# Geometric Theory and Control of Linear Parameter Varying Systems

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*Abstract: Linear Parameter Varying(LPV) systems appear in a form of LTI state space representations where the elements of the $A(\rho)$, $B(\rho)$, $C(\rho)$ matrices can depend on an unknown but at any time instant measurable vector parameter $\rho \in P$. This parameter can be a function of time or state variables. In the latter case we speak about quasi LPV (qLPV) systems. In some cases the parameters represent constant but unknown uncertainties or an external time signals. These properties show relations to the theory of uncertain systems with parametric uncertainties and to the theory of LTV systems, too. It can also be shown that many input affine nonlinear system representations and swiched linear systems can be rewritten also in the form of qLPV systems. These system representations appear in many modeling and control problems related to aerospace or vehicle system applications.*

*The application of LPV system representations appeared in relation to aerospace control and it represents a systematic approach to gain scheduling control for nonlinear systems (Shamma and Athans, 1991). Passivity and $H\infty$ theory has been extended to design robust controllers for LPV systems, see e.g. Lim and Ho (2002), Becker et al. (1993) and Becker and Packard (1994).*

*Moreover, the study of LPV systems provides additional insights into some longstanding and sophisticated problems in robust adaptive control, see Athans et. al. (2005) switching control systems, see Hespanha et. Al. (2003) and in intelligent control, see Feng and Ma (2001), Ravindranathan and Leitch(1999), Baranyi (2004).*

*This talk will propose a geometric view on the LPV systems. The geometric approach to dynamic systems appeared e.g. in Basile and Marro (1969), Wonham (1985) for LTI systems and in Isidori (1989) for input affine nonlinear systems where a central role is played by invariant subspaces like (A,B), (C,A) or unobservabilty subspaces and related algortithms like (C,A) - invariant subspace algorithm (CAISA), unobservibility subspace algorithm (UOSA) or their corresponding nonlinear versions using vector space distributions and codistributions.*

*Elaborating the geometric concepts and tools of parameter varying invariant subspaces, this paper investigates invariant subspace algorithms for a class of LPV systems where the $A(\rho)$, $B(\rho)$, $C(\rho)$ matrices are affine in $\rho$.*

*Fundamental concepts like controllability, observability of LPV, qLPV systems and related problems will be studied. It will be shown that a generalized Kalman-rank condition can be given as a necessary condition, but there are also conditions on the $\rho$ functions indicating*

*that their choice in describing nonlinear systems in LPV form can influence the above properties of the resulting model.*
*Using the geometric results and the associated invariant subspace algorithms, prototype control problems like disturbance decoupling problem (DDP), DDP with stability (Bokor et. al., 2002), dynamic decoupling, system inversion (Szabó et. al., 2003) and filter designs (Bokor and Balas, 2004) will be discussed for affine LPV systems using algorithms like UOSA, CAISA algorithms defined for LTI and input affine nonlinear systems. The advantage gained by using LPV formalism is that the solutions can be given in terms of linear algebraic manipulations like those elaborated for LTI systems in (Basile and Marro, 1969). This feature allows us to obtain solutions to some nonlinear problems rewritten into qLPV forms that could be hardly computable in the original nonlinear form.*
*Applications to aerospace control design and road vehicle control systems will be shown using MATLAB codes developed on the basis of geometric concepts elaborated for LPV applications and codes developed for LPV extensions of state space H∞ controller designs.*

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