# Experiments in Fuzzy Control of a Class of Servo Systems for Mobile Robots

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***Abstract:*** *The paper deals with experiments with Mamdani PI-fuzzy controllers (PI-FCs) to control a class of integral plants specific to servo systems playing the role of actuators for control systems (CSs) dedicated to mobile robots. In the first phase there are designed linear PI controllers tuned in terms of the Extended Symmetrical Optimum method to ensure the desired CS performance indices with respect to the step modifications of the set-point and of three possible types of load disturbance inputs. Then, there is presented an attractive development method for the PI-FCs based on the linear case results and on the modal equivalence principle. An example concerning the speed control of a nonlinear servo system with variable load, accompanied by real-time experimental results, validates the PI-FCs and the development method as low-cost solutions.*

***Keywords:*** *Servo Systems, PI-fuzzy Controllers, Mobile Robots*

## I INTRODUCTION

Nowadays although there have been achieved great progresses, mobile robots which are capable of performing various and complex tasks in an autonomous and intelligent way, have not yet been able to conquer wide ranges of applications. In this context, it is very important to develop high performance controllers to cope with the three navigation problems [1], tracking control (tracking a reference trajectory), path following and point stabilization. The tracking control problem can be further divided in local and global tracking problems [2].

The majority of controllers developed for nonholonomic mobile robots is based on either kinematic [3, 4], or dynamic models [5, 6]. One model that exploits the dynamics of the actuators, of the measuring devices and of the control equipment as part of the control system (CS) structure has been proposed in [7], and the simplified model of the controlled plant (CP) is characterized by the transfer function (t.f.) $P(s)$:

$$P(s) = k_P / [s(1 + T_\Sigma s)], \qquad (1)$$

where $k_P$ is the controlled plant gain and $T_\Sigma$ is the small time constant or the time constant corresponding to the sum of parasitic time constants.

Since the t.f. in (1) represents a simplified linearized model of CPs in nonlinear servo systems, the parameters $k_P$ and $T_\Sigma$ are time-variable within certain limits, so controlling the plants (1) is a challenging problem when very good CS performance indices are required in regulation and tracking.

The CS structure in the linear case is presented in Fig. 1, where: $r$ – reference input, $F(s)$ – t.f. of the reference filter, $r_1$ – filtered reference input, $y$ – controlled output, $u$ – control signal, $e = r_1 - y$ – control error, $d \in \{d_1, d_2, d_3\}$ – general disturbance input of threee types.

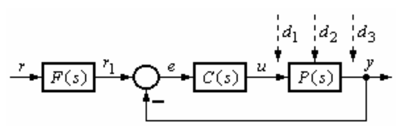


Figure 1
Linear control system structure

For this CS structure and the considered plant, in the linear case the use of PI controllers with the t.f. (2):

$$H_C(s) = [k_c / s](1 + sT_c), \qquad (2)$$

with the gain $k_c$ and the integration time constant $T_c$, tuned in terms of Kessler's Symmetrical Optimum method, can ensure acceptable CS performance indices [8].

But, in some practical applications the CS performance indices overshoot $\sigma_1$, settling time $t_s$, rise time $t_r$ and phase margin $\varphi_m$ prove to be rather unacceptable due to the large sensitivity with respect to the modification of the plant gain $k_P$ accompanied by a possible alleviation of $\varphi_m$. This shortcoming becomes serious when $T_\Sigma$ correspond to the sum of parasitic time constants generally taking only an approximate value.

A simple and efficient way to tune the parameters of the linear PI controller (2) controlling the plant (1) is represented by the ESO method [9], characterized by only one design parameter, $\beta$.

One solution to ensure CS performance enhancement is represented by fuzzy control. Due to the two-degree-of-freedom CS structure, the PI-fuzzy controllers (PI-FCs) presented in the paper ensure very good CS performance indices with respect to the two inputs, $r$ and $d$.

The paper is organized as follows. The following Section is dedicated to the presentation of the new development method based on the transfer of results from the linear case (in terms of the ESO method) to the fuzzy case based on the modal equivalence principle [10]. Then, there are presented in Section III real-time experimental results in a case study corresponding to nonlinear DC drive servo systems. The case study proves very good CS performance in both regulation and tracking, and validates the fuzzy control solution as low cost solution in mobile robots control. The conclusions are drawn in the end of the paper.

## II DEVELOPMENT METHOD FOR MAMDANI PI-FUZZY CONTROLLERS

The PI-FCs replace the linear PI controllers with the transfer function $C(s)$ in the CS structure presented in Fig. 1. The PI-FC is obtained by fuzzifying the linear PI controllers to ensure the aim of low-cost, and it represents a discrete-time controller involving a basic fuzzy controller (B-FC, without dynamics), with dynamics being added by the numerical differentiation of the control error $e_k$ expressed as the increment of control error, $\Delta e_k = e_k - e_{k-1}$, and by the numerical integration of the increment of control signal, $\Delta u_k$.

The structure of the PI-FC is presented in Fig. 2. The fuzzification is solved in terms of the regularly distributed input and output membership functions illustrated in Fig. 3. Other distributions and shapes of membership functions can modify in desired way the controller nonlinearities.

Fig. 3 highlights the three (strictly positive) parameters of the PI-FC to be tuned by the development method presented in the sequel, $B_e$, $B_{\Delta e}$ and $B_{\Delta u}$.

The inference engine in B-FC employs Mamdani's MAX-MIN compositional rule of inference assisted by the rule base presented in Table 1, and the defuzzificaton is done in terms of the centre of gravity method for singletons.

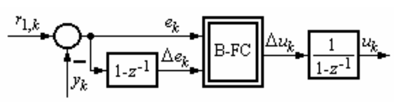


Figure 2
PI-fuzzy controller structure

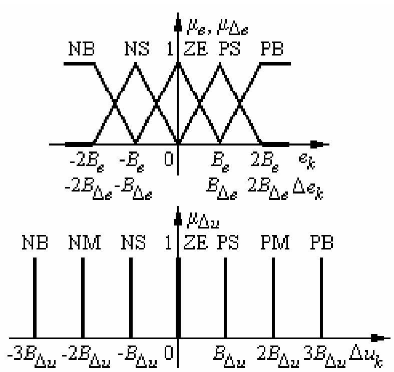


Figure 3
Membership function shapes and parameters

To develop the PI-FC it is necessary to do the linear case development focused on the ESO method characterized, as mentioned in Section I, by only one design parameter, β. The adequate choice of the parameter β within the domain 1 < β < 20, ensures the modification of CS performance indices ($\sigma_1$, $\hat{t}_r = t_r / T_\Sigma$ – normalized rise time, $\hat{t}_s = t_s / T_\Sigma$ – normalized settling time defined in the unit step modification of $r$, $\varphi_m$) according to designer's option and a compromise to these indices can be reached using the diagrams presented in Fig. 4, valid in the situation without $F(s)$. However, the presence of $F(s)$ improves further the CS performance indices.

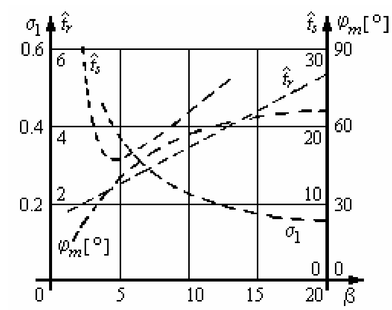


Figure 4
Control system performance indices versus β

Table 1
Decision table of B-FC

| $\Delta e_k$ | $e_k$: NB | NS | ZE | PS | PB |
|---|---|---|---|---|---|
| PB | ZE | PS | PM | PB | PB |
| PS | NS | ZE | PS | PM | PB |
| ZE | NM | NS | ZE | PS | PM |
| NS | NB | NM | NS | ZE | PS |
| NB | NB | NB | NM | NS | ZE |

The PI tuning conditions, specific to the ESO method, can be expressed in terms of (3):

$$k_c = 1/(\beta\sqrt{\beta}T_\Sigma^2 k_P),\ T_c = \beta T_\Sigma \,. \tag{3}$$

To apply the ESO method in case of fuzzy CSs it is necessary to discretize the continuous-time linear PI controller (2) resulting in the digital PI controller:

$$\Delta u_k = K_P \cdot \Delta e_k + K_I \cdot e_k = \\ = K_P(\Delta e_k + \alpha \cdot e_k) \quad (4)$$

where the parameters $K_P$, $K_I$ and α can be calculated, for example, in terms of (5) in case of Tustin's discretization method:

$$K_P = k_C[1 - T_s/(2T_c)], \\ K_I = k_C T_s / T_c, \quad (5) \\ \alpha = K_I / K_P = 2T_s/(2T_c - T_s),$$

with $T_s$ – sampling period.

The development method for the considered class of fuzzy CSs with PI-FCs controlling servo systems of type (1) consists of the phases A) and B):

A) The phase of linear controller development, referred to as the linear case, with the steps A1) … A3):

A1) Express the simplified mathematical model of the servo system in terms of the t.f. in (1) with the parameters $k_P$ and $T_\Sigma$.

A2) Choose the initial value of the design parameter β taking into account the desired / imposed CS performance indices and the diagrams presented in Fig. 4, design the reference filter, the simplest one having the t.f. $F(s)$ in the linear case:

$$F(s) = 1/(1 + \beta T_\Sigma s), \quad (6)$$

and discretize the reference filter.

A3) Set the value of $T_s$ in accordance with the requirements of quasi-continuous digital control and calculate the parameters in (5) corresponding to (4).

B) The phase of fuzzy controller development, referred to as the fuzzy case, with the modal equivalence principle resulting in (7):

$$B_{\Delta e} = \alpha B_e, \ B_{\Delta u} = K_I B_e, \quad (7)$$

where the free parameter $B_e$ represents the designer's option.

The choice of this parameter can be done to ensure the stability of the fuzzy CS [11, 12]. The sensitivity analysis with respect to the parametric variations of CP is recommended also in [13] taking into account the plant model in (1) as linearized simplified model and one of the aims of fuzzy control is to control complex plants.

## III EXPERIMENTAL RESULTS

To validate the development method presented in Section II it is considered a case study with the CP characterized in its linearized simplified form by the t.f. in (1).

The experimental setup corresponds to the speed control of a nonlinear laboratory DC drive (AMIRA DR300). The DC motor is loaded using a current controlled DC generator, mounted on the same shaft, and the drive has built-in analog current controllers for both DC machines having rated speed equal to 3000 rpm, rated power equal to 30 W, and rated current equal to 2 A. The speed control of the DC motor is digitally implemented using an A/D-D/A converter card. The speed sensors are a tacho generator and an additional incremental rotary encoder mounted at the free drive shaft. The picture and the schematic diagram of the hardware station are presented in Fig. 5.

The mathematical model of CP can be well approximated by the transfer function $P(s)$ in (1) with the nominal values of the parameters $k_P$=4900 and $T_\Sigma$=0.035 s.

Then, the proposed development method is applied, starting with the choice of the design parameter, β = 6. The parameters of the linear PI controller obtain the values $k_c$ = 0.0114 and $T_c$ = 0.21 s. The PI-FC tuning parameters obtain the values $B_e$ = 0.3, $B_{\Delta e}$ = 0.03 and $B_{\Delta u}$ = 0.0021.

Part of the real-time experimental results is presented in Fig. 6 and Fig. 7 for the CS with the original linear PI controller and with the developed PI-FC, respectively. The experimental scenario concerns the triangular-type modification (Fig. 6a and Fig. 7a), the sinusoidal variation of *r* (Fig. 6b and Fig. 7b), and a 5 s period of 10% $d_3$-type rated load in all four situations.

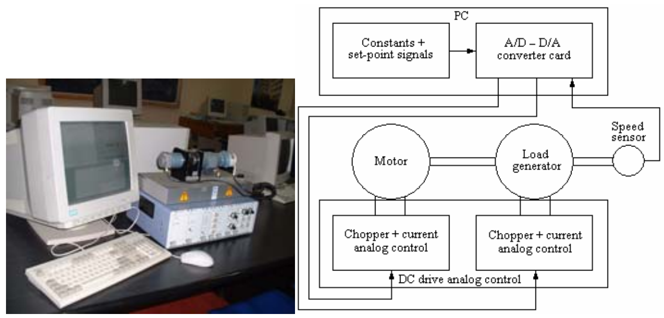


Figure 5
Picture and schematic diagram of hardware station

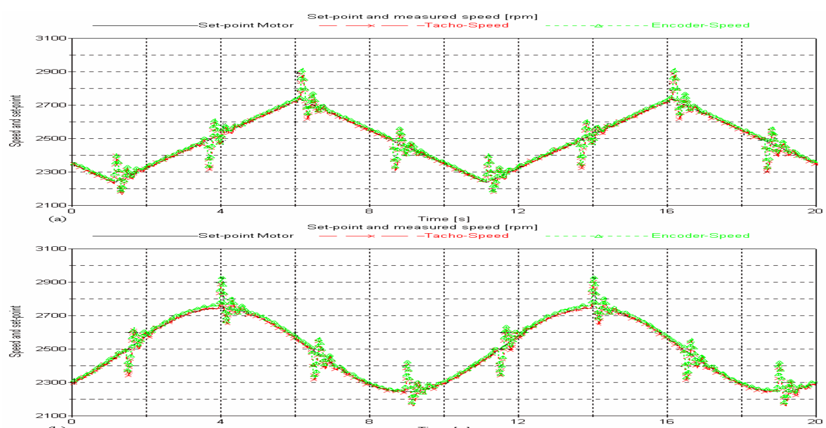


Figure 6
Speed response of control system with linear PI controller

## Conclusion

The paper presents a development method for a class of Mamdani PI-FCs dedicated to servo systems as actuators in mobile robots applications.The method can be applied with minor problems in case of PD-, PID-fuzzy controllers and of other complex fuzzy controller structures [14, 15, 16].

The case study presented in the paper, with real-time experimental results, highlights the CS performance enhancement ensured by the proposed PI-FCs in comparison with PI ones.

Future research will be focused on the automatic model-free development of fuzzy controllers.


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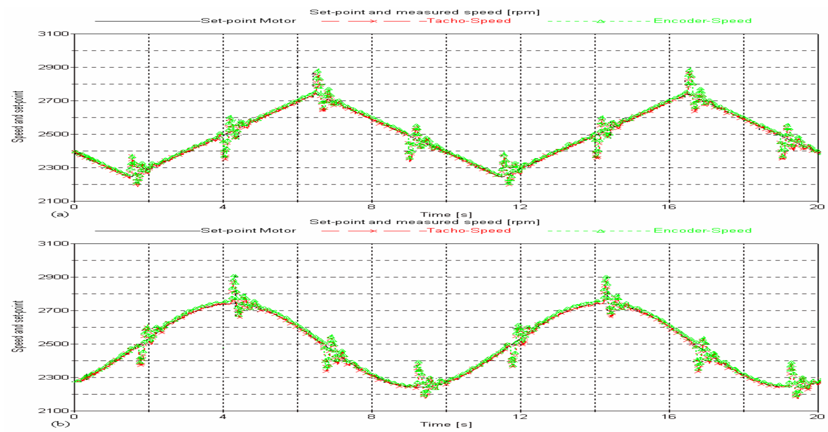


Figure 7
Speed response of control system with PI-fuzzy controller

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