attention and effort should be devoted to this design phase then in the case of classic mechanisms.
As to the design procedure at the beginning there is always the first intuitive proposal. It can be said that principal part of the compliant mechanisms is usually designed on the base of similarity with rigid-body mechanisms.

## II DESIGN OF COMPLIANT MECHANISMS

In principle, there are two ways how it is possible to realize desired small compliant motions:
- The kinematic approach i.e., where kinematics corresponds to classic mechanisms; only hinges are replaced by elastic joints.
- The approach based on compliant structures where motion is realized by flexural displacement of the whole structure.

The difference between these two approaches can be seen on examples of two RCC (Remote Center of Compliance) mechanisms used for robot wrists in Fig. 1 [3,6].

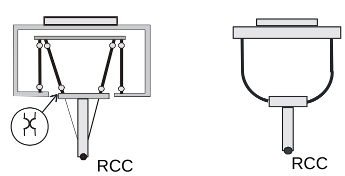



Figure 1
Two types of the RCC mechanisms

The design procedure usually consists of four principal steps according to general procedure in Fig. 2.

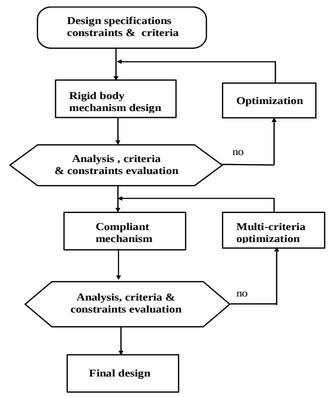



Figure 2
The design procedure

- The first step represents specification of criteria and constraints should be satisfied.

The design of mechanism in form of classic kinematics and its topology are main tasks in the second step. Here some principal criteria related to geometry and motion specifications are verified and parameters of mechanism are optimized.
The third step includes stiffness / compliance analysis of previously designed mechanism where joints of rigid links were replaced by flexural segments. Further criteria related to forces, displacement and dynamical performance are evaluated and the multi-criteria optimization procedure is applied to choose the best solution.

## III DESCRIPTION OF FLEXURAL CHARACTERISTICS

Let us describe flexural characteristics of an elastic segment / joint separated from a complex flexure as depicted in Fig. 3.

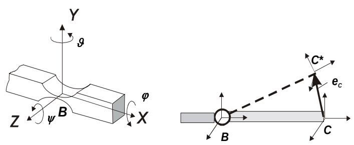



Figure 3
The elastic joint

For simplicity we suppose linear stress – strains dependence i.e. between internal / external forces and deflections. Then, the forces and deflections in the same reference system are related

$$\boldsymbol{d} = \boldsymbol{C} \cdot L = S^{-1} \cdot L \qquad (1)$$

where

- $\boldsymbol{d}$ is the six component vector of deflections,
- $\boldsymbol{L}$ is the vector of internal force and moment loads,
- $\boldsymbol{C}$ and $\boldsymbol{S}$ are, in general, the 6x6 compliance and stiffness matrices.

Components of these matrices are compliance / stiffness coefficients and can be calculated using FEM techniques, or, for some specified form of joints / arms applying methods of classic theory of elasticity.

The crucial problem in design is: how to compare various deformable segments as to their flexural characteristics. Usually, elastic joints are designed to have a given selective compliance in some desired directions. Comparing to classic revolute or prismatic joints, any elastic connection exhibits some cross-flexural effects that should be taken into account.

Providing linearity (1) it is possible to analyze and compare characteristics using method of singular value decomposition (SVD). The SVD of the compliance matrix $\boldsymbol{C}$ from (1) is expressed by transformation

$$\boldsymbol{C} = \boldsymbol{G} \cdot \boldsymbol{\Phi} . \boldsymbol{H}^T \qquad (2)$$

where $\boldsymbol{G}$, $\boldsymbol{H}^T$ are orthogonal matrices and $\boldsymbol{\Phi} = diag(\varphi_1, \varphi_2, \ldots \varphi_6)$ are singular values of the compliance matrix.

The geometric interpretation of such analysis is as follows:

Using transformation (1) the unit sphere in the force space is mapped into the deflection space as a general ellipsoid. The lengths of its main axes are singular values with orientation given by columns of the $\boldsymbol{G}$ matrix (See Fig. 4).

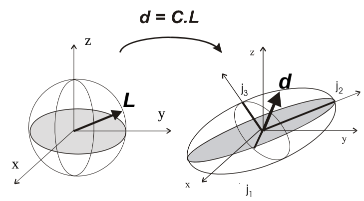



Figure 4
The compliance ellipsoid

Then, comparing elastic segments, two bodies have the same characteristics if they exhibit the same compliance ellipsoid: as to the length of its axes and as to their orientation. Different forms of elastic parts can exhibit a given selective compliance in particular directions [2].

It should be said, that any real elastic joint differs from “an ideal revolute or prismatic connection”. It always exhibits some cross flexural effects i.e. beside the desired motion it deflects in other directions too. Mathematically, the compliance matrix, except the dominant coefficient, includes another unwanted compliance elements. The compliance ellipsoid has some finite not negligible volume. These effects naturally deteriorate the accuracy of joint flexural motion and results in worse positional accuracy of a mechanism, as whole. Thus, considering the form and the volume of the compliance ellipsoid it is possible to evaluate "the kinematic quality" of a particular joint.

In order to satisfy maximal accuracy of the mechanisms, these effects of unwanted deflections should be considered in precise calculations. Then, they can be eliminated, minimized, or otherwise compensated.

We rewrite the vector of deflections from (1) into matrix form. Then the transformation of the deflected segment will be

$$ {}^{B}\boldsymbol{T}^{*}_{C*} = {}^{B}\boldsymbol{T}_{C} \cdot {}^{C}\boldsymbol{E}_{C*} \tag{3}$$

where, ${}^{C}\boldsymbol{E}_{C*}$ is the (6x6) matrix that represents transformation due to deflections of the joint in form

$$ {}^{C}E^{*}{}_{C*} = \begin{bmatrix} R_{def} & R_{def}.E_{def} \\ \boldsymbol{0} & R_{def} \end{bmatrix} \tag{4}$$

where $\boldsymbol{R_{def}}$ is the (3x3) matrix of rotations with dominant meaning of components that include functions of the desired rotation $\psi$ (see Figure 4); $c_{(.)}=\cos_{(.)}$, $s_{(.)} = \sin_{(.)}$

$$ \boldsymbol{R_{def}} = \boldsymbol{R}_z(\varphi)\,\boldsymbol{R}_y(\vartheta)\boldsymbol{R_x}(\psi) = $$

$$ = \begin{bmatrix} c_\varphi c_\vartheta & c_\varphi s_\vartheta s_\psi - s_\varphi c_\psi & c_\varphi s_\vartheta c_\psi + s_\varphi s_\psi \\ s_\varphi c_\vartheta & s_\varphi s_\vartheta s_\psi + c_\varphi c_\psi & s_\varphi s_\vartheta c_\psi - c_\varphi s_\psi \\ -s_\vartheta & c_\vartheta s_\psi & c_\vartheta c_\psi \end{bmatrix} \tag{5}$$

and translations errors due to compression / elongation and shear forces

$$ E_{def} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix} \tag{6}$$

The other components in (5), except those that contain function of $\psi$, represent cross-deflection effects should be compensated and the second order terms can be neglected.

Then, precise calculations of kinematic and force transmissions should include these deflected transformation matrices of all flexural bodies.

As obvious, in order to guarantee a desired life of such a part, there are given limits on maximal strain / stress for any elastic material loaded by cyclic way. These limits give constraints on maximal deflections of particular joints / segments and depend to their forms and dimensions, compared in [4,5]. This fact naturally corresponds to allowable load and working range of the whole mechanisms [Weight]. This can be shown on example of the planar mechanisms for two d.o.f. precise positioning table in Fig. For this purpose it is need to calculate stiffness of the whole mechanisms with respect to actuating forces. See [2].

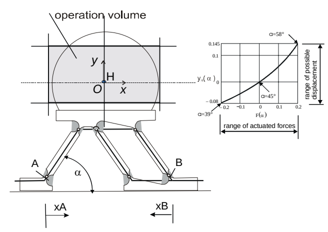



Figure 5
Limits and working space for the 2 d.o.f. micro-mechanism

As discussed previously, an optimal design can be formulated as a multi-objective optimization problem with the cost function taking into account several performance criteria can be written in the general form

$$ min\boldsymbol{F}(p) \tag{7}$$

subjected to

$$ \mathbf{g(X)} < 0\,;\, \mathbf{h(X)} < 0 \tag{8}$$

where: $\mathbf{X}$ is the vector of design variables; each component of the objective function is an expression of a design optimum criterion; each component $g_k$ (k=1,…,m) describes an inequality design constraint; and each component $h_l$ (l=1,…, n) describes an equality design constraint. Constraints can be formulated through the functions **g** and **h** to express design requirements but also limitations for the design variables and objective functions [1].

## IV A DESIGN OF THE COMPACT ELASTIC MICROGRIPPER

The design procedure will be shown on the example of an experimental gripper with parallel motion of fingers driven by SMA wires.

***Structure and topology of the mechanism.*** The performance can realize the mechanism with kinematic scheme in Fig. 6. The task for design is to find the geometry, i.e. all relevant dimensions in order to satisfy desired characteristics under specified constraints. There are:

- the range of motion for fingers
- maximal external dimensions
- available driving force and displacement that result in force and displacement transmissions
- characteristics of material.

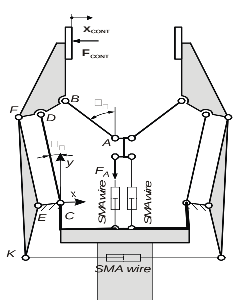



Figure 6
The kinematic structure

***Stiffness / compliance of the finger mechanism.*** The main characteristics that, beside the kinematic structure, specifies force transmission. Because of for the actual driving force we have:

$$F_A = n\left(S_A . d_A + \frac{F_{Cont}}{k}\right) \qquad (9)$$

where $n$ – is the number of fingers, $S_A$ is the stiffness of the finger mechanics related to the actuator displacement $d_A$, $F$ is the contact / grasping force and $k$ is the displacement transmission ratio between motion of finger and actuator.
Flexible / elastic joints in Fig. 7 will have the form of two circular notches made on both sides of the arm.

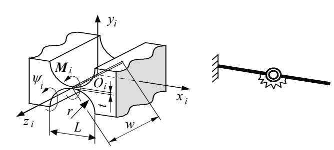



Figure 7
Detail of the joint

For such form where $w >> t$ , there is only one dominant compliance coefficient $c_{33} = c_\psi$ in the matrix $\boldsymbol{C_j}$, which corresponds to rotation $\psi_i$ about z-axis due to the internal torque $M_i$. This coefficient can be derived as function of geometrical parameters denoted in Fig. 7.

Varying dimensions and parameters of the joint it is possible to reach a desired flexibility (stiffness / compliance) of the joint.

The task is to calculate the compliance of the finger mechanism with respect to driving forces. As supposed, the finger mechanism in Fig. 8 is driven by SMA wires in the A point to close the finger and in the K point to open it. For the simplicity we consider that all flexural joints have the same characteristics represented by compliance $\boldsymbol{C_j}$ or, the stiffness $\boldsymbol{S_j}$ .

Structurally, this flexure consists of the part including parallel links with joints CD – EF which is serially connected to the link with joints B, A.

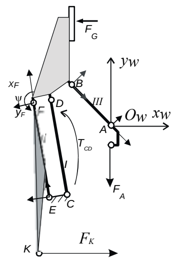


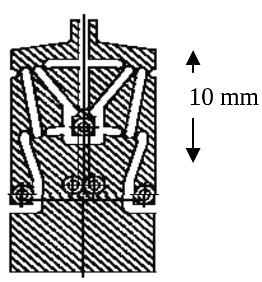

Figure 8
The finger mechanism

The compliance of such a half structure related to direction of the actuating force $F_A$ is given by superposition of compliances of particular joints transformed to $O_W$ references in A point.

$$\boldsymbol{C}_W = \boldsymbol{T}_{AW}(\boldsymbol{C}_{A(I+II)} + \boldsymbol{C}_{A(III)})\boldsymbol{T}_{AW}^T \qquad (10)$$

where $\boldsymbol{C}_{(I+II)}$ is the compliance of the parallelogram and $\boldsymbol{C}_{(III)}$ of the arm III. By the same way the compliance that corresponds to direction of closing force in K is calculated.

The final design of this gripper made from one piece of flat elastic material is shown in Fig. 9.




The compact elastic gripper

## Conclusion

The paper points out at some specific problems in design of micro-mechanical devices built on compact elastic structures. The evaluation of the compliance characteristics, modeling and optimization procedures have crucial importance in the whole design process.

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