Dynamic Modelling of Robot Manipulators by Zadeh-type Fuzzy Partitions

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Abstract:
This paper presents a novel fuzzy identification method for dynamic modelling of robot manipulators. The method is based on a special parameterisation of the antecedent part of fuzzy systems that results in fuzzy partitions for antecedents. Christoffel symbols, the derivatives of fuzzy systems are used for modelling the Coriolis effects and centrifugal forces. The majority of parameters, being linear, is evaluated by the least squares method. Those few non-linear parameters are subjected to an evolutionary global optimisation scheme and a gradient descent based local search.

Keywords: robot manipulator, dynamic modelling, fuzzy-partitions, unconstrained optimisation, ANFIS

1 Introduction

Dynamic modelling of robot manipulators (RMs), mapping the position, velocity and acceleration of joints into forces, torque exerted to the structure is based on the Lagrange formulation, which ensures the appropriate structure of the dynamic model that is commonly used in control algorithms. RMs are known to be highly non-linear multi-input multi-output systems. To preserve the known structure of the Lagrange formulation, common to all system equations of RMs and other dynamic systems such as missiles and aeroplanes, grey-box modelling is chosen. Forces exerted to joints of the RM are the sum of four components modelling consequently the torque resulting from the inertia (H), the Coriolis effects and centrifugal forces (C), the gravity forces (g) and the viscose friction (f). Individual knowledge of all these components is important for precise, model based robot control algorithms. Advantage is also taken of other commonly known facts of robotics like H and g are non-linear functions of joint positions and the driving torque is linear in the joint accelerations. The centrifugal and Coriolis effects are quadratic in the joint velocities and non-linear in the joint positions and f is linear in joint velocity [1]. The DM identification method uses the measured resultant torque and joint variables along suitably chosen paths for every joint.

Every single building block of the first, preliminary part of the identification is approximated by a constant using a singular value decomposition (SVD) based linear least squares (LS) method [9]. At the second step representative portions of the training data are extracted, on bases of input space coverage in the joint position space. The number of samples are reduced to a minimal value that is still representative which is monitored through the condition number of the sample matrix transformed for the SVD based linear LS method of parameter identification. These quantitatively reduced, but qualitatively representative portions are concatenated to give the new training data set. At the third step the possibility of improving the model is investigated in form of FLS building blocks for the non-linear functions of the joint positions [2]. Multi-input single-output complete first order Takagi-Sugeno-Kang (TSK) type FLSs are considered having membership functions (MFs) in form of Zadeh type fuzzy partitions [3]. Components of C - the Coriolis effects and centrifugal forces are evaluated as the Christoffel symbols of FLSs forming H- the inertia matrix. The non-linear parameters of Zadeh type fuzzy partitions are subjected to a multi-objective hybrid evolutionary optimisation method [4]. The number of non-linear parameters is reduced to its minimum, a single parameter is defining a Fuzzy partition (the number of fuzzy partitions - FLS inputs is defined by the geometry of the RM, thus cannot be reduced without significant loss in modelling precision). An ANFIS like gradient descent method is used for all the non-linear parameter of the dynamic model. The LS method is used for the remaining linear parameters of every fuzzy system, it's required Christoffel symbols and the remaining linear parameters of the RM.

This paper presents a novel method for dynamic modelling of robot manipulators by Zadeh-type fuzzy partitions. After this introduction the second paragraph describes the basic concepts and notations of robotics and fuzzy logic used throughout the paper. The third paragraph presents the new method for representing and unconstrained tuning of the dynamic model of RMs by fuzzy systems that for rule antecedents have Zadeh-type membership functions forming fuzzy partitions. The fourth paragraph describes some important details of the implementation. The fifth paragraph presents results obtained with the proposed method and its comparison to other methods. Finally concluding remarks are made and references are stated.

2 Robot Manipulators and Fuzzy Modelling

The application of Lagrange dynamic equations for a robot manipulator in the joint space formulates the resultant torque τ_i acting on the i^{th} joint from all the p joints of the RM as a function of following vectors: joint positions (q), velocities ($\dot{q}$) and accelerations ($\ddot{q}$):

$$\sum_j \left(D_{ij}(q)\cdot \ddot{q}_j\right)+\sum_j\sum_k \left(\dot{q}_j \cdot D_{ijk}(q)\cdot \dot{q}_k\right)+D_i(q)+f_i\cdot \dot{q}_i = \tau_i,\ i, j, k = 1,2,\ldots,p, \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i$ stand for the joint variables and their derivatives. The first component of (1) is shortly referred to as $H \cdot \ddot{q}$ describing the inertia, the second as $C \cdot \dot{q}$ describing the Coriolis effects and centrifugal forces, the third as g for the gravitational forces and the fourth as $f \cdot \dot{q}$ for the viscous friction:

$$H_{ik} = D_{ik}(q),\ \mathrm{C}_{ik} = \sum_{j=1}^{p} \dot{q}_j \cdot D_{ijk}(q),\ g_i = D_i(q), f_i = const, \quad (2)$$

where $D_{ik}, \mathrm{D}_{ijk}, \mathrm{D}_i$ are in general, highly non-linear scalar functions of joint positions. They may contain sin(·) and cos(·) functions of joint positions and/or of their products and sums defined by the geometry of the RM. There are well known general relations that can be used for reducing the number of unknown elements, like D_{ijk} are the Christoffel symbols of D_{ij} [1]:

$$D_{ijk} = \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k}+\frac{\partial D_{ik}}{\partial q_j}-\frac{\partial D_{jk}}{\partial q_i}\right), D_{ijk} = D_{ikj}, D_{kij} = -D_{jik}, D_{kjk} = 0 \forall i, k \geq j \quad (3)$$

It should be noted that direct measurement of any single component from (2) is not possible, the only information on the output of the system is the resultant torque (1). The identification of all non-linear functions under these terms is a considerable problem.

The identification method to be proposed uses Zadeh-formed membership functions (MFs) for antecedents in a Takagi-Sugeno-Kang (TSK) type FLS having *n* inputs and 1 output. The antecedent, the premise part of a fuzzy rule is:

$$\omega_l(x) = \prod_{i=1}^{n} \mu_{F_{l(i)}}(x_i), \quad (4)$$

where $\mu_{F_{l(i)}}(x_i)$ is the membership function of the i^{th} input variable in the l^{th} rule. If a linguistic variable can be assigned *K* different linguistic values, each described by a MF $\mu_k(x)$ such that for every input *x* it holds that $\sum_{k=1}^{K}\mu_k(x)=1$, the MFs are said to form a fuzzy-partition. Assuming that the rule base is complete in the sense that it covers the whole input domain, it immediately follows that the TSK model structure simplifies to [6]:

$$f(x) = \sum_{l=1}^{M} \omega_l(x)\cdot y_l(x). \quad (5)$$

Zadeh-formed MFs are the Z-, the S-, and the π-functions (named after their shape) defined respectively as:

$$mfz(x,b_1,b_2)=\begin{cases}1 & x\le b_1\\ 1-2((x-b_1)/(b_2-b_1)) & b_1<x\le \tfrac{1}{2}(b_2+b_1)\\ 2((x-b_1)/(b_2-b_1)) & \tfrac{1}{2}(b_2+b_1)<x\le b_2\\ 0 & x>b_2\end{cases}$$

$$mfs(x,b_1,b_2)=1-mfz(x,b_1,b_2)\qquad , \qquad (5)$$

$$mf\pi(x,b_1,b_2,b_3,b_4)=\begin{cases}mfs(x,b_1,b_2) & x\le b_2\\ 1 & b_2<x\le b_3\\ mfz(x,b_3,b_4) & x>b_3\end{cases}$$

where $b_1 \le b_2 \le b_3 \le b_4$ are the parameters defining the MFs. If there is more than one value x such that the degree of membership of x is equal to one, the interval where the $\mu_k(x,b)=1$ (the interval $[b_2, b_3]$ for $mf\pi$ type μ_k) is called the plateau of the μ_k MF [7].

3 Forming the Dynamic Model of a Robot Manipulator by Fuzzy-partitions

The nature of Zadeh-formed MFs is such that simply making equal the last two parameters of the preceding MF to the first two parameters of the succeeding MF easily forms fuzzy partitions. Let our input space be normalised ($x_{min} = 0$ and $x_{max} = 1$). If we do not want to allow any plateaux, parameter b_2 must be equal to b_3 in (5) and the number of parameters is $K-1$, where K is the number different linguistic values, MFs assigned to a linguistic variable, one input of the FLS.

If we take into consideration all of the constraints (5) we end up with a series of strictly ordered parameters

$$0<b_1<b_2<\ldots<b_{K-1}<1 \qquad (7)$$

Let us define the first MF to be $mfz(x,0,b_1)$ and the K^{th}, the last one to be $mfs(x,b_{K-1},1)$. Let all intermediate k^{th} MFs be $mf\pi(x,b_{k-1},b_k,b_k,b_{k+1})$ for $k = 2, \ldots, K-1$. This way the ordered series of $K-1$ parameters together with constants 1 and 0 are the minimal number of parameters to define a fuzzy-partition of Zadeh-formed MFs.

This minimal number of non-linear parameters is a very important issue for optimisation as over parameterised systems are hard to optimise.

Let us represent the b_k parameters in a different manner [3]. Let us consider K pieces of rational, positive or zero parameters $a_\kappa \in R_0^+, \kappa = 1, ..., K$. If we simply form b_k as:

$$b_k = \sum_{j=1}^{k} a_j \Big/ \sum_{\kappa=1}^{K} a_\kappa \quad (8)$$

for every k = 1, …, K all the constraints (7) are automatically fulfilled for every a_κ without any further restrictions on a_κ.

The proposal is to identify the D_{ij} and D_i components of the dynamic model defined as (1), using the simplifications of (3) as FLSs of form (5) with (8), where x will be equal to q. Forming all the D_{ijk} components as Christoffel symbols is possible simply as as:

$$\frac{\delta f(q)}{\delta q_i} = \sum_{l=1}^{M} \left(\frac{\delta \omega_l(q)}{\delta q_i} \cdot y_l(q) + \omega_l(q) \cdot \frac{\delta y_l(q)}{\delta q_i}\right) \quad (9)$$

The proposal is to use an ANFIS like optimisation of all parameters, gradient based optimisation for antecedent parameters a_K of all FLSs and linear least square (LS) method for consequent parts of FLSs and all remaining linear parameters of the dynamic model. To avoid the trap of finding only a local optimum with the gradient descent method, a fast and efficient evolutionary search is to be performed to approach the global optimum of non-linear a_K parameters.

5 Implementation

The proposed method is tested for a SCARA type RM. The training data set is reduced to 179 points [8]. The input space is normalised to the unit hyper-cube. The fuzzy-partition representation has been incorporated into a multi-objective hybrid genetic algorithm [4]. One chromosome consists of only four a_K integer parameters. One parameter defines a complete fuzzy partition of three MFs. There are only two D_{ij}s (D_{11} and D_{12}) that are non-linear functions of only two inputs q_2 and q_4 [2]. These four a_K parameters are all that is required to model the non-linearity of a SCARA RM. The remaining twelve linear parameters of the RM and the two times twenty seven linear parameters of the two TSK FLSs having two inputs, nine rules each is determined by the LS method.

6 Result

The result of a quick, small evolutionary search is a chromosome of (37522, 32020, 65333, 53411) for the four non-linear a_K parameters. Linear parameters of the dynamic model are listed in Table 1. below. The first column names the parameter, the second contains its exact value, results of geometrical an mechanical analysis of joints and its configuration. The third column contains an earlier result [2]. Result of the proposed method is listed in the fourth column of Table 1. Table 2. Contains the parameters of MFs forming Zadeh-type fuzzy partitions. Table 3. lists all the linear parameters that form consequent parts of all nine rules of both FLSs for D_{11} and D_{12}.

	Exact	LSQ [2]	Propsed
D_{11}	$Func_{11}$	FLS_{11}	FLS_{11}
D_{12}	$Func_{12}$	FLS_{12}	FLS_{12}
D_{14}	0.004	1.1315	0.0098776
D_{22}	1.1454	0.19586	1.1388
D_{24}	0.004	-0.25933	0.005855
D_{33}	130.2521	130.2521	130.2521
D_{44}	0.409	-0.45519	0.40839
D_{112}	$f(Func_{11})$	FLS_{112}	$f(FLS_{11})$
D_{114}	0	0.0022199	0
D_{214}	0	0.00024719	0.023923
D_3	67.1985	67.1985	67.1985
f_1	14.5031	14.5031	14.5031
f_2	13.8	13.7869	13.5866
f_3	3948.9	3948.9	3948.9
f_4	13.4	13.4001	13.4008

Table 1. Linear parameters of the dynamic model.

	b_1	b_2	b_3
$_{11}zMF_{11}(q_2)$	0	0.5725	-
$_{11}\pi MF_{12}(q_2)$	0	0.5725	1
$_{11}sMF_{13}(q_2)$	0.5725	1	-
$_{11}zMF_{21}(q_4)$	0	0.48862	-
$_{11}\pi MF_{22}(q_4)$	0	0.48862	1

${}_{11}sMF_{23}(q_4)$	0.48862	1	-
${}_{12}zMF_{11}(q_2)$	0	0.99693	-
${}_{12}\pi MF_{12}(q_2)$	0	0.99693	1
${}_{12}sMF_{13}(q_2)$	0.99693	1	-
${}_{12}zMF_{21}(q_4)$	0	0.81503	-
${}_{12}\pi MF_{22}(q_4)$	0	0.81503	1
${}_{12}sMF_{23}(q_4)$	0.81503	1	-

Table 2. Non-linear parameters ${}_{11}xMF_{xx}$ for FLS_{11} and ${}_{12}xMF_{xx}$ for FLS_{12}

	c_0	c_1	c_2
${}_{11}y_{11}(q_2, q_4)$	1.5836	-0.31855	0.0059997
${}_{11}y_{12}(q_2, q_4)$	-0.072244	0.014675	-0.033884
${}_{11}y_{13}(q_2, q_4)$	1.5836	-0.31855	0.0059997
${}_{11}y_{21}(q_2, q_4)$	0.43328	-0.25865	0.10659
${}_{11}y_{22}(q_2, q_4)$	0.95086	-0.42537	-0.80804
${}_{11}y_{23}(q_2, q_4)$	0.43328	-0.25865	0.10659
${}_{11}y_{31}(q_2, q_4)$	1.5836	-0.31855	0.0059997
${}_{11}y_{32}(q_2, q_4)$	-0.072244	0.014675	-0.033884
${}_{11}y_{33}(q_2, q_4)$	1.5836	-0.31855	0.0059997
${}_{12}y_{11}(q_2, q_4)$	0.3856	0.15897	-0.0065901
${}_{12}y_{12}(q_2, q_4)$	0.098905	-0.43382	0.022196
${}_{12}y_{13}(q_2, q_4)$	0.3856	0.15897	-0.0065901
${}_{12}y_{21}(q_2, q_4)$	-0.79427	-0.34648	0.0096337
${}_{12}y_{22}(q_2, q_4)$	0.12912	0.77157	-0.053574
${}_{12}y_{23}(q_2, q_4)$	-0.79427	-0.34648	0.0096337
${}_{12}y_{31}(q_2, q_4)$	0.3856	0.15897	-0.0065901
${}_{12}y_{32}(q_2, q_4)$	0.098905	-0.43382	0.022196
${}_{12}y_{33}(q_2, q_4)$	0.3856	0.15897	-0.0065901

Table 3. Linear parameters of rule consequent ${}_{11}y_{xx}$ for FLS_{11} and ${}_{12}y_{xx}$ for FLS_{12}

The identification error of torque acting on joints 1-4 is presented in Figure 1. The mean square error is 0.05209, the maximal absolute error is 2.4423 Nm and there are no more than nine such points where the error is greater than three times the standard deviation of the error. The relative value of the maximal error is 2.8314%, 2.2275%, 0%, 0.66065% for joints 1, 2, 3 and 4 respectively.

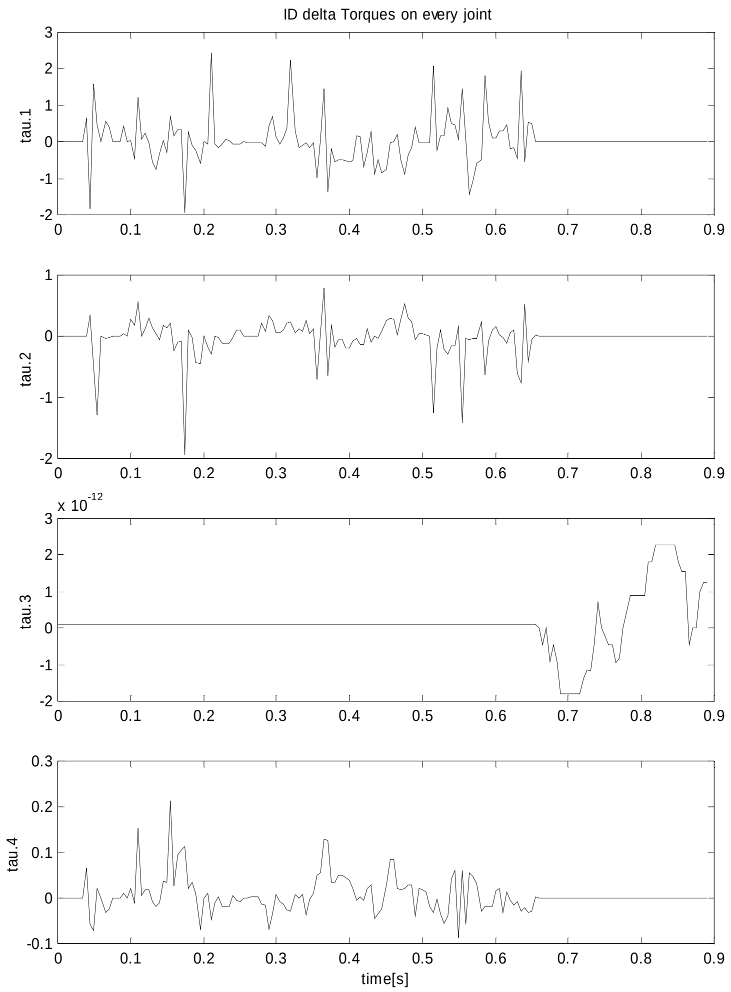

Figure 1. SCARA RM torque identification error in Nm for joints 1,2,3 and 4.

7 Conclusions

The proposed identification method is capable of forming and fine-tuning a soft computing, fuzzy system based dynamic model for a robot manipulator. The number of non-linear parameters can be kept to minimal and optimised by evolutionary and gradient based methods, too. The value of the linear parameters can be determined by a least squares method. After an initial evaluation [8] the complete identification method is capable of running on-line with a control algorithm if we use an on-line iterative least squares method for the linear parameters [9], while from the background a hybrid evolutionary and gradient based method [4] periodically updates the non-linear parameters.

The relative value of the maximal error is well within the tolerance level of a model based control algorithm [1]. Parameters identified by this method can be considered as real physical values, in contras to previous results where some negative numbers appeared for inertia terms.

References

[1] The ZODIAC, 'Theory of Robot Control', Springer-Verlag London Ltd., 1996, ISBN 3540760547

[2] A.Nemes, B.Lantos, Optimization of Fuzzy Logic Systems for Gray-Box Dynamic Modeling of Robot Manipulators by Genetic Algorithms", Proceedings of IEEE INES'99, pp. 353-358

[3] A.Nemes, Function Identification by Unconstrained Tuning of Zadeh-type Fuzzy Partitions, Proceedings of the International Symposium of Hungarian Researchers on Computational Intelligence, 2001

[4] A.Nemes, System Identification Based on Multi-Objective Optimisation and Unconstrained Tuning of Zadeh-type Fuzzy Partitions, Proceedings IEEE SISY, 2003

[5] Jyh-Shing Roger Jang, Chuen-Tsai Sun, Eiji Mizutani, Neuro-Fuzzy and Soft Computing, A Computational Approach to learning and Machine Intelligence, Prentice-Hall, 1997, ISBN 0-13-287467-9

[6] J.-S. R Jang, C.-T. Sun, E. Mizutani, *Neuro-Fuzzy and Soft Computing*, Prentice-Hall International, Inc., 1997, ISBN 0-13-287467

[7] Hans Hellendron, Dimiter Driankov (Eds.), Fuzzy Model Identification, Selected approaches, Springer, 1997, ISBN 3-540-62721-9

[8] A.Nemes, B.Lantos, Training Data Reduction for Optimisation of Fuzzy Logic Systems for Dynamic Modelling of Robot Manipulators by Genetic Algorithms, Proceedings of IEEE IMTC 2001

[9] W. H. Press, et. all, *Numerical Recipes in C, The Art of Scientific Computing*, Cambridge University Press, Cambridge, 1990, ISBN 0-521-35465-X.