Decision-making with Distance-based Operators in Fuzzy Logic Control

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Abstract: The norms and conorms family tree root is uninorm. It will be shown, that distance based operators satisfy most of properties of generally defined parametrical operators. Based on this theory some new types of fuzzy integrals are introduced theoretically. It can be shown, that the reason used for fuzzy integral introduction is similar to the reasons in decision-making by fuzzy control logic.

*Keywords:*distance-based operators, fuzzy integrals, FLC

1 Introduction

In practical representation of the fuzzy logic theory it is very important to be visually recognized. If we examine an FLC model with triangular or trapesoid membership functions, we can recognize the relationships between classical integral calculus and the decision making in FLC. It is very important in the cases of non-continuous tnorms and conorms, where the use of classical degree of fiering is not correct in all cases. So the new groups of tnorms and conorms must find their place in theoretical system of uninorms and parametrical norms, and based on theoretical background step should be made for further applications.

The uninorms were introduced , as a generalization of t-norms and t-conorms. For uninorms, the neutral element is not forced to be either 0 or 1, but can be any value in unit interval. The recent results on uninorms we can find in [1]. New trends in information aggregation, starting from the classical Zadehian operators through the group of entropy-based and evolutionary operators, to distance-based operators which can found in [2] and [4], with the proposition that this non-classic group of norms and conorms are guided by uninorm theory. Paper [2] described the main idea of using these norms in fuzzy logic control (FLC), where the fuzzy rule output is nothing else, but a fuzzy set weighted with the degree of coincidence of the rule premise and system input. You can read the same reason at introducing a new type of fuzzy integral, based on uninorms theory in [3]. Fuzzy measure theory needed in fuzzy integral calculus can be found in [5], [6] and [3].

2 Theoretical background

2.1 Fuzzy Logic Control

In control theory much of the knowledge of a controller can be stated in the form of *if-then* rules, involving some variables. The fuzzy logic control has been carried out searching for different mathematical models in order to supply these rules. In most sources it was suggested to represent an

IF x is $\boldsymbol{A}$ THEN y is $\boldsymbol{B}$ (1)

rule in the form of *fuzzy relation.*

On the other hand, this relation can be constructed as a special fuzzy operator: the fuzzy implication (***Imp***). In the definition of this connection or implication lie the most significant differences between the models of Fuzzy Logic Controllers (FLC-s).

The other important part of a rule-based system is the inference mechanism. The inference y is ***B'*** is obtained when the proposition is: the rule IF x is $\boldsymbol{A}$ THEN y is $\boldsymbol{B}$, and the system input x is ***A'***.

The connection ***Imp***(***A,B***) is generally defined, and it can be some type of t-norm. Generally Modus Ppnens sees the real influences of the implication choice on the inference mechanisms in fuzzy systems, of course, where the general rule consequence is obtained by

$$B'(y) = \sup_{x \in X} T\left(A'(x), \boldsymbol{Imp}\left(A(x), B(y)\right)\right). \quad (2)$$

The FLC rule base output is constructed as a crisp value calculated from associative using t-conorm on all rule outputs $B'(y)$.

2.2. Fuzzy measures

Fuzzy measure is defined as a function $m : \Sigma \to [a,b]$, where Σ is a σ algebra of fuzzy subsets of X (X is a non empty set). The interval $[a,b]$ can be modified in interval $[0,1]$, as usually in FLC. Function m must have properties, sometimes generalized properties, described as

M1. boundary condition, $m(\varnothing)=0$,

M2. monotonicity, for every A and B from set of fuzzy subsets, where $A \subseteq B$, then $m(A) \leq m(B)$.

M3. continuity, that for either $A_1 \subseteq A_2 \subseteq ...$ it is $\lim_{i\to\infty} m(A_i) = m \lim_{i\to\infty} A_i$.

In order to generalize fuzzy measure theory we can find the so called S measure types, with properties

MP1. $m(A \cup B) = S(m(A), m(B))$, for $A \cap B = \varnothing$, i.e. A and B are separable, if:

MP2. $m(A \cap B) = T(m(A), m(B))$.

(T,S) is a pair of t-norm and t-conorm. It is very important, that the parametrical (T,S) pair, with parameter e has further conditions:

MP3. If e=0, we have a probability measure,

MP4. If e=1, we have a possibility measure.

MP5. For parameter $e \in [0,1]$, and for every A and B from set of fuzzy subsets

$$m(A \cup B) = \begin{cases} m(A) + m(B) - e & \text{if } m(A) > e, m(B) > e, \\ \max(m(A), m(B)) & \textit{otherwise} \end{cases}$$

2.3.Fuzzy integrals

In [3] (S,U) integral was introduced.

Def.1. Let $m : \Sigma \to [a,b]$ be a Sm faithful S measure.

Given an Sm faithful partition $\beta = \{B_k | B_k \in \Sigma, k \in N\}$, the ($S$,$U$) integral of a measurable function $\rho : X \to [0,1]$ is defined by:

$$\int_X^{(S,U)} \varphi dm = \underset{k\in K}{S} \left(\underset{i=1}{\overset{n}{S}} U(a_i, m(A_i \cap Bk)) \right) \tag{4}$$

where U is a uninorm distribuitive over S t-conorm.

3 (*S*,*U*) Integral as the decision making in FLC with distance based operators

3.1. Distance based and evolutionary operators

The maximum distance minimum operator with respect to parameter $e \in [0,1]$ is defined as

$$T_e^{\max} = \begin{cases} \max(x,y) & if \quad y-e > e-x \\ \min(x,y) & if \quad y-e < e-x \\ \min(x,y) & if \quad y=x \text{ or } y-e=e-x \end{cases}$$

The maximum distance maximum operator with respect to $e \in [0,1]$ is defined as

$$S_e^{\max} = \begin{cases} \max(x,y) & if \quad y-e > e-x \\ \min(x,y) & if \quad y-e < e-x \\ \max(x,y) & if \quad y=x \text{ or } y-e=e-x \end{cases}.$$

The minimum distance minimum operator with respect to $e \in [0,1]$ is defined as

$$T_e^{\min} = \begin{cases} \min(x,y) & if \quad y-e > e-x \\ \max(x,y) & if \quad y-e < e-x \\ \min(x,y) & if \quad y=x \text{ or } y-e=e-x \end{cases}$$

The minimum distance maximum operator with respect to $e \in [0,1]$ is defined as

$$S_e^{\min} = \begin{cases} \min(x,y) & if \quad y-e > e-x \\ \max(x,y) & if \quad y-e < e-x \\ \max(x,y) & if \quad y=x \text{ or } y-e=e-x \end{cases}$$

The structures of the evolutionary operators are illustrated in 3D, in [4].

Lemma1.

The pairs $(T_e^{\max}, S_e^{\max})$ and $(T_e^{\min}, S_e^{\min})$ satisfy conditions $S_e^{\max} = 1 - T_e^{\max}(1-x, 1-y)$ and $S_e^{\min} = 1 - T_e^{\min}(1-x, 1-y)$.

Proof. Based on [2].

Lemma2.

The pairs $T_e^{\max}$ and $T_e^{\min}$ are commutative, associative, monotone binary operationson the unit interval [0,1] and for $0 < e < 1$ $T_e^{\max}(x,e) = x$ and $T_e^{\min}(x,e) = x$, i.e. $T_e^{\max}$ and $T_e^{\min}$ are uniniroms.

Proof. Based on [2].

From lemmas and based on [6] we conclude that $T_e^{\max}\left(x, S_e^{\max}(y,z)\right) = S_e^{\max}\left(T_e^{\max}(x,y), T_e^{\max}(x,z)\right)$.

3.2. Decision making

The t-norms family-tree root is uninorm, and its main branch is the parametrical group. Using these norms for combining fuzzy sets, we obtain compensation between small and large degrees of memberships, and the "pessimistic" (intersection-type, see MP3), and "optimistic" (union-type, MP4) connection between fuzzy sets make even.

In system control, however, intuitively quite the opposite is expected: let's make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker. So we used the distance-based and entropy-based norms and conorms as *evolutionary operators*.

The modified generalised entropy-based operator satisfies evolutionary conditions. Using a novel inference mechanism, in well-known rule base system the generalised modus ponens remains but the coincidence of the rule premise and the system input appears in a new form.

Because of the non-monotonic property of entropy-based operators, it was unreasonable to use the classical degree of firing, to give expression to coincidence of the rule premise (fuzzy set *A*), and system input (fuzzy set *A'*), therefore a degree of coincidence (*Doc*) for these fuzzy sets has been initiated. It is nothing else, but the proportion of area under membership function of the modified entropy-based intersection of these fuzzy sets, and the area under membership function of the their union (using *max* as the fuzzy union). This reason has two advantages: it considered the width of coincident of *A* and *A'*, and not only the height, andhe rule output is weighted with a measure of coincident of *A* and *A'* in each rule.

The rule output fuzzy set (*B'*) is not achieved as a cut of rule consequence (*B*) with *Doc*.

$$B'(y) = T_e^{\min\, or\, \max}\left(B(y), Doc\right) \tag{5}$$

where *Doc* is the degree of coincidence, and gives expression to coincidence of the rule premise (fuzzy set *A*), and system input (fuzzy set *A'*)

$$Doc = \frac{\int_X T_e(A, A')dx}{\int_X \max(A, A')dx}$$

It is easy to prove, that $Doc \in [0,1]$, and $Doc = 1$ if *A* and *A'* cover over each other, and $Doc = 0$ if *A* and *A'* have no point of contact.

This system was tested. The conclusion was, in simplified rule base system, using this type of evolutionary operators and novel *Doc* as the fuzzy sets coincidence expression, that the controlled system obtains the desired state better then by classical approach.

The FLC rule base output is constructed as crisp value calculated from associative using t-conorm on all rule outputs $B_i'(y)$.

3.3. Decision making with (S,U) integral

Let's introduce an *m* measure, for fuzzy subsets *A, B* and *A′* like in *I.B*, as the measure number related to area under membership function, which describes this fuzzy set. It is easy to prove, that this measure satisfies all properties M1-M3. Conditions MP1-MP5. can be prove, if we for every point of the kernel of fuzzy set *A* imagine the $\mu(x)$ (membership function of *A*) as the basic of *m* measure for the *A*.

In this case (see MP1., MP2.)

$$m(A_i \cap A') = T(m(A_i), m(A')) = Doc \cdot \int_X \max(A_i, A)$$

as the weight measure for computing rule output B_i' in i^{th} rule in rule base.

In i^{th} one rule output is obtained as

$$B_i' = \mathop{S}_{j=1}^{p} T(B(y), m(A_i \cap A))$$

where we need $\mathop{S}_{j=1}^{p}$ if we have MISO system with *p* rule premisse. This form is in accordance with Eq. (5) and with the defiinition of (*S,U*) integral.

The FLC rule base output is constructed as crisp value calculated from associative using (*S,U*) integrals for rule outputs, based on Eq. 4., as follows:

$$B_i^{'} = \underset{forallrules}{S}\left(\overset{p}{\underset{j=1}{S}} T\left(B(y), m\left(A_i \cap A\right)\right)\right)$$

It is in fact an area, and from that area we obtain a crisp FLC system output with a defuzzification method.

As S *t*-conorm in integral, and T *t*-norm for calculating measure m (see Introductions)¸ we can use pair ($T_e^{\max}$, $S_e^{\max}$), because they are parametrical norms, and satisfy all conditions required for fuzzy measures and fuzzy integrals.

4 Conclusion

Using that idea, we obtain a method, which is close to the visual description of decision-making in FLC, for triangular or trapezoid membership function. It is very important too, that this method has strong theoretical background in fuzzy integrals (*S,U*), and finally, this fuzzy integral theory was applied, and can be used for others, conditional t-norms and t-conorms.

5 References

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