

# Volumetric Error Correction in Coordinate Measurement

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*Abstract: The paper presents a general error model for coordinate measuring machine is given. To demonstrate the generalaty of the mode lit is applied to a coordinate measuring machine. Description of the calibration for determining the error map of the measurin volume is given. In order obtain continouosdescription of the error a regression spline volume is fitted to the discrete values.*

*Keywords: error model, coordinate measuring machine, calibration, data fitting, regression splines*

## 1 Introduction

Substantial work has been carried out in the last years on the development of error models for geometric, kinematic, and thermal errors. However, most of the earlier applied analytic geometry, vector representation and error matrices. More recent works relied on the use of rigid-body kinematics in connection with with homogenous transformation matrices [5, 9, 10]. Although, in principle, all these techniques could yield a model for a given machine, only the latter has the potential to facilitate a simple error model formulation for an arbitrary machine configuration.

In this paper the general kinematic error model will applied to a coordinate measuring machine and an algorithm for determining the parameters for calculating the eror description functions will be given.

## 2 General Kinematic Error Model

A linear stage of precision machinery is expected to travel along a straight line and stop at a predefined position. However in the practiced the actual path deviate

from the straight line due to the geometric errors of the guideways and it results also in angular errors as it is given in Fig. 1.

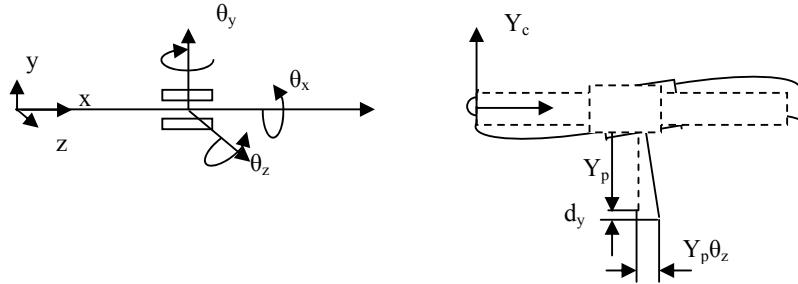


Figure 1  
Representation of the error components of a linear stage (abstract and actual carriage)

For each axis a transformation matrix can be used to describe in homogenous coordinates the deviations from the ideal motion. The general form of a transformation is given by:

$$dT(x, \phi, \theta, \psi) = \begin{bmatrix} c(\theta_y)c(\theta_z) & -c(\theta_y)s(\theta_z) & s(\theta_y) & d_x \\ c(\theta_x)s(\theta_z) + s(\theta_x)s(\theta_y)s(\theta_z) & c(\theta_x)c(\theta_z) - s(\theta_x)s(\theta_y)s(\theta_z) & -s(\theta_x)c(\theta_y) & d_y \\ s(\theta_x)s(\theta_z) - c(\theta_x)s(\theta_y)c(\theta_z) & s(\theta_x)c(\theta_z) - c(\theta_x)s(\theta_y)s(\theta_z) & c(\theta_x)c(\theta_y) & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where  $d_x$ ,  $d_y$  and  $d_z$  are the translational and  $\Phi$ ,  $\theta$  and  $\Psi$  are the rotational components and  $s$  respectively  $c$  are short for  $\sin$  and  $\cos$ .

In case of the coordinate table the angular errors are very small, and all the errors are position dependent the following approximation can be made:

$$dT(x, \phi, \theta, \psi) = \begin{bmatrix} 1 & -\theta_z(x) & \theta_z(x) & d_x(x) \\ \theta_z(x) & 1 & -\theta_x(x) & d_y(x) \\ -\theta_y(x) & \theta_x(x) & 1 & d_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analog results can be derived for the  $y$  and  $z$  axis. Note that only the translational kinematic components are described in the form of homogenous transformations, but the rotational components can be presented on a similar way. These components are called guided element. The guided elements are linked by so-called connecting elements, which can be represented by matrices with similar structure with only constant elements. The squariness or perpendicularity error can be represented by a so-called shear matrix of the following form, dependent on which axis is taken as a reference. Here the  $z$  axis was taken as a reference.

$$dT(x) = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 1 & 1 & sh_y & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resulting error matrix can be obtained by multiplying the individual matrices in the sequence as they follow each other in the kinematic chain.

A traditional co-ordinate measuring machine consists of three translational components x, y and z, and a probe is attached to the end of the z component. Usually the probe can be considered as a constant translational transformation.

Performing distance measurement in the measuring volume of the co-ordinate measuring machine we observe the relative change of the probe tip, therefore only the relative deviations are of interest.

### 3 Outline of the Investigated Construction

The conventional practice of stacking linear guided systems to form an X-Y table has inherent errors which are difficult to compensate. For many reasons ultra precise X-Y tables are guided by a flat reference surface. These tables usually employ vacuum or magnetically preloaded air bearings. The advantage of preloaded air bearings is that only one flat guide surface is necessary, whereas the opposed bearing preloading requires two flat guide surfaces that are parallel. Other advantages of the air-bearing are the zero static friction which makes infinite resolution and the very high repeatability possible. Contrary to rolling element bearings air bearings average the errors of the guide surface finish and irregularities.

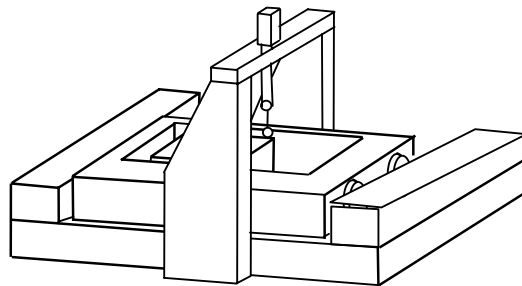


Figure 2

The construction of the CMM to be investigated

The carriage consists of two nested tables; each of them has four legs which in turn rests on vacuum preloaded air bearings. The reference plate is lapped to an accuracy of  $0,5 \mu\text{m}$ . The axes are driven by piezomotors. The position are determined by an incremental two co-ordinate optoelectronic measuring system, having  $0,05 \mu\text{m}$  resolution, attached to the bottom face of the inner table. This results in a light weight construction, which in turn ensures fast (acceleration up to  $20 \text{ m/s}^2$ ) and accurate positioning (less than  $1 \mu\text{m}$ ).

The probe is attached to a pinole running in an air bushing driven by a piezo motor, with approximately  $5 \text{ nm}$  resolution. The displacement is measured by a linear optical scale.

## 4 Kinematic Error Model of the CMM

### 4.1 The Carriage

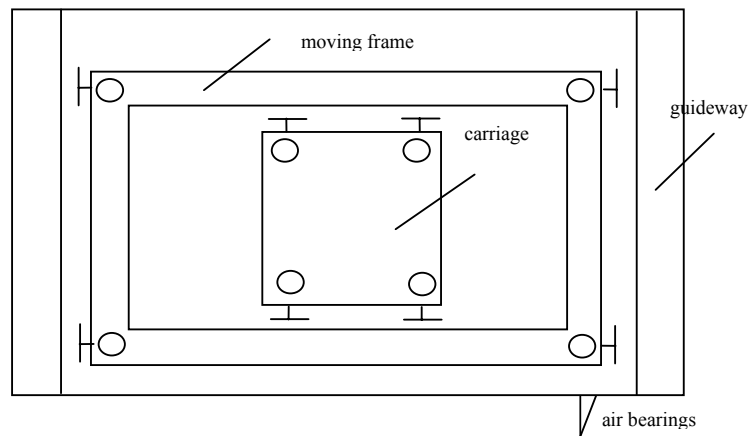


Figure 3  
Schema of the carriage

The error motions of the carriage are determined by the surface flatness of the granite reference surface. The vacuum preloaded air bearings average the errors of the guide surface finish and irregularities. The reference surface determines the roll and pitch error of the carriages. The straightness of the guideways determine the yaw error. However because the carriage is supported on both side by air-bearings the errors compensate each other to a certain extend.

The reading of the measuring system in the x- and y-directions does not exactly represent the relative axial displacement. Due to pitch variations between the dots and the electronic interpolation between these dots on the scale, a position dependent error is caused. The error of the two coordinate measuring system are included in the translational terms.

The soul squariness or perpendicularity error in the horizontal (x,y) plane is caused by the the perpendicularity error between the dot-line in the x and y directions. Usually this can be neglected.

## **4.2 The Pinole**

The pinole itself can be considered as a linear stage load with most of its errors. Only yaw do not introduce any error due to the spherical tip of the probe. This can be represented by the appropriate matrix. The errors of the measuring scale is a function of z displacement and is represented by the term  $T_z(z)$ .

# **5 Calibration**

To obtain the desired data for software error correction the carriage and the pinole are calibrated individually.

## **5.1 Determination of the Carriage Errors**

The errors of errors of the carriage are determined by using a single and a dual beam laser interferometer and three capacitive sensors as it is given in the subsequent figure. Hereby the six degrees of freedom of the carriage are taken away and the components of the position dependent error matrix can be computed in measuring point. The measuring points form a grid, with equal distance between the points along the main coordinate directions.

## **5.2 Determination of the Pinole Errors**

To determine the error of the measuring system, a single beam laser interferometer is used. The angular errors are measured by two pairs of capacitive sensors.

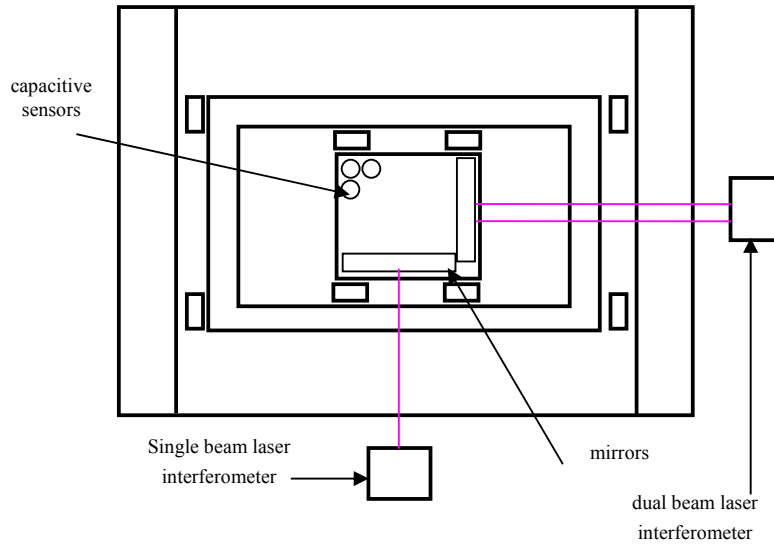


Figure 4  
Calibration setup for the carriage

## 6 Software Error Correction

B spline tensor surfaces play an important role in computer aided surface design. A B-spline surface can be written as:

$$S(t, u) = \sum_i \sum_j d_{ij} N_i^3(t) N_j^3(u)$$

where it was assumed that one knot sequence is along u-line while the other one in the t direction. This description can be extended to volumes:

$$V(t, u, v) = \sum_i \sum_j \sum_k d_{ij} N_i^3(t) N_j^3(u) N_k^3(v)$$

If we consider the values  $V$  as the error vector and the parameters  $t$ ,  $u$  and  $v$  as the coordinates of the ideal position then by determining the weights a continuous volumetric representation of the resulting errors can be obtained. The weights are computed using a least square procedure.

### Conclusions

In the paper the application of volumetric error correction method is outlined for a specific coordinate measuring machine. The simultaneous determination of the error components of the carriage and the pinole was given. This measurement setup seems to be practical relieving us from some calculations and at the same time it includes the effect of gravitational and spring forces into the model. A novell technique for derivating the continuous parametric error description from the measured discrete values was introduced. After running the first tests this technique seems to promising.

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