Error Analysis and Correction of a High Precision Co-ordinate Table

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Abstract: A small scale, high precision co-ordinate table was developed as part of a project to solve measurement problems originating from microtechnic. Construction with a single reference surface was selected and vacuum preloaded air bearing applied in order to enable smooth motion and eliminate stick slip. Laser interferometers were applied to measure both the displacement and the angular error. The paper describes the mathematical evaluation of the measuring system's error and how that is taken into account in the evaluation software.

Keywords: Co-ordinate table, measuring system, evaluation software, error computation

1 Introduction

Co-ordinate metrology is today a firmly established technique in industry. The universal applicability and high degree of automation accounts for it's succes. Two other factors are equally important: tolerances are becomming closer as a result of interchangeable manufacturing and the increase of subcontracting is obliging manufacturers to assume responsibility for the dimensional integrity of their parts. With the appearance of micromachined mechanical elements increased demand can be observed for higher resolution and more accurate inspection of these parts.

2 The Table Construction

For many reasons ultra precise X-Y tables are guided by a flat reference surface. These tables usually employ vacuum or magnetically preloaded air bearings. Their advantage is that only one flat guide surface is necessary, whereas the opposed bearing preloading requires two flat guide surfaces that are parallel. Other advantages of the air bearings are the zero static friction, which makes infinite resolution and the very high repeatability possible. Moreover porous air bearings average the errors of the guide surface finish and irregularities.

In our case, the table has four legs and rests on four vacuum preloaded flat porous air bearings. The porous carbon in the bearing produces a uniform air pressure across the face of the bearing while automatically restricting and damping the air flow at the same time. As a consequence the possibility of collapse is reduced and a higher pitch moment stiffness is reached.

As a reference surface a 000 class granite[1] control surface plate, having an overall flatness of less then 2 μm, is used and the axes are driven by linear motors.

The window in the moving part of the table enables the installation of a back lighting LED panel.

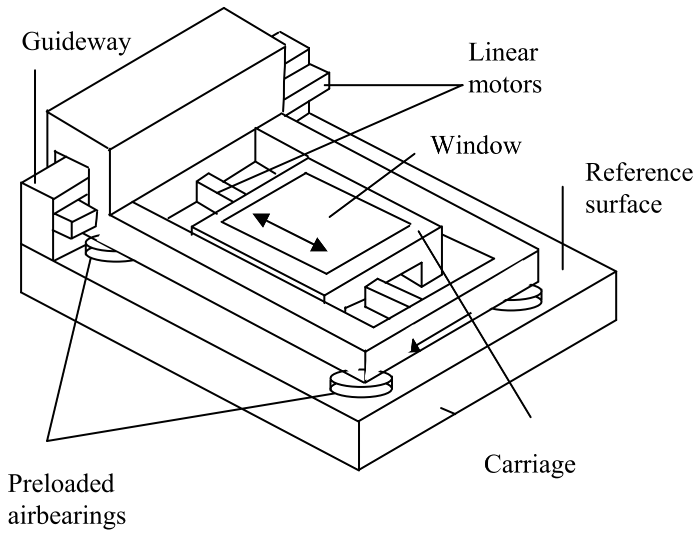

Figure 1

The mechanical construction

3 The Measuring System

The table position is determined by two plane mirror laser interferometers, a single- and a dualbeam, relative to the measuring frame fixed to the granite base. The laser sources are coupled to the sensor heads via fiberoptic cables.

The single beam interferometer enables a displacement measurement along one axis with a resolution of 1 nm (under ideal environmental conditions) while the dualbeam interferometer allows the simultaneous displacement and angular measurement. The angular resolution is approximately 0.05 arcsec. The measuring range is 100 mm and ±2 arcmin respectively.

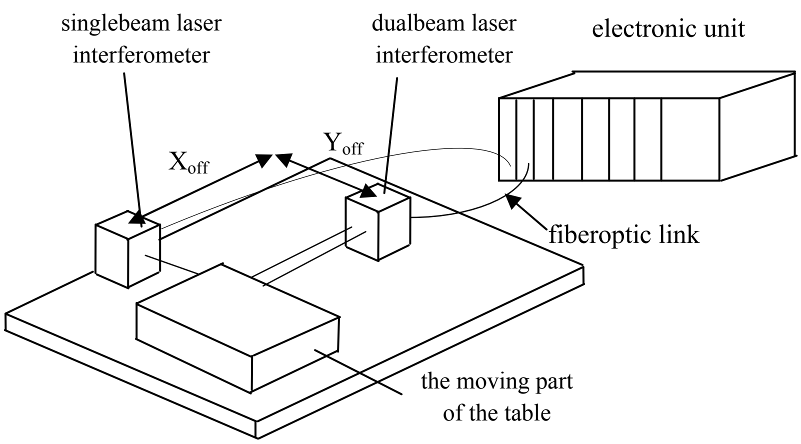

Figure 2
The outline of the measuring system

4 The Geometric Model of the Measuring System

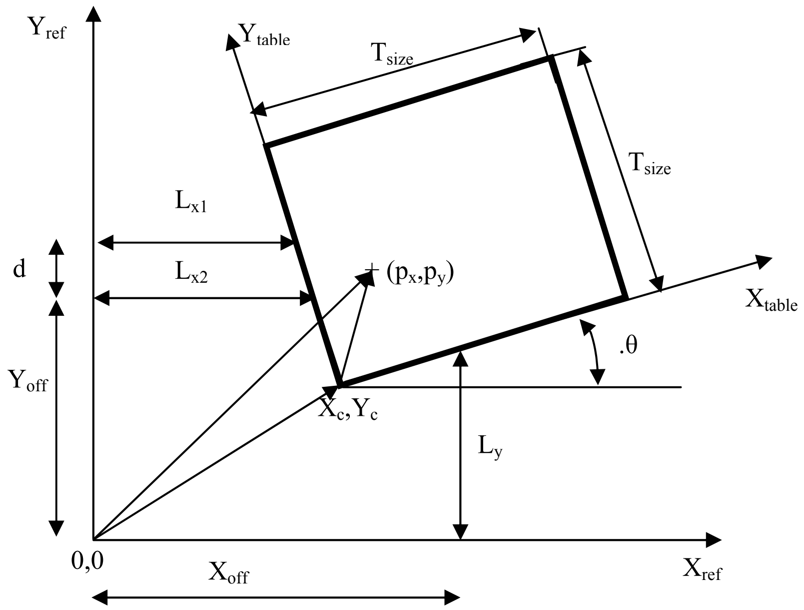

Figure 3

The geometric model

Where:

- X_{ref} and Y_{ref} are X the Y axis of the measuring frame co-ordinate system
- X_{table} and Y_{table} are X the Y axis of the table co-ordinate system
- X_c and Y_c are the co-ordinates of the table coordinate system's origo
- X_{off} and Y_{off} are the positions of the interferometers along the measuring frame axis
- d is the separation between the two beams in the dualbeam interferometer
- L_{x1}, L_{x2} and L_y are the displacements measured by the interferometer
- T_{size} is the table size

Let us in the first instance assume that the roll and pitch error of the table is negligable. From Fig. 3 the following two equation for the origo of the table co-ordinate system axis can be seen:

$$\frac{X_c - L_{x2}}{Y_{off} - Y_c} = tg\theta \tag{1}$$

$$\frac{L_y - Y_c}{X_{off} - X_c} = \mathrm{tg}\theta \tag{2}$$

Where

$$tg\theta = \frac{L_{x1}-L_{x2}}{d} \tag{3}$$

Hereout follows for the co-ordinates of the origo:

$$X_c = \frac{(X_{off} - L_{x2})tg^2\theta}{1+tg^2\theta} + \frac{(Y_{off} - L_y)tg\theta}{1+tg^2\theta} + L_{x2} \tag{4}$$

$$Y_c = \frac{Y_{off} \cdot tg^2\theta}{1+tg^2\theta} + \frac{tg\theta(L_{x2} - X_{off})}{1+tg^2\theta} + \frac{Ly}{1+tg^2\theta} \tag{5}$$

The co-ordinates of an arbitrary point on the table can be expressed by using the following equation:

$$\begin{pmatrix} P_x \\ P_y \end{pmatrix} = \begin{pmatrix} \cos & -\sin \\ \sin & \cos \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} X_c \\ Y_c \end{pmatrix} \Longrightarrow \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} \begin{pmatrix} P_x - X_c \\ P_y - Y_c \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \tag{6}$$

5 The Error Model

In the above outlined measuring system we have to take into account the following error sources:

- displacement measurement error as a result of wavelength variation
- angle measurement error
- perpendicularity error of the laserbeams and the reflecting mirrors
- flatness error of the mirrors
- reference surface flatness error resulting in role and pitch error on the table

Let us consider these error components one by one:

The displacement of the reflecting mirror is given by the following equation:

$$D_x = \frac{c}{2f}\left(\mathrm{N} + \frac{\varphi}{2\pi}\right) \tag{7}$$

Where:

- c is the speed of light
- f is the laser frequency

- N is the number 2πs and
- Φ is the phase angle

The laser frequency stability, after warmup is better than 3 x 10^{-7}. The speed of light is influenced by the temperature, the humidity, the turbulence and the contamination of the air. The frequency of the laser can be stabilized and the effect of temperature and humidity can be compensated digitaly by measuring both air and object temperature and the humidity.

The error equation for the angle is:

$$\Delta tg\theta \leq \frac{\Delta(L_{x2}-L_{x1})}{d} + \frac{\Delta d}{d} tg\theta \qquad (8)$$

Hereout follows that the angular error is determined by the relative error of the beamdistance of the dualbeam interferometer.

The deviation from the prependicularity of the reflecting mirrors and that of the directions of the interferometers contribute to the overall error of the system. Also the perpendicularity error of the reference system (X_{ref},Y_{ref}) has a similar effect.

The angle between the mirrors is φ, whereas the angles between the reference axis and the axis of the interferometers are given by α and β respectively.

From Fig. 4 it can be immediatly seen that:

$$\Delta = 90° - \varphi = \theta - \psi \qquad (9)$$

Where $\Delta = 90° - \varphi$ is the perpendicularity error of the table (the reflecting mirrors)

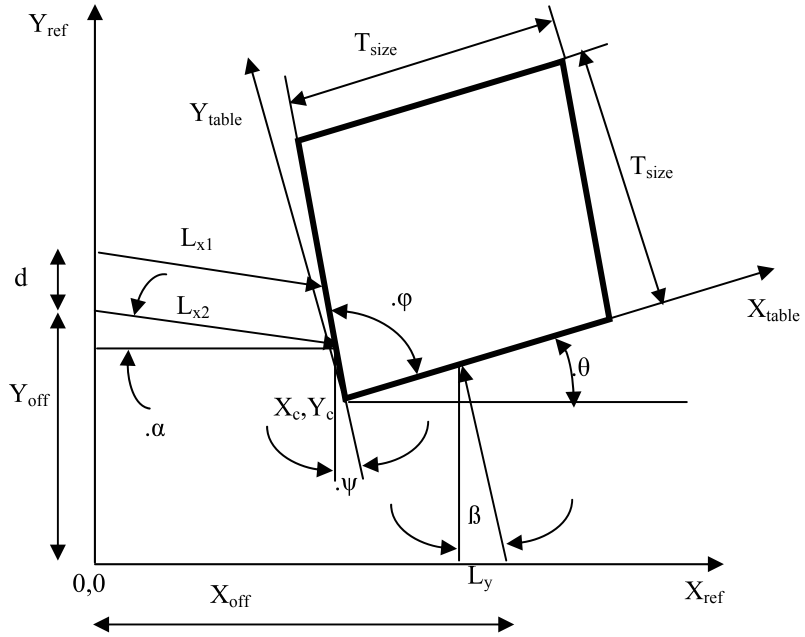

Figure 4
Perpendicularity error model

$$\frac{X_c - L_{x2}\cos\alpha}{Y_{off} - L_{x2}\sin\alpha - Y_c} = \mathrm{tg}\ \psi \tag{10}$$

$$\frac{L_y\cos\beta - Y_c}{X_{off} - L_y\sin\beta - X_c} = \mathrm{tg}\ \theta \tag{11}$$

$$\frac{(L_{x2} - L_{x1})\cos\alpha}{d + (L_{x2} - L_{x1})\sin\alpha} = \mathrm{tg}\ \psi \tag{12}$$

From hereout follows by substituting

$\sin\alpha = \alpha$, $\cos\alpha = 1$, $\sin\beta = \beta$, $\cos\beta = 1$,

and neglecting the higher order small terms we get:

$$tg\ \theta_{meas} = \frac{L_{x2}-L_{x1}}{d} \quad \text{and} \quad tg\ \theta = \frac{tg(90^o-\varphi) + tg\psi}{1 - \mathrm{tg}(90^o-\varphi)\mathrm{tg}\psi}$$

$$\theta = 90° - (\varphi + \psi) = \Delta - \mathrm{arctg}\frac{(L_{x2}-L_{x1})\cos\alpha}{d + (L_{x2}-L_{x1})\sin\alpha} = \Delta - \frac{\mathrm{arctg\ tg}\theta_{meas}\cos\alpha}{1+\mathrm{tg}\theta_{meas}\sin\alpha} \tag{13}$$

$$X_c = \frac{L_{x2}}{1+\alpha\mathrm{tg}\theta_{meas}} + \frac{tg\theta_{meas}\,(\Delta+tg\theta_{meas})(X_{off} - L_{x2})}{1+(3\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas}} - \frac{(1+2\alpha\mathrm{tg}\theta_{meas})\mathrm{tg}\theta_{meas}\,L_y}{1+(3\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas}} +$$

$$+ \frac{\alpha tg^3\theta_{meas}\,X_{off}}{1+(3\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas}} - \frac{(1+(2\alpha+\Delta)\mathrm{tg}\theta_{meas}-\Delta\mathrm{tg}\theta_{meas})\mathrm{tg}\theta_{meas}\,Y_{off}}{1+(3\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas}} \tag{14}$$

$$Y_c = \frac{(1+2\alpha \mathrm{tg}\theta_{meas})\, L_y}{(\mathit{1}+(2\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas})} + \frac{(\Delta+\mathrm{tg}\theta_{meas})(L_{x2} - X_{off})}{(\mathit{1}+(2\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas})} +$$

$$+ \frac{\alpha \mathrm{tg}^2\theta_{meas} X_{off}}{(\mathit{1}+(2\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas})} + \frac{\mathrm{tg}\theta_{meas}\,(\Delta+\mathrm{tg}\theta_{meas})\, Y_{off}}{(\mathit{1}+(2\alpha+\Delta)\mathrm{tg}\theta_{meas}+\mathrm{tg}^2\theta_{meas})} \tag{15}$$

Using these results the co-ordinates of a point in the table co-ordinate system can be expresed using equation [6]. The geometric constant values Δ, α and β used in the computation can be measured by conventional optical techniques.

If we consider the laserbeam as ray with nearly zero diameter then the flatness error of the mirrors results in an error of the displacement measurement. However the finite diameter of the beam has an integration property, so the local flatness error can be compensated to a certain extend. Waveness errors larger then the beam diameter have to be taken into account. A map of the mirror surface is the necessary and sufficient information for the compensation of this type of errors.

The map can be captured by interferometric techniques. The applied technologies in manufacturing optical surfaces ensure that the functions describing the surface is continoues into the second derivatives. Therefore a spline model of the surface is space and time saving.

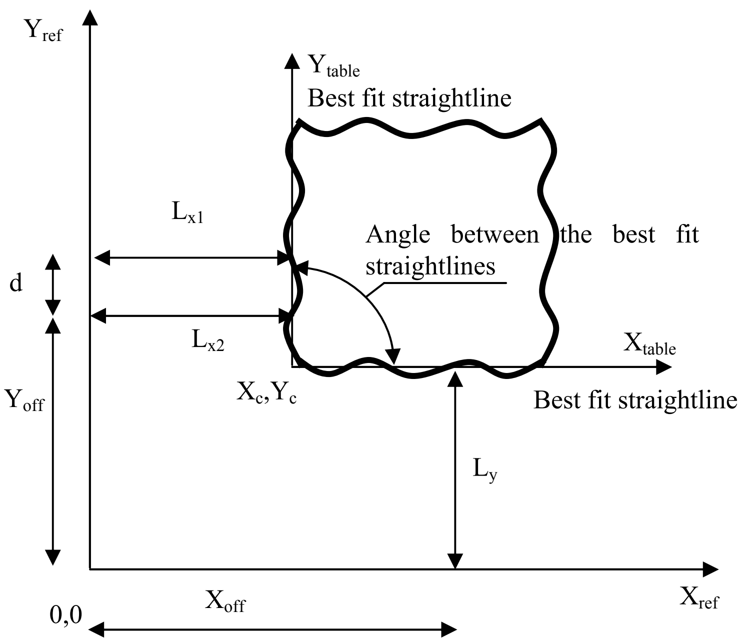

Figure 5

Model of the straightness (flatness) error

6 The Software Implementation

The interferometer delivers through USB ports two current co-ordinate and a current angle value. These signals are connected to a personal computer responsible for the calculations. As high presion measurements are connected with relavtively slow displacements the computational time is not a critical factor and as we have seen the amount of data involved is also small. The compensation for the speed of light deviation is performed by the electronic unit of the interferometer. There is no need for any additional hardware. The program performing the above mentioned calculations is written in C++ and runs on the Window XP operating system. Based on an analysis of the formulas at some point multilenght arithmetic is used in order to ensure the required accuracy. The measurement result wil be published in separate paper.

Conclusions

The research a part of which presented here proved the usefulness of software error compensation in co-ordinate measurement. According to our expectation combined with new construction the accuracy can be improved with an order of magnitude. The initial results are promising and the technique can extended into the third dimension.

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