Aspects Concerning the Development of Fuzzy Controllers for Servo Systems

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1 Introduction

The class of plants having the transfer functions $H_P(s)$ expressed in terms of (1) or (2):

$$H_P(s)=\frac{k_P}{s(1+sT_\Sigma)}\ ,\ \ H_P(s)=\frac{k_P}{s(1+sT_\Sigma)(1+sT_1)}\ , \tag{1}$$

$$H_P(s)=\frac{k_P}{(1+sT_\Sigma)(1+sT_1)}\ ,\ \ H_P(s)=\frac{k_P}{(1+sT_\Sigma)(1+sT_1)(1+sT_2)}\ , \tag{2}$$

with T_Σ – small time constant or time constant corresponding to the sum of parasitic time constants and $T_\Sigma < T_2 < T_1$, characterize sufficiently well the controlled plants as part of servo systems.

The paper is organized as follows. There are presented two methods (in Sections 2 and 3) for optimal tuning of controller parameters in the case of controlling the plants (1) and (2). Then, Section 4 is dedicated to the presentation of a development method for a Mamdani fuzzy controller with dynamics. By accepting that the simplified dynamics of a mobile robot can correspond to the first transfer function in (1), there are developed in Section 5 three controllers, and there is performed the validation of the development methods and of the controller structures by digital simulations.

2 Extended Symmetrical Optimum (ESO) Method

In the case plants with the transfer functions of the forms (1) the use of a PI or PID controller having the transfer functions (3) and (4), respectively:

$$H_C(s) = \frac{k_c}{s}(1 + sT_c) \ , \tag{3}$$

$$H_C(s) = \frac{k_c}{s}(1 + sT_c)(1 + sT_c^{'}) \ , \tag{4}$$

tuned in terms of Kessler's Symmetrical Optimum (SO) method, can ensure acceptable performance [1].

In both cases, the open-loop transfer function $H_0(s)$ can be expressed as:

$$H_0(s) = H_C(s)H_P(s) = \frac{k_0(1 + sT_c)}{s^2(1 + sT_\Sigma)} \ , \ k_0(s) = k_c k_P \ , \tag{5}$$

However, in some practical applications the control system performance – with the well-known quality performance indices: overshoot $\sigma_1 \approx 43\%$, settling time $t_s \approx 16.3T_\Sigma$, first settling time $t_1 \approx 3.7T_\Sigma$ and phase margin $\varphi_r \approx 36^o$ – prove to be rather unacceptable. But, if the plant contains nonlinearities or variable parameters (this is the case of k_P – the plant gain), the use of fuzzy controllers with dynamics can ensure control system performance enhancement.

The values of these quality performance indices become unacceptable due to a large sensitivity with respect to the modification of k_P accompanied by an alleviation of φ_r. This shortcoming can be much stronger if T_Σ corresponds to the sum of parasitic time constants generally having taking only an approximated value.

In [2] there was proposed an extension of Kessler's SO method with large possibilities for application in the field of servo systems, in particular of electrical drives with possible variable moment of inertia. In this case the controlled plant is described by a transfer function of the form (1) with k_P – often variable and T_Σ – usually constant. The closed-loop transfer function $H_w(s)$ with w (the reference input) as input and y (the controlled output) as output can be expressed in terms of (6):

$$H_w(s) = \frac{H_0(s)}{1 + H_0(s)} = \frac{b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \ , \ b_1 = a_1 \ , \ b_0 = a_0 \ . \tag{6}$$

The optimization conditions according to Kessler's SO method were generalized in the following form [2]:

$$\beta^{1/2} a_0 a_2 = a_1^2 \ , \ \beta^{1/2} a_1 a_3 = a_2^2 \ . \tag{7}$$

By using the equations (7), the transfer functions $H_0(s)$ and $H_w(s)$ are re-expressed in their optimal forms (8):

$$H_0(s)_{opt} = \frac{1+\beta T_\Sigma s}{\sqrt{\beta^3}\, s^2 (1+T_\Sigma s)}\,,\; H_w(s)_{opt} = \frac{1+\beta T_\Sigma s}{\sqrt{\beta^3} T_\Sigma^3 s^3 + \sqrt{\beta^3} T_\Sigma^2 s^2 + \beta T_\Sigma s + 1}\,. \quad (8)$$

It is fully justified to consider the expressions (8) as optimal ones because an optimization occurs indeed by the maximization of the phase margin in the case of constant values of k_P.

By the choice of the parameter β in the domain $\beta \in (1, 20)$, the control CSPIs $\{\sigma_1, t_s^{\wedge} = t_s/T_\Sigma, t_1^{\wedge} = t_1/T_\Sigma, \varphi_r\}$ can be accordingly modified and a compromise between these performance indices can be reached by using the diagrams shown in Fig.1.

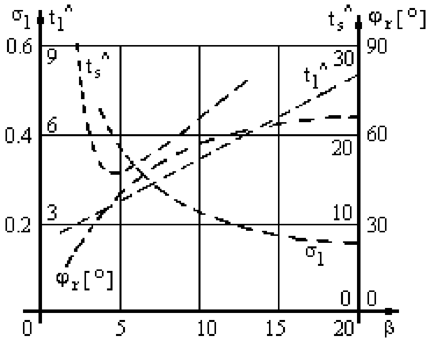

Fig.1. Control system performance indices versus β.

It can be observed that all coefficients in the closed-loop transfer function $H_w(s)_{opt}$ depend on the only one parameter β and that the PI and PID controller development is reduced to the choice of this parameter, β.

On the basis of the generalized optimization conditions (7), the favorable equations for tuning the controller parameters are obtained:

$$k_c = \frac{1}{\sqrt{\beta^3} T_\Sigma^2 k_P}\;,\; T_c = \beta T_\Sigma\,,\; T_c^{'} = T_1\;. \quad (9)$$

The ESO method guarantees a minimum phase margin in the case of variable k_P, $k_P \in D_P = [k_{Pm}, k_{PM}]$, and the possibility for computing the value of k_P^*, the “medium” value of k_P, in terms of [2]:

$$k_P^* = \sqrt{k_{Pm} k_{pM}}\;, \quad (10)$$

exemplified for the speed control of variable inertia drives [3]. The application of the ESO method in this case enables the determination of the value of β that, for a variation domain of the parameter k_P, guarantees a minimum accepted value φ_{rm} of the phase margin, $\varphi_r \geq \varphi_{rm}$.

The PI controller development with respect to w has been also performed and applied in [4], [5], where the behavior with respect to the disturbance input has been only reported.

The ESO method can be applied also in the case of controllers with non-homogenous information processing with respect to the controller input channels (w and y) [6] by suppressing the action of the zero in (8). In the case of controllers with homogenous information processing this can be done by adding two versions of reference filters [2].

3 Further Extension in Applying the ESO Method

If the plant transfer function has one of the forms of (2), the use of a PI or PID controller tuned according to Kessler's Modulus Optimum (MO) method [1] leads to good CS performance. Exceptions occur in the following two situations:

- the time constant T_1 has very large value, $T_1 >> T_2 > T_\Sigma$, when the controller implementation can rise some problems, and a PID controller is used, the second time constant of the plant, T_2, is compensated by the time constant T_C' of the controller:

$$T_c' = T_2 \ ; \tag{11}$$

- the disturbance is fed to the plant input, when the rejection of the disturbance effects is done very slowly.

For both plants in (2), the open-loop transfer function $H_0(s)$ obtains the form (12):

$$H_0(s) = \frac{k_0(1+sT_c)}{s(1+sT_\Sigma)(1+sT_1)} \ , \ \ k_0(s) = k_c k_P \ . \tag{12}$$

In such situations there are proposed two basic versions for treating the problem [7]:

- the use of a P or PD (eventually, lead-lag) controller tuned according to the Modulus Optimum (MO) method; the result is in a control system with non-zero static coefficient, which is often not acceptable;

- the use of a PI or PID controller tuned according to the Kessler's SO method [1]; the large overshoot in this case is not a generally acceptable solution.

An alternative solution with much better results consists in use of PI or PID controllers in the conditions of applying the optimization relations of the ESO method. These relations can be applied in the case of controllers with both homogenous and non-homogenous information processing. There will be presented as follows aspects concerning the development method in the case with homogenous information processing.

For the plants (2) and $H_C(s)$ having the form (3) and (4), the coefficients in the transfer function $H_w(s)$ (6) can be expressed as:

$$a_0 = k_c k_P \ , \ \ a_1 = 1 + k_c k_P T_c \ , \ \ a_2 = T_1 + T_2 \ , \ \ a_3 = T_1 T_\Sigma \ . \tag{13}$$

By applying the optimization equations (7) and introducing the notation (14):

$$m = T_\Sigma / T_1 \ , \ \ m << 1 \ , \tag{14}$$

the equations for tuning the controller parameters result in terms of (15):

$$k_c = \frac{(1+m)^2}{\sqrt{\beta^3 T_\Sigma' k_P m}} \ , \ \ T_c = \frac{\beta T_\Sigma' \Delta(m)}{(1+m)^2} \ , \tag{15}$$

where:

$$T_\Sigma' = \frac{T_\Sigma}{1+m} \ , \ \ \Delta(m) = m^2 + (2 - \sqrt{\beta})m + 1 \ . \tag{16}$$

For several values $m \in [0, 0.25]$ and $\beta \in [4, 16]$ the expressions of k_c and T_c as function of m can be organized in tabled forms; on the basis of these tables there can be drawn the correction diagrams of controller tuning parameters (k_c and T_c), illustrated in Fig.2. These two diagrams outline the necessity for essential modifications in the values of the tuning parameters with respect to the case presented in the previous Section.

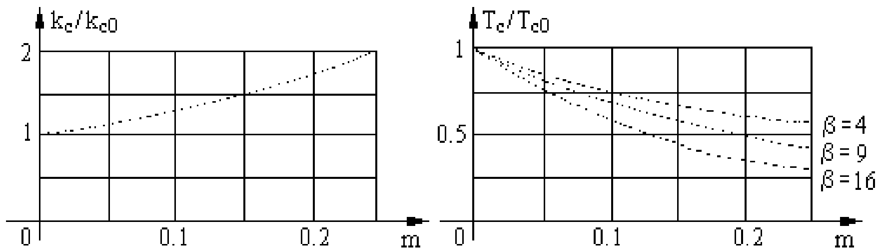

Fig.2. Diagrams of k_c and T_c versus m (k_{c0} and T_{c0} correspond to m = 0).

There are presented in [7] the expressions of the transfer functions $H_0(s)_{opt}$ and $H_w(s)_{opt}$, and aspects concerning the analysis with respect to the modification of the disturbance inputs and the application of the method to the case of non-homogenous information processing.

4 Development Method for a Mamdani PI-Fuzzy Controller

The structure of the standard version of PI-fuzzy controller (PI-FC) with integration of the control signal is presented in Fig.3, and it is based on:

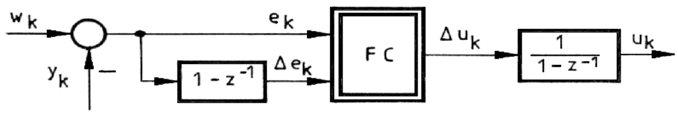

Fig.3. Block diagram of standard PI-fuzzy controller.

- the numerical differentiation of the control error e_k under the form of the increment of control error, Δe_k:

$$\Delta e_k = e_k - e_{k-1} \ ; \tag{17}$$

with k – the index of the current sampling interval;

- the numerical integration of the increment of control signal, Δu_k.

The development of this controller starts with the development of a linear PI controller by using methods dedicated to conventional control systems; in the case of the plants (1), (2) it is recommended to use the methods presented in Section 2 and Section 3. Then, there is expressed the discrete equation of the PI quasi-continuous digital controller in its incremental (velocity type) version (18):

$$\Delta u_k = K_P \Delta e_k + K_I e_k = K_P (\Delta e_k + \alpha \cdot e_k) \ , \tag{18}$$

where the parameters $\{K_P, K_I, \alpha\}$ are functions of $\{k_c, T_c\}$:

$$K_P = k_c T_c \left(1 - \frac{T_s}{2T_c}\right) \ , \ K_I = k_c T_s \ , \ \alpha = \frac{K_I}{K_P} = \frac{2T_s}{2T_c - T_s} \ , \tag{19}$$

where T_s stands for the sampling period.

For the strictly speaking fuzzy controller (the block FC in Fig.3), the fuzzification can be solved in the initial phase as follows:

- for the input variables e_k, Δe_k: there are chosen 5 (or more, but an odd number) linguistic terms with regularly distributed triangular type membership functions having an overlap of 1;

- for the output variable Δu_k there are chosen 7 linguistic terms with regularly distributed singleton type membership functions, Fig.4.

Other shapes of membership functions can contribute to control system performance enhancement.

The considered PI-FC represents a type-II fuzzy system according to [8], [9], having the specific parameters $\{B_e, B_{\Delta e}, B_{\Delta u}\}$. These strictly positive parameters are in correlation with the shapes of the membership functions of the linguistic terms corresponding to the input and output linguistic variables.

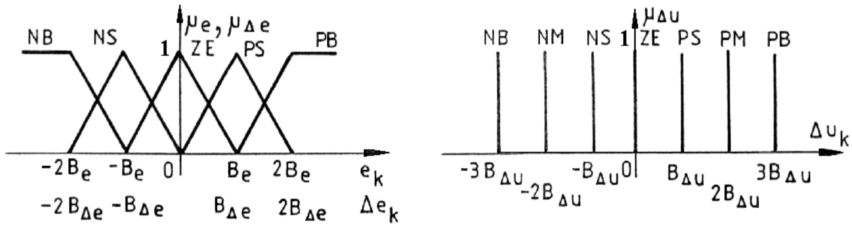

Fig.4. Shapes of membership functions for the standard PI-FC.

The inference engine of the block FC employs the Mamdani's MAX-MIN compositional rule of inference assisted by a complete rule base. The rule base of the block FC is expressed as decision table, and it is illustrated in Table 1.

Table 1. Decision table of standard PI-FC-OI.

$\Delta e_k \setminus e_k$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

For the standard version of PI-FC the defuzzification as part of the block FC is done by the center of gravity method.

It has to be pointed out that the parameters of the basic linear PI controller (3), k_c and T_c, are taken into consideration in $\{B_e, B_{\Delta e}, B_{\Delta u}\}$ by applying this method for tuning the FC parameters. The choice of the inference method and of the defuzzification method as well represents the user's option.

The development of the PI-FC is finalized by applying the modal equivalences principle [10] which can be expressed in terms of the development equations (20):

$$B_{\Delta e} = K_P B_e \ , \ \ B_{\Delta u} = K_I \ B_e \ , \qquad (20)$$

where the value of the parameter B_e is chosen in accordance with the experience of the control systems specialist.

5 Application

For control problems in mobile robots including [11] trajectory tracking, path following and point stabilization, there are widely used several mathematical models including:

- kinematic models [12];
- dynamic models [13-16].

These control problems belong to the general class of controlling nonsmooth or nonholonomic systems [17], [18].

Since the first model in (1) appears in more complex or simpler forms in the mentioned dynamic models, the robot control problems can be simulated by considering it as benchmark-type one-dimensional model [19]. On the other hand, the first transfer function in (1) is necessary for including the model of actuator dynamics and other supplementary dynamics because unmodeled dynamics is considered the major cause for chattering in real-life applications when sliding mode control is employed in robot control [20-22].

The considered case study is characterized by the first transfer function in (1), with the parameters k_P =1 and T_Σ = 1 sec. For controlling this plant, the considered control system structure is a conventional one, presented in Fig.5, where: C – controller, P – controlled plant, Fw – reference filter, w – reference input, $w^{\sim}$ – filtered reference input, e – control error, u – control signal, y – controlled output, v_1, v_2, v_3, v_4 – possible disturbance inputs.

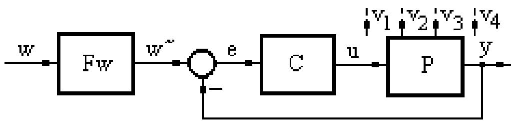

Fig.5. Control system structure.

The ESO method is applied in this case. It starts with choosing β = 2, ans the parameters of the PI controller will obtain the values k_c = 0.3536, and T_i = 2 sec. Part of the digital simulation results are illustrated in Fig.6, in the following simulation conditions: a sinusoidal modification of w followed by a –0.5 step modification of v_4 (after 80 sec), without using the filter Fw. The dash dotted line is used for w, the continuous line for y, and the dotted line for u.

The sinusoidal modification of the reference input is suggestive for control problems in mobile robots.

For the accepted case study, there is developed a PI-FC in terms of Section 4. The parameters of the PI-FC are B_e = 0.3, $B_{\Delta e}$ = 0.03, $B_{\Delta u}$ = 0.0021, and the digital simulation results are presented in Fig.7 in the same simulation conditions as in the case of using the PI controller.

The sliding mode controller is with PI action (sliding mode-PI controller, SM-PI-C), with the structure presented in Fig.8, and the parameters are tuned to guarantee the sliding mode existence condition ($\sigma \cdot \dot{\sigma} < 0$, with σ – the switching variable): c=1, U_0=1, T_i=2. The digital simulation results for the control system with SM-PI-C are presented in Fig.9 and Fig.10 in the accepted simulation conditions.

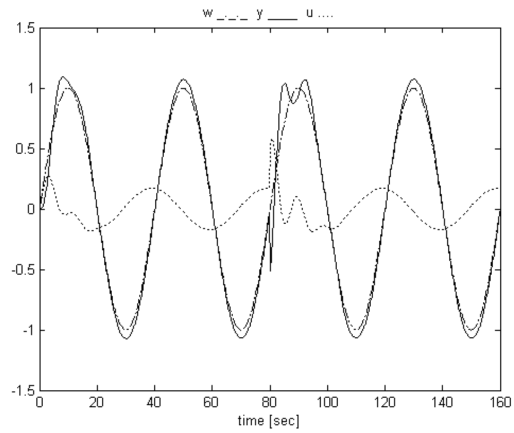

Fig.6. Simulation results for the control system with PI controller.

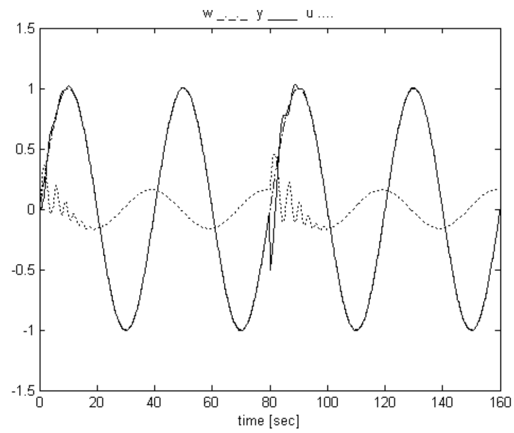

Fig.7. Simulation results for the control system with PI-fuzzy controller.

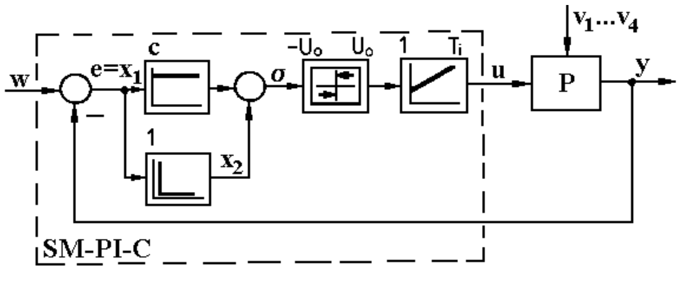

Fig.8. Structure of control system with SM-PI-C.

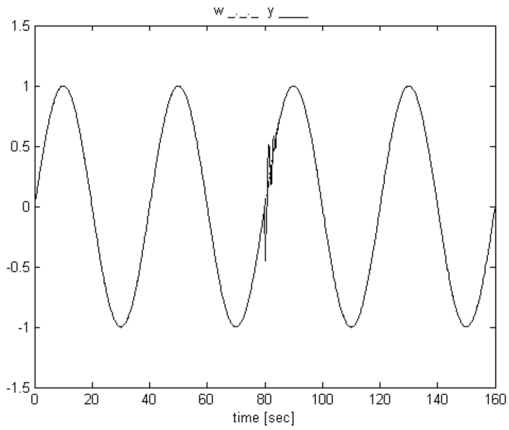

Fig.9. w and y versus time for the control system with SM-PI-C.

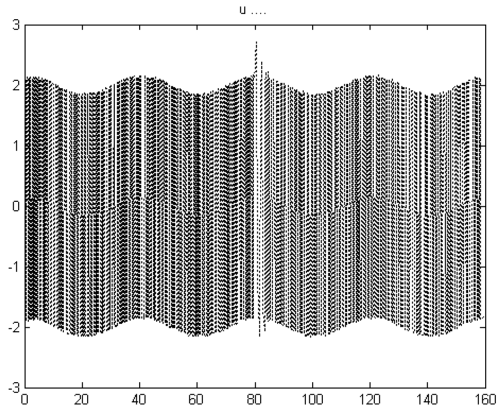

Fig.10. Control signal versus time for the control system with SM-PI-C.

6 Conclusions

The paper presents aspects concerning the development of fuzzy controllers for servo systems. The development is based on applying two methods dedicated to conventional control systems and on applying the modal equivalences principle. The presentation is focused on PI-fuzzy controllers.

The application illustrated in the paper can correspond to a servo system used in control problems related to mobile robots, and validates the presented development methods and controllers for further use in control of servo systems.

Digital simulation results prove that when coping with sinusoidal modifications of the reference input and to step modifications of the disturbance input the PI-fuzzy controllers ensure the control system performance enhancement. This is also the problem of sliding mode controllers, but the chattering alleviation must be performed in this case; solutions for the given control system structure are the use of boundary layer approach [22] or of the discrete-time sliding mode approach [23], [24].

The paper proves the potential of the ESO method, of fuzzy controllers and of sliding mode controllers as attractive solutions in problems of mobile robot control.

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