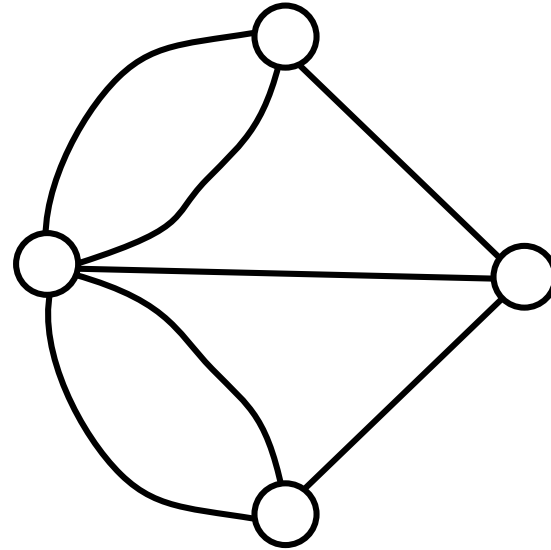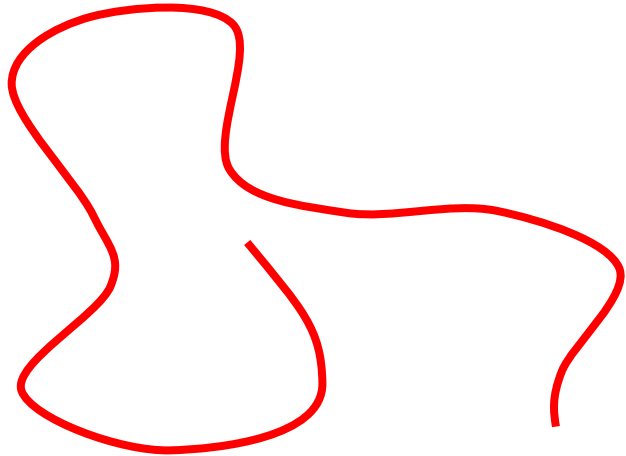# Graph theory
from puzzles to a new paradigm

LÁSZLÓ LOVÁSZ

Eötvös Loránd University
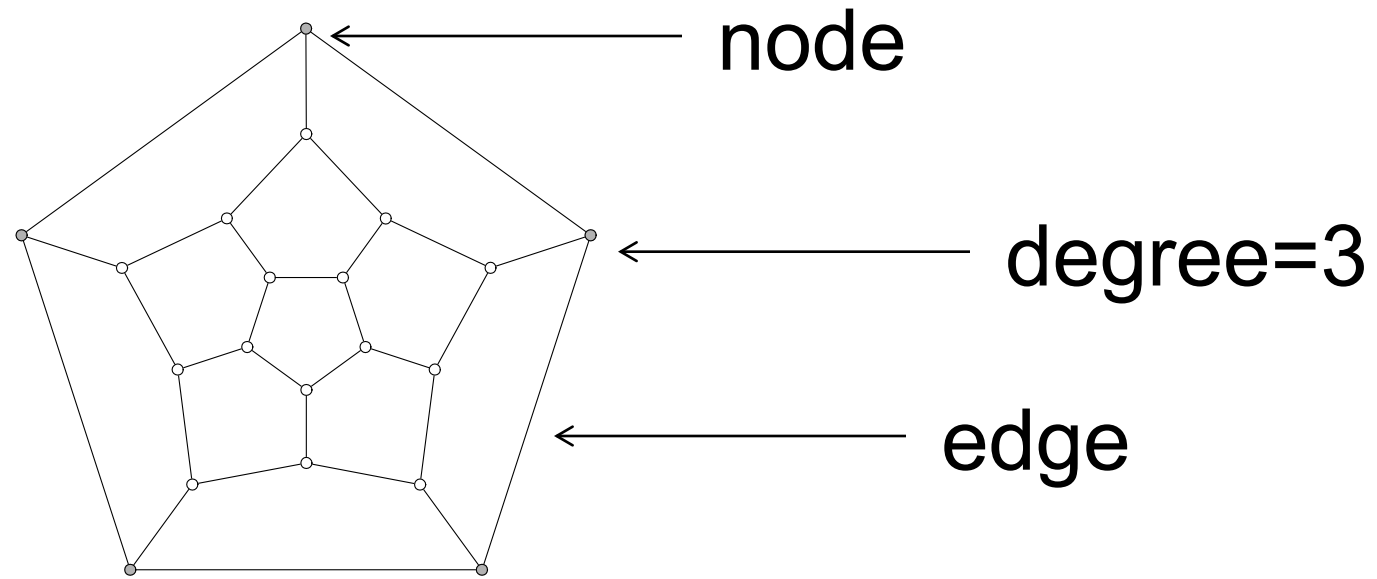
Alfréd Rényi Institute of Mathematics

Königsberg bridges

Leonard Euler
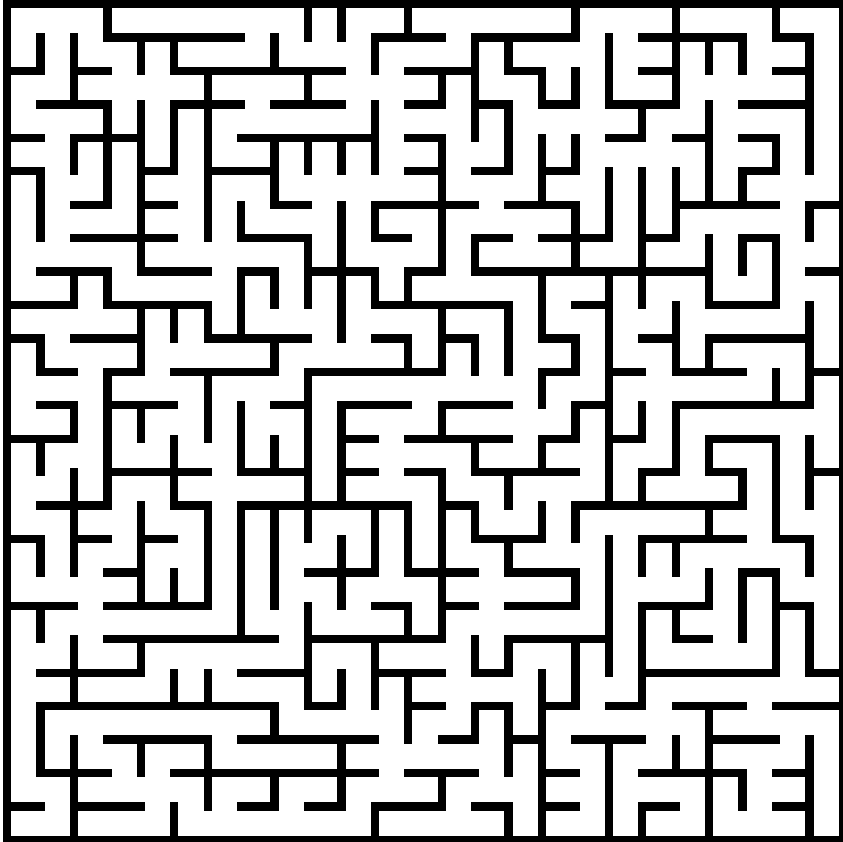
Can you draw this without lifting the pencil?

node

degree=3

edge

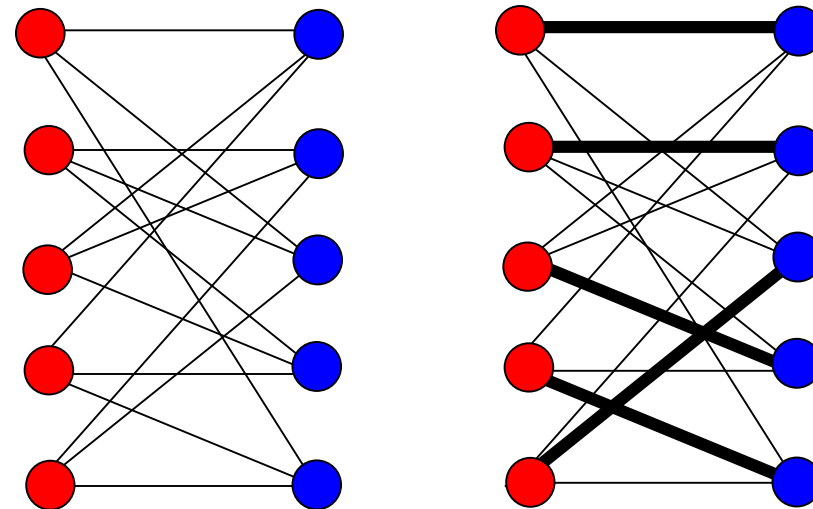graph/network

Trémaux 1882

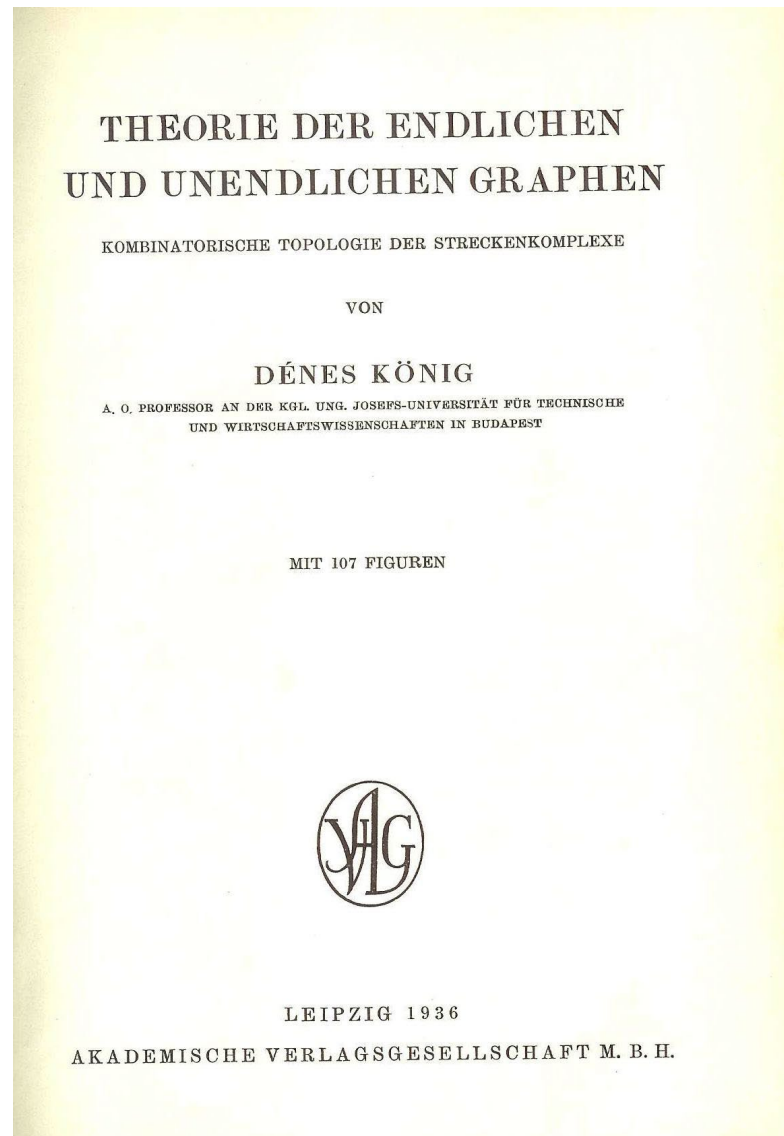Tarry 1895

Dijkstra 1959
shortest path
(GPS)

A school dance is attended by 100 boys and 100 girls. Every girl knows 24 boys, and ever boy knows 24 girls.
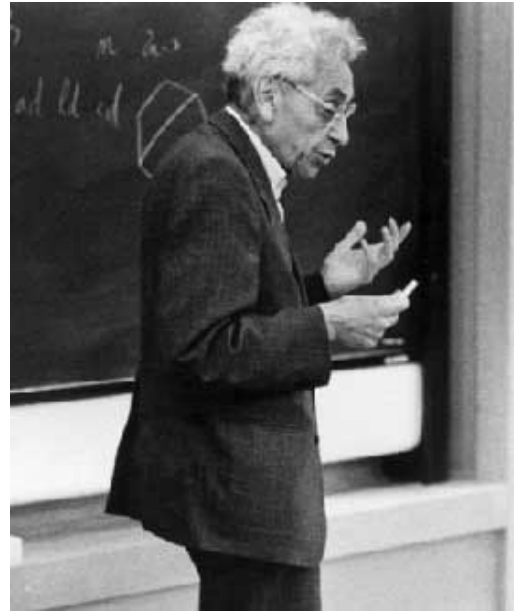*Claim:* they can all dance so that everybody dances with somebody he/she knows.

Dénes Kőnig

THEORIE DER ENDLICHEN
UND UNENDLICHEN GRAPHEN

KOMBINATORISCHE TOPOLOGIE DER STRECKENKOMPLEXE

VON

DÉNES KÖNIG

A. O. PROFESSOR AN DER KGL. UNG. JOSEFS-UNIVERSITÄT FÜR TECHNISCHE
UND WIRTSCHAFTSWISSENSCHAFTEN IN BUDAPEST

MIT 107 FIGUREN

LEIPZIG 1936
AKADEMISCHE VERLAGSGESELLSCHAFT M. B. H.
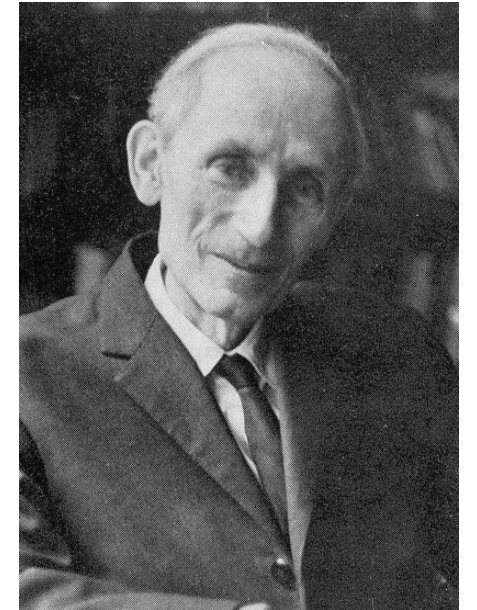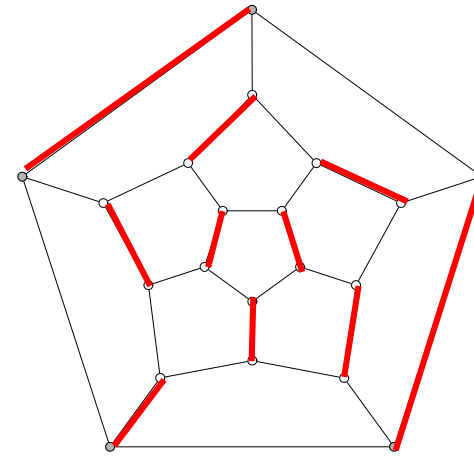
Dénes Kőnig

Paul Erdős   Lajos Pósa

The edges of any graph with n nodes

can be covered by $n^2/4$ edges and triangles.

(Erdős-Goodman-Pósa)

Perfect matching problem:
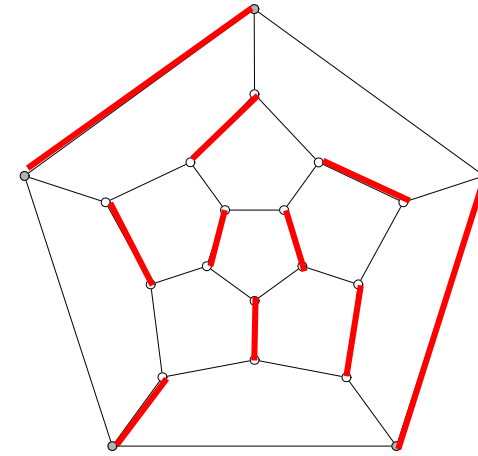
Can the nodes of a graph

be paired up by edges?

Tibor Gallai

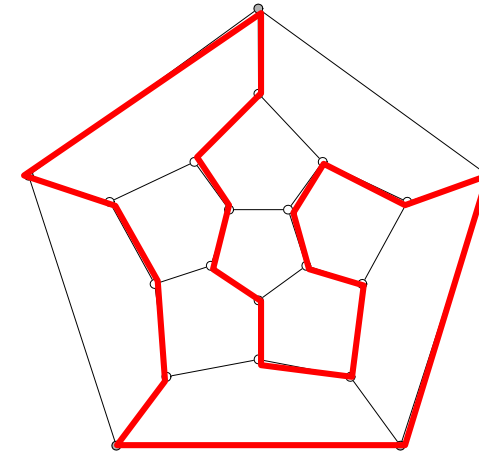Frobenius 1917, Kőnig 1931, Egerváry 1931,

Tutte 1947, Edmonds 1963

Perfect Matching Problem

Can the nodes of a graph

be paired up by edges?

Traveling Salesman Problem

Can all nodes be traversed

exactly once (Hamilton cycle)?

Perfect M_____blem

_____ph

_____ edges?

Traveli_____em

_____versed

___ce (Hamilton cycle)?

"Easy": efficient
algorithm,
nice structure

Hard: no efficient
algorithm,
no nice structure

Perfect M... ...blem

..ph

..y edge?
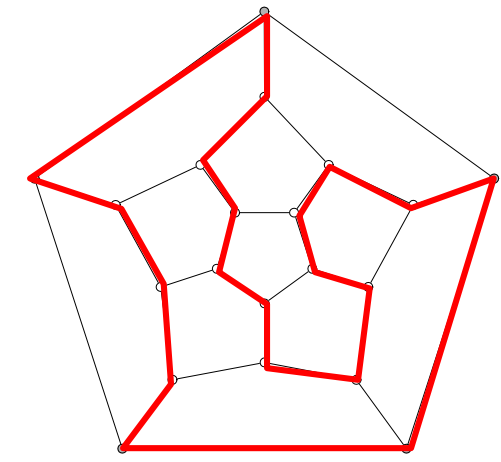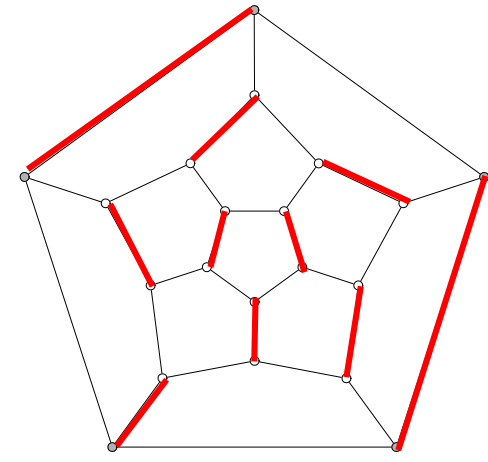
"Easy": efficient algorithm, nice structure

Traveli...

..versed

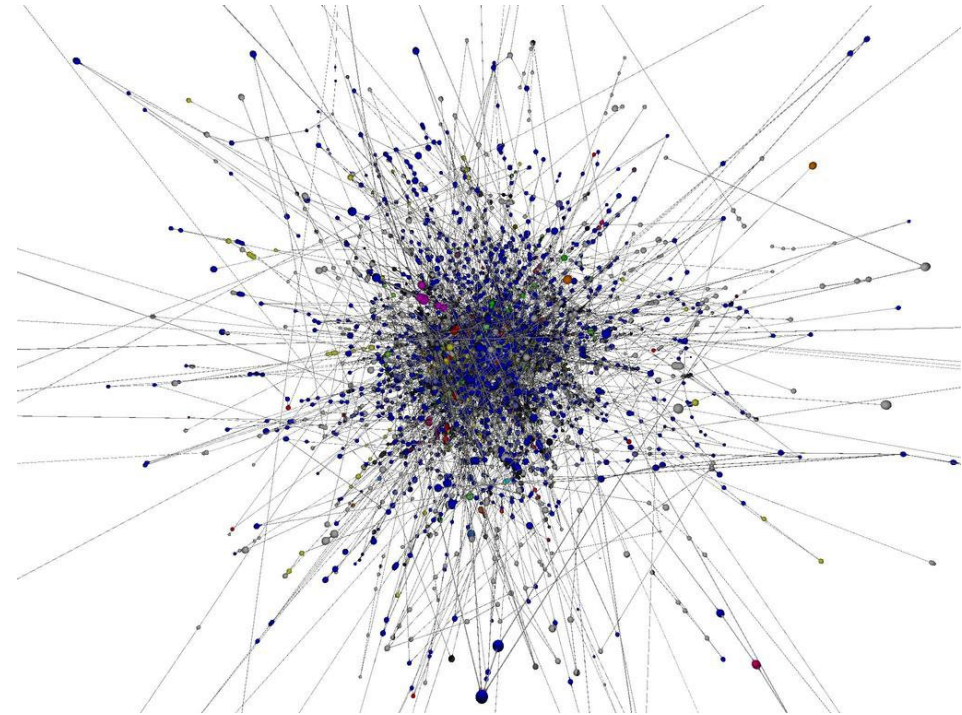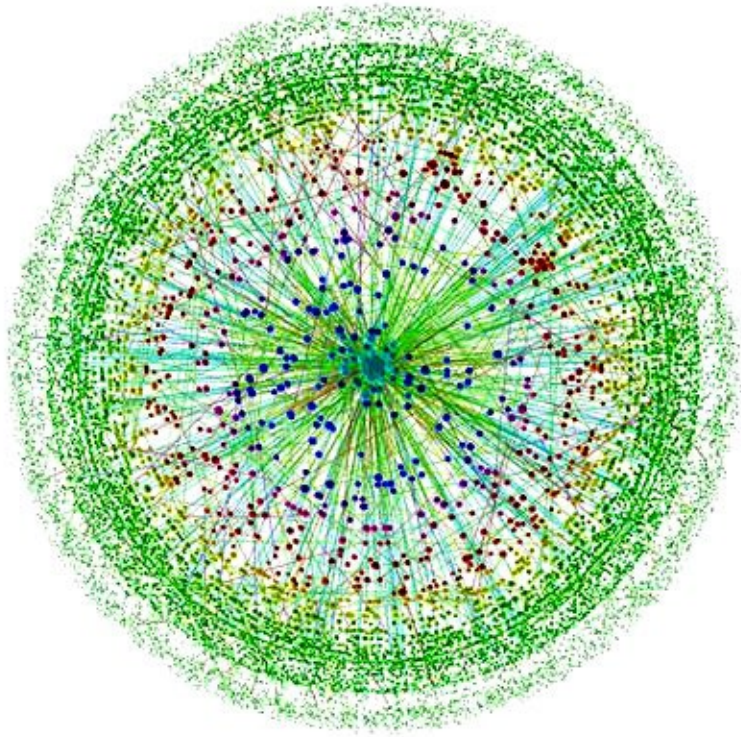..e (Hamilton cycle)?

Hard: no efficient algorithm, no nice structure

NP-completeness, the P=NP problem (one of the 7 Milleneum Problems), computational complexity
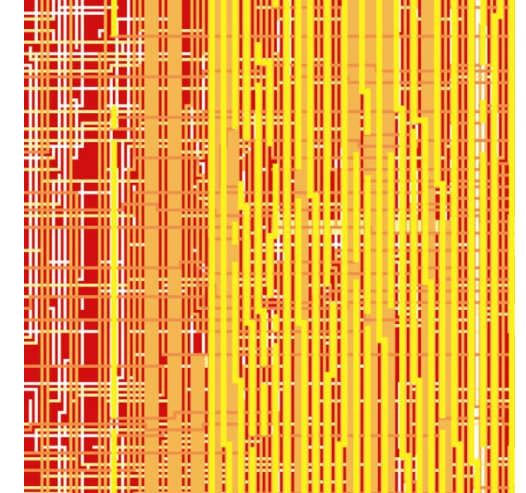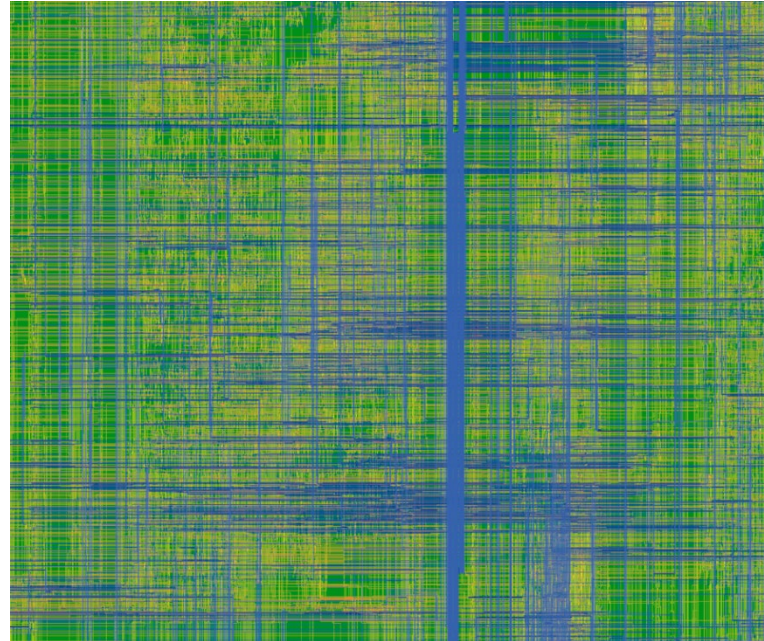
– internet

Very large graphs/networks

- – internet

- – social networks

- – ecological networks

- – chip design

Institut f. Diskrete Mathematik
Universität Bonn

Very large graphs/networks

- internet
- social networks

- ecological networks

- chip design

- crystals

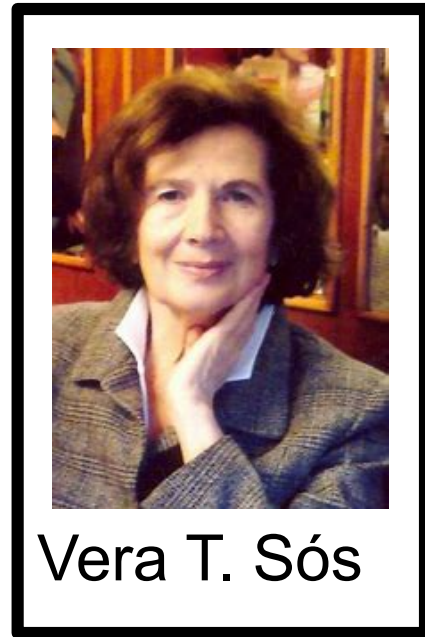- brain

- universe

New mathematical language of science

↑

What can math say about it?

The infinite is an approximation of the very large finite.
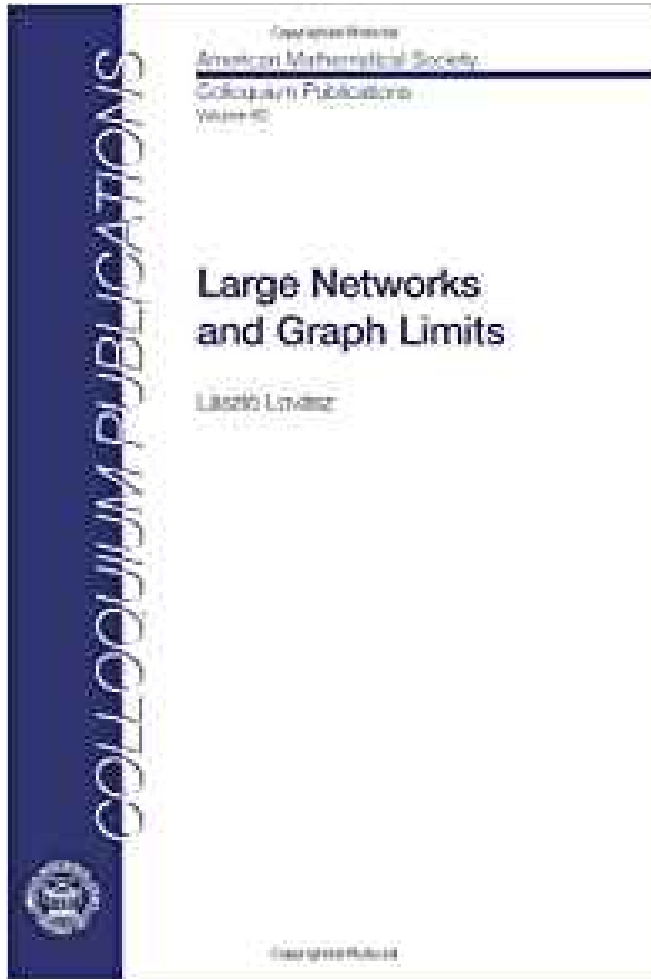

Christian Borgs, Jennifer Chayes

Vera T. Sós

Balázs Szegedy

Katalin Vesztergombi, LL
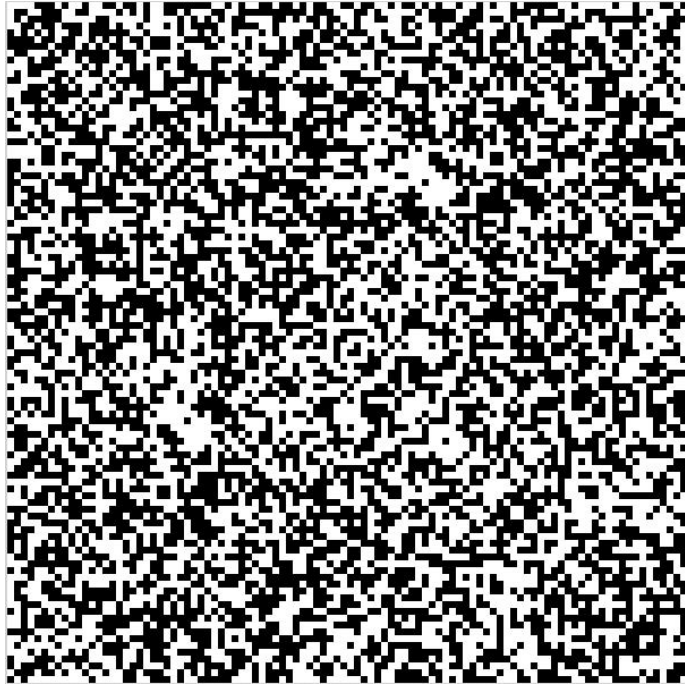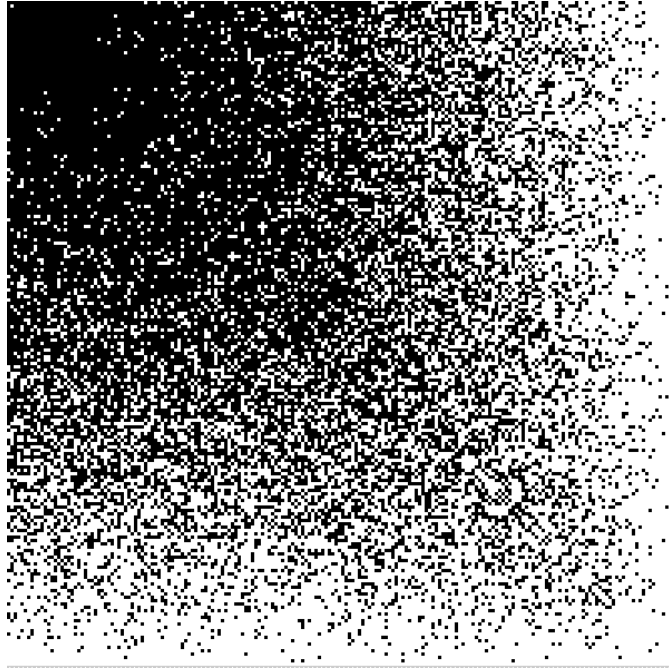
Kunszenti Kovács Dávid

Abért Miklós

Csóka Endre

A random graph

with 100 nodes and 2500 edges

$$W(x,y) \equiv \frac{1}{2}$$
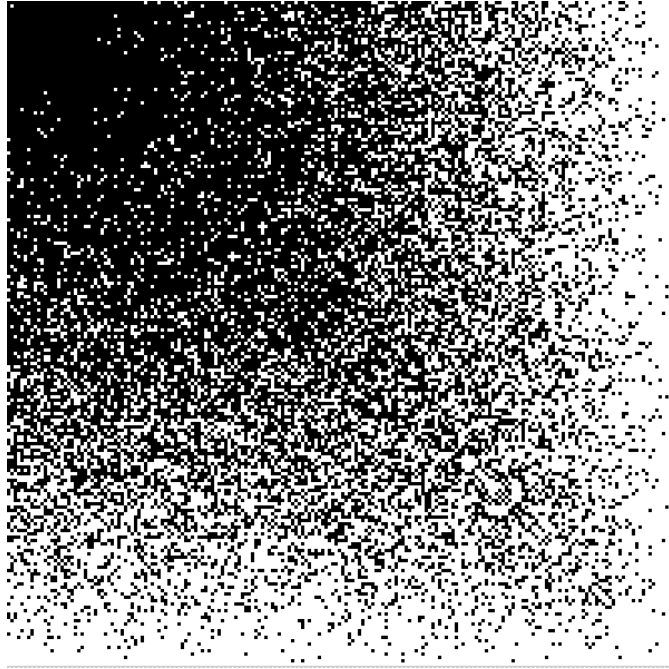
Randomly grown uniform attachment graph with 200 nodes

$$W(x,y) = 1 - \max(x,y)$$

density of triangles $\rightarrow \iiint W(x,y)W(y,z)W(z,x)\,dx\,dy\,dz$

Thank you for your attention!