A Youla-parameterization Approach for Controller Design based on ESO and 2E-SO Methods for Low Order Benchmarks

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Abstract: The Youla parameterization (also the Q – parametrization) is a modern control design method suitable for both stable and unstable plants. In the design phase systems with electrical drives can be well approximated through low order linear models called benchmarks. For such system it can be advantageous to use control design based on modulus optimum criterion (Modulus Optimum MO-m and Symmetrical Optimum SO-m) or methods derived from this one, such as ESO-m and 2E-SO-m. In addition to the original, classical SO-m, the 2E-SO-m has the advantage of providing a better phase margin and improved robustness. The paper presents briefly the development and tuning solutions of PI and PID controllers based on ESO-m and 2E-SO-m meant for electrical drive systems. Based on these a Youla parameterization interpretation of the methods is given.

Keywords: drive systems, servo-system, benchmark models, ESO and 2E-SO method, Youla parametrization design.

1 Introduction

The Youla parameterization, also called as Q-parameterization, is a design method applied for both stable and unstable plants [1]. It differs from classical design methods that impose controller types and structures, the Youla parameterization requires polynomials relative to the system's properties. The disadvantage of the method consists in fact that in case of high order, non-minimum phase or unstable plants the controller results as a nonconventional one, which is often denied by practicians. In the paper there is presented a Youla-parameterization interpretation of a certain controller design method for stable benchmark type models, and can be applied for control of electrical drives.

The paper is organized as follows. Paragraph 2 presents two extensions of the SO-m, section 3 deals with a Youla-parameterization approach of the previously introduced methods. Chapter 4 presents possibilities for application of the method and conclusions.

2 Extensions of the Symmetrical Optimum Method for Plants with and without Integral Component

Based on optimality conditions imposed for the amplitude-frequency characteristics [2], in papers [3] and [4] Kessler introduced two design methods, the modulus optimum method (MO-m) and the symmetrical optimum method (SO-m). The main advantages of the methods are:

- The use of conventional PI and PID controllers;
- Simple tuning relations for controller parameters, that are very useful in practice;
- Favourable placement of poles (MO-m) or relatively favourable (SO-m), implying very good or pretty good dynamical behaviour. This can be anticipated in the design phase (these can be partially corrected by adequate reference filters).

Consequently, these methods are welcome, being interpreted and extended under different particular formulations (see for example [4],[5],[6]).

One of disadvantages of the SO-m consists in a relatively small phase margin of the system ($\varphi_r \approx 36^0$), which leads to an increased sensibility towards parameter changes and low robustness. In other situations even the use of pole-zero cancelation technique can worsen the system's behaviour regarding load disturbances. These disadvantages can be avoided by using the so called "Extended Symmetrical Optimum method" (ESO-m) [7] and "Extension based on ESO-m", (2E-SO-m). The first method [7] is dedicated to plants with integral components, while the second one [8] to plants without integral components.

The MO-m and SO-m refer mainly to benchmark type models – synthesised in table 2.1. These models correspond to transfer functions (t.f.) which can be used e.g. for electrical driving systems, speed- and positoning-control.

Table 2.1

Rel. no.	Plant transfer (t.f.) functions, *P(s)*	
	Speed-control	Positioning control
	(a)	(b)
(2.1)	$\dfrac{k_P}{(1+sT_\Sigma)}$	$\dfrac{k_P}{s(1+sT_\Sigma)}$
(2.2)	$\dfrac{k_P}{(1+sT_1)(1+sT_\Sigma)}$	$\dfrac{k_P}{s(1+sT_1)(1+sT_\Sigma)}$
(2.3)	$\dfrac{k_P}{(1+sT_1)(1+sT_2)(1+sT_\Sigma)}$	$\dfrac{k_P}{s(1+sT_1)(1+sT_2)(1+sT_\Sigma)}$

Remarks: 1. T_Σ characterises the small time constant or the equivalent of small time constants; if the plant contains a relatively small dead-time component, this can be included into T_Σ:

$$\hat{P}(s) = P(s)e^{-sT_D} \quad T_1 > T_2 > T_\Sigma \quad , \quad T_\Sigma = \sum_1^k \tau_\nu + T_D \tag{2.4}$$

2. The parameter k_P, constant or variable gain, characterizes well enough many control applications with electrical drives (as controlled plants).

In case of using PI, PID controllers and pole-zero cancellation in controller design, one of the methods indicated in table 2.2 is used.

The closed loop t.f. regarding the reference signal $H_r(s)$, results:

- MO-m: $H_r(s) = \dfrac{b_0}{a_0 + a_1 s + a_2 s^2}$ with $b_0 = a_0$ (2.5)

- SO-m(ESO,2E-SO): $H_r(s) = \dfrac{b_0 + b_1 s}{a_0 + a_1 s + a_2 s^2 + a_3 s^3}$, $b_0 = a_0$, $b_1 = a_1$ *or* $b_1 \neq a_1$

(2.6)

In order to fulfill the design requirements based on modulus optimality conditions, between the coefficients of the transfer functions the following relations are imposed

MO-m: $2a_0a_2 = a_1^2$ (a) ; SO-m: $2a_0a_2 = a_1^2$, $2a_1a_3 = a_2^2$ (b) (2.7)

Table 2.2

The method	Controller type, t.f. *C(s)* and design relations regarded to the plant t.f. *P(s)*		
	PI (or I)	PID	PID^2
	(a)	(b)	(c)
MO-m	(2.1-a): I (2.2-a): PI	(2.3-a): PID	Fourth order t.f.
SO-m	(2.1-b): PI	(2.2-b): PID	(2.3-b): PID^2
ESO-m	(2.1-b): PI	(2.2-b): PI	(2.3-b): PID^2
2E-SO-m	(2.2-a): PI	(2.3-a): PID	Fourth order t.f.
t.f. *C(s)* (2.8)	$\frac{k_r}{s}(1+sT_r)$	$\frac{k_r}{s}(1+sT_r)(1+sT_r')$	$\frac{k_r}{s}(1+sT_r)(1+sT_r')\frac{1+sT_d}{1+sT_f}$
MO-m	$k_c=\frac{1}{2k_pT_\Sigma}$ $T_c=T_1$	$k_c=\frac{1}{2k_pT_\Sigma}$ $T_c=T_1;\quad T_c'=T_2$	Further pole-zero cancelation is used
SO-m ESO-m 2E-SO-m	See table 2.3		

Table 2.2 contains the idealized forms for the controllers. In some of the cases these forms are physically unrealizable, in this case small time constants are additionally introduced in the denominator.

Starting from the optimality condition (2.7), in [7] and [8] between the t.f. coefficients (2.5), (2.6) the following conditions have been imposed:

$$\beta a_0 a_2 = a_1^2 \quad , \quad \beta a_1 a_3 = a_2^2 \tag{2.9}$$

where β is a design parameter at the choice of the system designer. For the particular case of $\beta = 4$ one gets the SO-m tuning relations.

Table 2.3 synthesizes the main information regarding the two extensions of the SO-m, based on papers [7] and [8]. By applying the ESO-m or 2E-SO-m, more favourable closed system pole-zero placement, controlled through the β parameter is obtained. As a consequence, favourable polynomial forms for *T(s)* and *S(s)* are obtained (*T(s)* is the complementary sensitivity function, *S(s)* is the sensitivity function), used in the design phase. For β = 4, 9, 16 simple tuning relations are obtained. By increasing the value of β two important effects for the system are gained:

- increase of the system phase margin,
- favourable modification of the value of $M_{S0} = \max\{|S_0(j\omega_s)|\}$, of system robustness, which is an advantage in case of systems with variable parameters.

Table 2.3

Rel.	Descriptor	method	Relations
(a) (2.10)	$L_0(s)= C(s)P(s)$	E-SO	$\dfrac{(1+\beta T_\Sigma s)}{\beta^{3/2}T_\Sigma^2 s^2(1+sT_\Sigma)}$
(b)		2E-SO	$\dfrac{1+\beta T_{\Sigma m} s}{\beta^{3/2}T_\Sigma' \dfrac{m}{(1+m)^2} s(1+sT_1)(1+sT_\Sigma)}$
(a) (2.11)	$H_r(s)$ $(T_0(s))$	E-SO	$\dfrac{1+\beta T_\Sigma s}{\beta^{3/2}T_\Sigma^3 s^3+\beta^{3/2}T_\Sigma^2 s^2+\beta T_\Sigma s+1}$
(b)		2E-SO	$\dfrac{(1+\beta T_{\Sigma m} s)}{\beta^{3/2}T_\Sigma'^3 s^3+\beta^{3/2}T_\Sigma'^2 s^2+\beta T_\Sigma' s+1}$
(a) (2.12)	Poles and zero	ESO	$z_1=-\dfrac{1}{\beta T_\Sigma}$, $p_1=-\dfrac{1}{\beta^{1/2}T_\Sigma}$ $p_{2,3}=\dfrac{-(\beta-\beta^{1/2})T_\Sigma \pm\left[(\beta-\beta^{1/2})^2 T_\Sigma^2-4\beta T_\Sigma^2\right]^{1/2}}{2\beta T_\Sigma^2}$
(b)		2E-SO	$z_1=-\dfrac{1}{\beta T_{\Sigma m}}$, $p_1=-\dfrac{1}{\beta^{1/2}T_\Sigma}$ $p_{2,3}=\dfrac{-(\beta-\beta^{1/2})T_\Sigma' \pm\left[(\beta-\beta^{1/2})^2 T_\Sigma'^2-4\beta T_\Sigma'^2\right]^{1/2}}{2\beta T_\Sigma'^2}$
(a) (2.13)	$S_0(s)$	ESO	$\dfrac{\beta^{3/2}T_\Sigma^2 s^2(1+sT_\Sigma)}{1+\beta T_\Sigma s+\beta^{3/2}T_\Sigma^2 s^2+\beta^{3/2}T_\Sigma^3 s^3}$
(b)		2E-SO	$\dfrac{\beta^{3/2}T_\Sigma' \dfrac{m}{(1+m)^2} s(1+sT_1)(1+sT_\Sigma)}{\beta^{3/2}T_\Sigma'^3 s^3+\beta^{3/2}T_\Sigma'^2 s^2+\beta T_\Sigma' s+1}$
(a) (2.14)	Design relations	ESO	$k_c=\dfrac{1}{\beta^{3/2}k_p T_\Sigma^2}$ $T_c=\beta T_\Sigma$, $T_c'=T_2$
(b)		2E-SO	$k_c=\dfrac{(1+m)^2}{m}\dfrac{1}{\beta^{3/2}k_p T_\Sigma'}=\dfrac{(1+m)^3}{m}\dfrac{1}{\beta^{3/2}k_p T_\Sigma}$, $T_c=\beta T_{\Sigma m}$, $T_c'=T_2$

(a)	Suppl. relations	ESO	$\varphi_r = arctg(\beta T_\Sigma \omega_c) - arctg(T_\Sigma \omega_c)$
(2.15) (b)		2E-SO	$T_{\Sigma m} = T_\Sigma^{'} \frac{1+(2-\beta^{1/2})m+m^2}{(1+m)^2}$, $T_\Sigma = T_\Sigma^{'}(1+m)$ $\varphi_r = \pi/2 + arctg(\beta T_{\Sigma m}\omega_c) - arctg(T_1\omega_c) - arctg(T_\Sigma \omega_c)$

Here $m = \frac{T_\Sigma}{T_1}$. The β parameterization was introduced in paper [7], and the parameterization β and m in paper [8].

3 Youla - parameterization Design based on Results of ESO and 2E-SO Methods

In [9] it is presented that if $G(s)$ has a bounded rational form, with real coefficients, $|G(j\omega)| < \infty$, there exists a coprime factorization over the set of all bounded rational forms having real coefficients:

$$G(s) = \frac{N(s)}{M(s)} \quad \text{with} \quad G(s) \in \varphi \tag{3.1-a}$$

$$N(s)X(s) + M(s)Y(s) = 1 \quad \text{(Bezout's identity)} \tag{3.1-b}$$

where: $N(s), X(s), M(s), Y(s) \in \varphi$, and φ – set of all bounded rational forms having real coefficients. The all stabilizing controllers of the t.f. (3.1-a) can be specified as:

$$C(s) = \frac{X(s) + M(s)Q(s)}{Y(s) - N(s)Q(s)} \quad \text{where} \quad Q(s) \in \varphi \tag{3.2}$$

$Q(s)$ - a parameteric rational form. If *P(s)* is stable, the coprime factorization (3.1-a) can be particularized as:

$$N(s) = P(s) \quad , \quad M(s) = 1 \quad , \quad X(s) = 0 \quad , \quad Y(s) = 1 \quad . \tag{3.3}$$

Consequently, controller (3.2) is calculated with:

$$C(s) = \frac{Q(s)}{1 - P(s)Q(s)} \quad \text{or} \quad Q(s) = C(s)[1 - P(s)Q(s)] . \tag{3.4}$$

This can be represented as shown in figure 3.1. Further, the following relations are established:

$$y(s) = S(s)P(s)C(s)r(s) + S(s)d_1(s) + S(s)P(s)d_2(s) \tag{3.5}$$

where: $S(s) = \dfrac{1}{1+L(s)} = \dfrac{1}{1+C(s)P(s)}$ is the sensitivity function, (3.6-a)

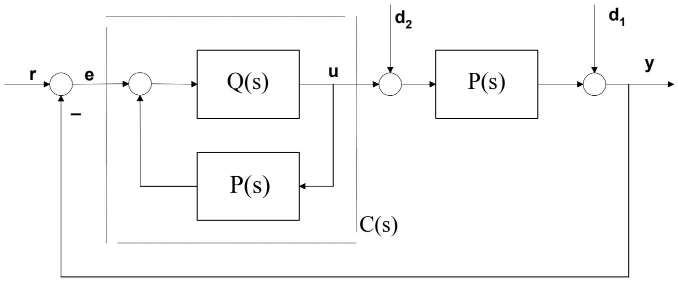

Figure 3.1
Youla-parameterization based control system diagram

$$T(s) = \frac{C(s)P(s)}{1+C(s)P(s)} = H_r(s)$$, the complementary sensitivity function, (3.6-b)

with the following connection:

$S(s)+T(s)=1$, $T(s)=1-S(s)$ $L(s)=C(s)P(s)$ the open loop transfer function, (3.7)

The design using Youla parameterization consists in establishing *Q(s)* so that well stated requirements are fulfilled for *S(s)* or *T(s).* From relation (3.4) results that *C(s)* depend only on *Q(s)* and *P(s).*

Remark: The design specifications can be also established for behaviour regarding the load disturbance d_2, using t.f. H_{d2}:

$$H_{d2}(s) = \frac{P(s)}{1+C(s)P(s)} \quad (3.6\text{-c})$$

The more restrictive these conditions are the more complicated the controller structure is.

Based on this, it results the Youla parameterization design principle.

If Q(s) is a stable t.f., then the Youla parameterization establishes the controller family C(s) that stabilises the P(s) plant.

As consequence, in controller design the following steps are made:

Step (1): For the given stable plant *P(s),* calculus of *C(s)*, having *Q(s)* as parameter. Calculus of *S(s)* and *T(s).*

Step (2): Establishing of a *Q(s)* through which the imposed performances for *S(s)* or

T(s) are ensured. Parametric forms for *Q(s)* are advantageous.

Step (3): Establishing the controller *C(s)* that fulfills the imposed requirements.

Step (4): Verification of desired performances, sensitivity analysis of the system.

The use of Youla-parameterization in case of ESO-m and 2E-SO-m is justified by the possibility of imposing favourable forms of the expressions of *S(s)* and *T(s)* that confers the system performances.

3.1 Youla Parameterization Design based on Results of MO-m

This case is presented as a first example of Youla parameterization design, exemplified only for t.f. (2.2-a), the other cases being solved similarly.

Step (1): For the given *P(s)*, calculus of *C(s)*, *S(s)* or *T(s)* having *Q(s)* as parameter results in form:

$$C(s) = Q(s)\frac{(1+sT_\Sigma)(1+sT_1)}{(1+sT_\Sigma)(1+sT_1)-k_pQ(s)} \tag{3.8}$$

$$S(s) = \frac{(1+sT_\Sigma)(1+sT_1)-k_pQ(s)}{(1+sT_\Sigma)(1+sT_1)} \quad \text{and} \quad T(s) = \frac{k_pQ(s)}{(1+sT_\Sigma)(1+sT_1)} \tag{3.9}$$

Step (2): Imposing a favourable *T(s)* in the specific form for MO-m*:*

$$T(s) = \frac{1}{1+2T_\Sigma s+2T_\Sigma^2 s^2} \quad \text{and} \quad Q(s) = \frac{1}{k_p}\frac{(1+sT_\Sigma)(1+sT_1)}{1+2T_\Sigma s+2T_\Sigma^2 s^2} \tag{3.10}$$

Step (3): Establish the controller *C(s)*. Replacing (3.10) into (3.8) results:

$$C(s) = \frac{1}{2k_pT_\Sigma s}(1+T_1 s) \quad \text{with parameters:} \quad k_c = \frac{1}{2k_pT_\Sigma}\ ,\ T_c = T_1\ . \tag{3.11}$$

Remarks: 1. The design requirement can be imposed in other forms as well. For example, if only steady-state error is required to be zero then:

$$S(s)\big|_{s=0} = 0 \quad \Leftrightarrow \quad \left.\frac{(1+sT_\Sigma)(1+sT_1)-k_pQ(s)}{(1+sT_\Sigma)(1+sT_1)}\right|_{s=0} = 0 \quad \text{and results: } Q(0) = \frac{1}{k_p}\ .$$

Substituting this into (3.8), a PID-controller is obtained, with complete pole-zero cancellation:

$$C(s) = \frac{Q(s)}{1-P(s)Q(s)} = \frac{1}{k_p(T_1+T_\Sigma)s}\frac{(1+sT_\Sigma)(1+sT_1)}{(1+s\frac{T_1T_\Sigma}{T_1+T_\Sigma})} \quad \text{with parameters:}$$

$$k_c = \frac{1}{k_p(T_1+T_\Sigma)}\ , \quad T_c = T_1\ , \quad T_c' = T_\Sigma \quad \text{and} \quad T_f = \frac{T_1T_\Sigma}{T_1+T_\Sigma}\ .$$

A detailed analysis of the system performances reveales that even if the system becomes faster, its global properties are not necessarily better.

2. The controller structure becomes more complicated if the design requirement reffers to a very restrictive t.f. $H_{d2}(s)$.

3.2 Youla Parameterization Design based on Results of ESO-m

The plant t.f. is (2.1) … (2.3) (b); the design is exemplified only for t.f. (2.1) (b), the second and third case being solved similarly.

Step (1): For the given *P(s)* the calculus of *C(s)*, *S(s)* and *T(s)* having *Q(s)* as parameter ensures:

$$C(s) = Q(s)\frac{s(1+sT_\Sigma)}{s(1+sT_\Sigma)-k_p Q(s)} \tag{3.12}$$

$$S(s) = \frac{s(1+sT_\Sigma)-k_p Q(s)}{s(1+sT_\Sigma)} \quad \text{and} \quad T(s) = \frac{k_p Q(s)}{s(1+sT_\Sigma)} \tag{3.13}$$

Step (2): The system performances are imposed through *T(s),* which is specific for ESO-m*:*

$$T(s) = \frac{1+\beta T_\Sigma}{1+\beta T_\Sigma s+\beta^{3/2}T_\Sigma{}^2 s^2+\beta^{3/2}T_\Sigma{}^3 s^3} \quad \text{and}$$

$$Q(s) = \frac{1}{k_p}\frac{(1+\beta T_\Sigma)s(1+sT_\Sigma)}{1+\beta T_\Sigma s+\beta^{3/2}T_\Sigma{}^2 s^2+\beta^{3/2}T_\Sigma{}^3 s^3}\,. \tag{3.14}$$

Step (3): Establish the controller *C(s)*; replacing into (3.10) in (3.8) results:

$$C(s) = \frac{1}{k_p}\frac{s(1+sT_\Sigma)}{\beta^{3/2}T_\Sigma{}^2 s^2(1+sT_\Sigma)}(1+\beta T_\Sigma s) = \frac{1}{\beta^{3/2}k_p T_\Sigma{}^2 s}(1+\beta T_\Sigma s)\ , \tag{3.15}$$

where the controller parameters are:

$$k_c = \frac{1}{\beta^{3/2}k_p T_\Sigma{}^2} \quad T_c = \beta T_\Sigma\,. \tag{3.16}$$

3.3 Youla Parameterization Design based on Results of 2E-SO-m

The plant t.f. is (2.1) … (2.3) (a); the design steps mentioned at point 3.1 are made. The design is exemplified only for t.f. (2.1) (a), the second and third case being solved similarly. It must be remarked that the parameterization introduced by relation (2.9) must be imposed already at the stage when *T(s)* is fixed.

Step (1): For the given *P(s)* the calculus of *C(s)*, *S(s)* and *T(s)* having *Q(s)* as parameter ensures:

$$C(s) = Q(s)\frac{(1+sT_\Sigma)(1+sT_1)}{(1+sT_\Sigma)(1+sT_1)-k_pQ(s)} \tag{3.17}$$

$$S(s) = \frac{(1+sT_1)(1+sT_\Sigma)-k_pQ(s)}{(1+sT_1)(1+sT_\Sigma)} \quad \text{and} \quad T(s) = \frac{k_pQ(s)}{(1+sT_1)(1+sT_\Sigma)} \ . \tag{3.18}$$

Step (2): The expression (3.18) is used:

$$Q(s) = \frac{1}{k_p}T(s)(1+sT_1)(1+sT_\Sigma) \ . \tag{3.19}$$

Taking into account the need for a controller *C(s)* as simple as possible, the system performances must be imposed through a *T(s)* (or S(s) or $H_{d2}(s)$) with specific form. For example, in this case, the form of a proportional-derivative-with 3rd order lag (PDL[3]) model. It is the case of 2E-SO-m, where the use of a PI (or PID or PID[2]) controller is imposed. Correspondingly:

$$T(s) = \frac{b_1s+1}{a_3s^3+a_2s^2+a_1s+1} \tag{3.20}$$

where

$$a_3 = \frac{T_\Sigma T_1}{k_pk_c}, \quad a_2 = \frac{T_\Sigma + T_1}{k_pk_c}, \quad a_1 = \frac{1+k_pk_cT_c}{k_pk_c}, \quad a_0 = 1 \quad , b_1 = T_c \tag{3.21}$$

Step (3): Establish a controller *C(s)* (a PI form for simplicity) which fulfills imposed requirements through a desired form of *T(s);* this form must satisfy conditions (2.9). Replacing into (3.19) relation (3.20), results:

$$Q(s) = \frac{1}{k_p}\frac{(1+sT_1)(1+sT_\Sigma)(1+sT_c)}{a_3s^3+a_2s^2+a_1s+1} \ . \tag{3.22}$$

Replacing (3.22) into (3.17), *C(s)* results as:

$$C(s) = \frac{1}{k_p}\frac{(1+sT_1)(1+sT_\Sigma)(1+sT_c)}{(a_1-T_c)s[\frac{a_3}{a_1-T_c}s^2+\frac{a_2}{a_1-T_c}s+1]} \ . \tag{3.23}$$

If conditions (2.9) are imposed on (3.21), it results:

$$\beta^{1/2}k_ck_p(T_1+T_\Sigma) = (1+k_ck_pT_c)^2 \ , \quad \beta^{1/2}(1+k_ck_pT_c)T_1T_\Sigma = (T_1+T_\Sigma)^2 \tag{3.24}$$

Further on, noting with $m = T_\Sigma / T_1$, after successive replacements one gets

$$a_3 = \frac{T_\Sigma^3}{(1+m)^3}\beta^{3/2} = T_\Sigma^{'3}\beta^{3/2},\ a_2 = \frac{T_\Sigma^2}{(1+m)^2}\beta^{3/2} = T_\Sigma^{'2}\beta^{3/2},\ a_1 = T_\Sigma^{'}\beta,\ a_0 = 1 \tag{3.25}$$

Further replacements lead to the controller parameters:

$$k_c = \frac{(1+m)^2}{m}\frac{1}{\beta^{3/2}k_p T_\Sigma^{'}} \quad \text{and} \quad T_c = \beta T_{\Sigma m} = \beta T_\Sigma^{'}\frac{[1+(2-\beta^{1/2})+m^2]}{(1+m)^2}\ \mathrm{T} \tag{3.26}$$

and the expressions of *T(s)* and *S(s)* according to 2E-SO-m are obtained:

$$T(s) = \frac{(1+\beta T_{\Sigma m}s)}{\beta^{3/2}T_\Sigma^{'3}s^3 + \beta^{3/2}T_\Sigma^{'2}s^2 + \beta T_\Sigma^{'}s + 1},\ Q(s) = \frac{1}{k_p}\frac{(1+sT_1)(1+sT_\Sigma)(1+\beta T_{\Sigma m}s)}{\beta^{3/2}T_\Sigma^{'3}s^3 + \beta^{3/2}T_\Sigma^{'2}s^2 + \beta T_\Sigma^{'}s + 1} \tag{3.27}$$

For the second and third case only a previously performed pole-zero cancellation yields to the same result.

So, for on Youla parameterization based design, the following choice can be made:

- Place the zero $z_3 = -\frac{1}{\beta T_{\Sigma m}}$ as a function of the values of {*β*, T_Σ, *m* };
- The poles of the characteristic equation

$$\Delta(s) = \beta^{3/2}T_\Sigma^{'3}s^3 + \beta^{3/2}T_\Sigma^{'2}s^2 + \beta T_\Sigma^{'}s + 1 \tag{3.28}$$

  can be chosen as function of {*β*, T_Σ, *m*}.

In this context the poles' placement given by (3.28) leads to a *Q(s)* of form (3.27), and finally in step (3), the controller's t.f. can be expressed:

$$C(s) = \frac{1}{k_p}\frac{s(1+sT_\Sigma)}{\beta^{3/2}T_\Sigma^{2}s^2(1+sT_\Sigma)}(1+\beta T_{\Sigma m}s) = \frac{1}{\beta^{3/2}k_p T_\Sigma^{2}s}(1+\beta T_{\Sigma m}s)\ , \tag{3.29}$$

3.4 Final Remarks

The main task in case of controller design based on the Youla-parameterization consists in fixing some rational conditions through t.f. *T(s), S(s)* or $H_{d2}(s)$. This choice influences the form of *Q(s)* and of controller t.f. *C(s)*. If these forms are unproperly chosen – too simple or too restrictive – the controller that stabilizes the class of plants *P(s)* becomes too complicated, often unsuitable for handling the control task. A quasi-continuously (QC) operating implementation of the control solution can be easily performed. In this case as well the presence of integral component can lead to a need for the Anti-Windup Reset (AWR) measure.

Conclusions

In the paper an interpretation of the Youla-parameterization design is presented, for controller design for two special cases of electrical driving systems characterized by benchmark type stable models *P(s)*.

As a difference from classical design methods which impose certain controller types and structures, in the Youla-parameterization based design there are certain polynomials relatively to the t.f. imposed which caracterize the properties of the system.

The paper presents in detail the way of transposing the positive results gained from classical design methods based on modulus conditions (MO-m, SO-m) or conditions derived from these (ESO-m si 2E-SO-m) into a Youla-parameterization formulation. If the imposed conditions are adequately chosen, the controller is easy to implement. If the conditions are inadequate then the controller structure results as more difficult to comprehend, such solutions are less accepted in the practice by engineers.

In case of non-minimum phase systems or unstable systems, the inconvenience of the method consists in the fact that the resulting controller is complicated. For these plants the design can only be solved by the general formulation of the coprime factorization.

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