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Comparison of the Length of the Catenary Curve and its Parabolic Approximation in the Span of an Overhead Line

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Nomenclature

$y_{\text{cat}}(x)$	Catenary curve	
$y_{\text{par}}(x)$	Basic parabola curve	
$y_{\text{par}\,\psi}(x)$	Modified parabola curve	
L_{cat}	Length of the catenary	(m)
L_{par}	Length of the basic parabola	(m)
$L_{\text{par}\,\psi}$	Length of the modified parabola	(m)
S	Span length	(m)
h_1	Height of the left–hand side support point	(m)
h_2	Height of the right–hand side support point	(m)
c	Catenary parameter	(m)
a	Leading coefficient of the basic parabola	(m^{-1})
a_ψ	Leading coefficient of the modified parabola	(m^{-1})
ψ	Angle of the span inclination	$(^{\circ})$
x_{MIN}	x–coordinate of the vertex point	(m)
y_{MIN}	y–coordinate of the vertex point	(m)
D	Maximum sag of the basic parabola	(m)
D_ψ	Maximum sag of the modified parabola	(m)
MIN	Vertex point (low point)	

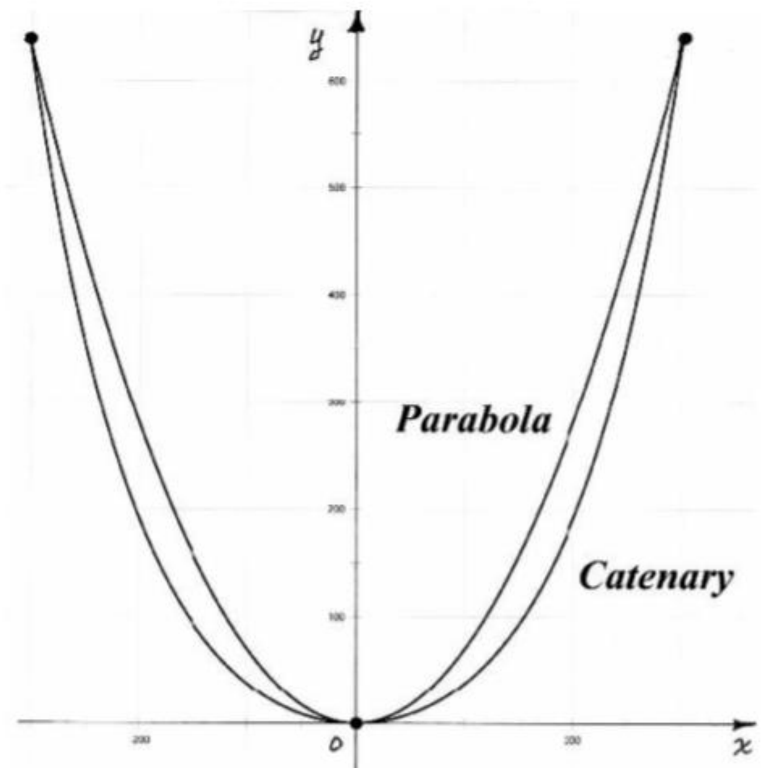
Leveled spans

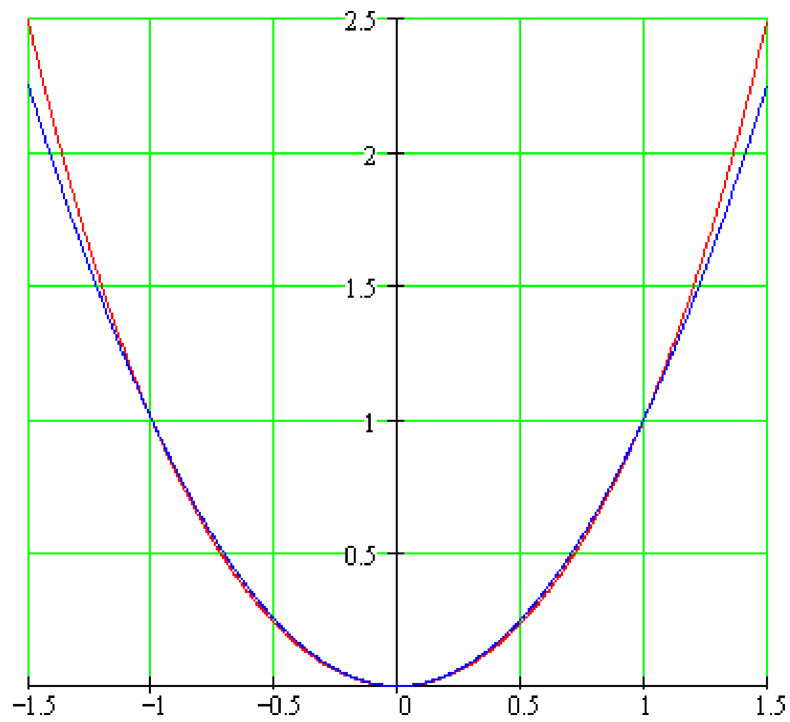
Inclined spans

Necessary Equations, Algorithms, Formulas:

- Universal Equation of the Catenary Conductor Curve
- Universal Equation of the Parabolic Conductor Curve
- Universal Algorithm for the Parabolic Approximation of the Catenary
- Universal Formula for Computation of the Length of the Catenary Conductor Curve
- Universal Formula for Computation of the Length of the Parabolic Conductor Curve

Catenary and Parabola Curves with the Common Vertex Point

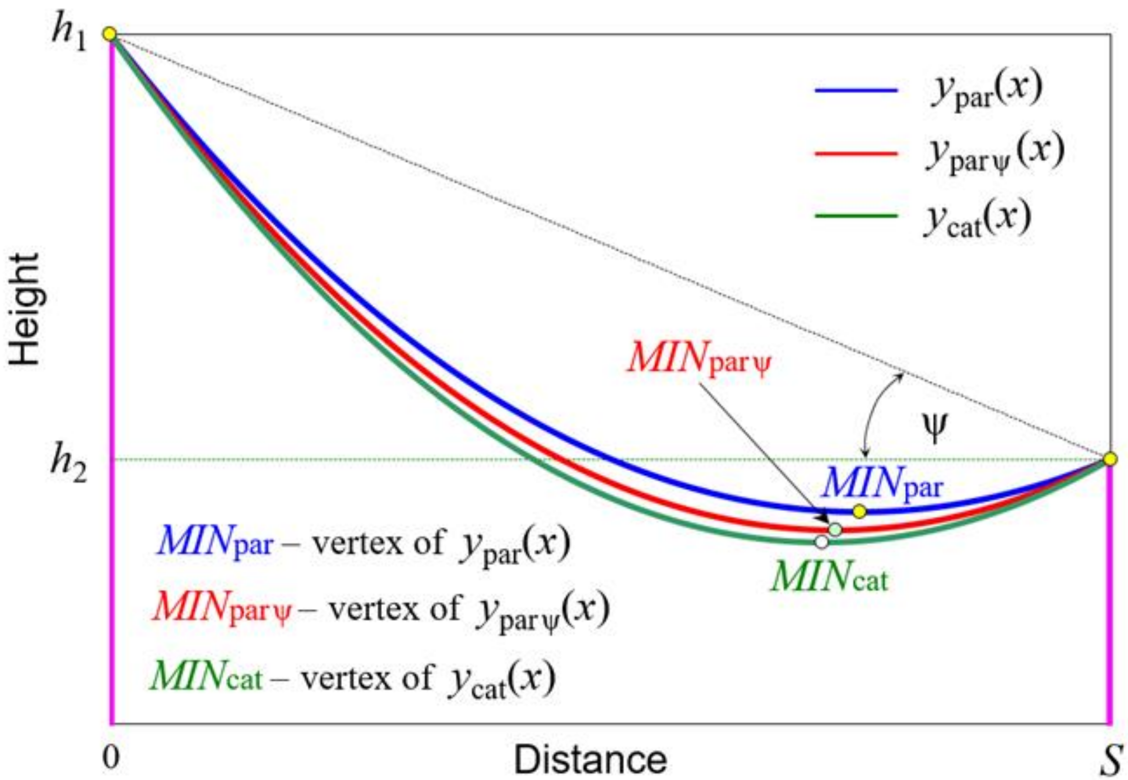




https://link.springer.com/content/pdf/10.1007/s00004-006-0017-7.pdf

http://mathforum.org/library/drmath/view/65729.html

Catenary, Modified Parabola and Basic Parabola Curves in an Inclined Span (type: $h_1>h_2$) and their Vertex Points



$$\frac{1}{\cos\psi} = \left[1 + \left(\frac{h_2 - h_1}{S}\right)^2\right]^{0.5}$$

Universal Equation of the Catenary Conductor Curve

$$y_{\text{cat}}(x) = 2c \cdot \text{sh}^2 \left\{ \frac{1}{2c} \left[x - \frac{S}{2} + c \cdot \text{arsh} \frac{h_2 - h_1}{2c \cdot \text{sh}(S/2c)} \right] \right\} + h_1 - 2c \cdot \text{sh}^2 \left[\frac{S}{4c} - \frac{1}{2} \text{arsh} \frac{h_2 - h_1}{2c \cdot \text{sh}(S/2c)} \right]$$

$$y_{\text{cat}}(x) = 2c \cdot \text{sh}^2 \frac{x - x_{\text{MIN}}}{2c} + y_{\text{MIN}}$$

Universal Equation of the Parabolic Conductor Curve

$$y_{\text{par}}(x) = \frac{4D}{S^2} \left[x - \frac{S}{2} \left(1 - \frac{h_2 - h_1}{4D} \right) \right]^2 + h_1 - D \left(1 - \frac{h_2 - h_1}{4D} \right)^2 \qquad y_{\text{par}}(x) = a \cdot (x - x_{\text{MIN}})^2 + y_{\text{MIN}}$$

Universal Algorithm for the Parabolic Approximation of the Catenary without Applying the Multiplier $1/\cos\psi$

$$y_{\text{par}}(x) = \frac{1}{2c} \left\{ x - \frac{S}{2} \left[1 - \frac{2c}{S^2} (h_2 - h_1) \right] \right\}^2 + h_1 - \frac{1}{2c} \left\{ \frac{S}{2} \left[1 - \frac{2c}{S^2} (h_2 - h_1) \right] \right\}^2 \qquad D = \frac{S^2}{8c}$$

Universal Algorithm for the Parabolic Approximation of the Catenary Applying the Multiplier $1/\cos\psi$

$$y_{\text{par}\,\psi}(x) = \frac{1}{2c \cdot \cos\psi} \left\{ x - \frac{S}{2} \left[1 - \frac{2c}{S^2} (h_2 - h_1) \cos\psi \right] \right\}^2 + h_1 - \frac{1}{2c \cdot \cos\psi} \left\{ \frac{S}{2} \left[1 - \frac{2c}{S^2} (h_2 - h_1) \cos\psi \right] \right\}^2$$

$$a_\psi = \frac{1}{\cos\psi} \cdot a = \frac{1}{2c \cdot \cos\psi} \qquad D_\psi = \frac{1}{\cos\psi} \cdot D = \frac{1}{\cos\psi} \cdot \frac{S^2}{8c}$$

Universal Formula for the Length of the Catenary

$$L_{\text{cat}} = \sqrt{4c^2 \text{sh}^2 (S/2c) + (h_2 - h_1)^2}$$

Universal Formula for Computation of the Length of the Basic Parabola Conductor Curve in a Span

$$L_{\text{par}} = \frac{c}{2} \text{arsh}\left(\frac{S}{2c} + \frac{h_2 - h_1}{S} \right) + \frac{c}{2} \text{arsh}\left(\frac{S}{2c} - \frac{h_2 - h_1}{S} \right) + \\ + \left[\frac{S}{4} + \frac{c(h_2 - h_1)}{2S} \right] \cdot \sqrt{1 + \left(\frac{S}{2c} + \frac{h_2 - h_1}{S} \right)^2} + \left[\frac{S}{4} - \frac{c(h_2 - h_1)}{2S} \right] \cdot \sqrt{1 + \left(\frac{S}{2c} - \frac{h_2 - h_1}{S} \right)^2}$$

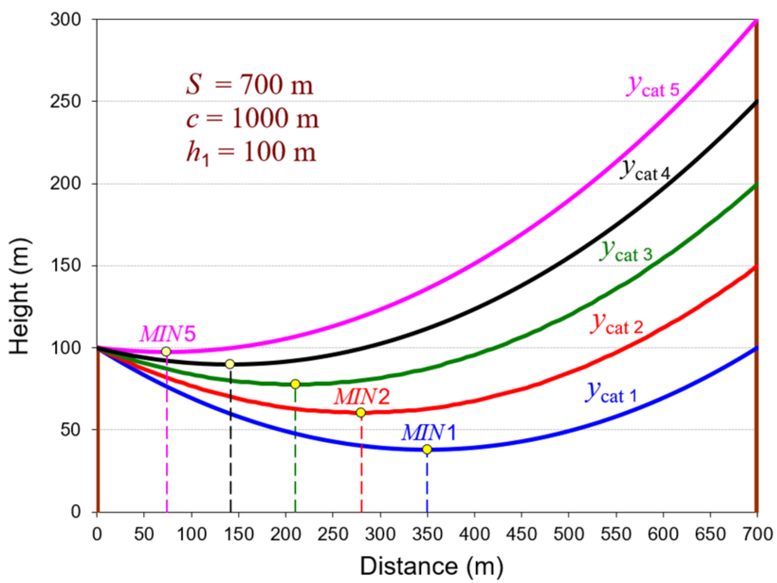
Universal Formula for Computation of the Length of the Modified Parabola Conductor Curve in a Span

$$L_{\text{par}\,\psi} = \frac{c \cdot \cos\psi}{2} \text{arsh}\left(\frac{S}{2c \cdot \cos\psi} + \frac{h_2 - h_1}{S} \right) + \frac{c \cdot \cos\psi}{2} \text{arsh}\left(\frac{S}{2c \cdot \cos\psi} - \frac{h_2 - h_1}{S} \right) + \\ + \left[\frac{S}{4} + \frac{c(h_2 - h_1) \cdot \cos\psi}{2S} \right] \cdot \sqrt{1 + \left(\frac{S}{2c \cdot \cos\psi} + \frac{h_2 - h_1}{S} \right)^2} + \left[\frac{S}{4} - \frac{c(h_2 - h_1) \cdot \cos\psi}{2S} \right] \cdot \sqrt{1 + \left(\frac{S}{2c \cdot \cos\psi} - \frac{h_2 - h_1}{S} \right)^2}$$

Numerical example:

TABLE I. INPUT DATA FOR THE CATENARY CURVES

Data	S (m)	h_1 (m)	h_2 (m)	c (m)
Case 1	700	100	100	10^3
Case 2	700	100	150	10^3
Case 3	700	100	200	10^3
Case 4	700	100	250	10^3
Case 5	700	100	300	10^3



Catenary curves in the spans of the same length but with different inclinations, shown in the same figure related to a numerical example

Catenaries

$$y_{\text{cat}\,1}(x) = 2\cdot 10^3 \cdot \text{sh}^2\left(\frac{x-350}{2\cdot 10^3}\right) + 38.12218$$

$$y_{\text{cat}\,2}(x) = 2\cdot 10^3 \cdot \text{sh}^2\left(\frac{x-280.06620}{2\cdot 10^3}\right) + 60.52444$$

$$y_{\text{cat}\,3}(x) = 2\cdot 10^3 \cdot \text{sh}^2\left(\frac{x-210.47154}{2\cdot 10^3}\right) + 77.76898$$

$$y_{\text{cat}\,4}(x) = 2\cdot 10^3 \cdot \text{sh}^2\left(\frac{x-141.54062}{2\cdot 10^3}\right) + 89.96639$$

$$y_{\text{cat}\,5}(x) = 2\cdot 10^3 \cdot \text{sh}^2\left(\frac{x-73.57069}{2\cdot 10^3}\right) + 97.29246$$

Basic parabolas

$$y_{\text{par}\,1}(x) = 5\cdot 10^{-4}\cdot (x-350)^2 + 38.75$$

$$y_{\text{par}\,2}(x) = 5\cdot 10^{-4}\cdot (x-278.57143)^2 + 61.19898$$

$$y_{\text{par}\,3}(x) = 5\cdot 10^{-4}\cdot (x-207.14286)^2 + 78.54592$$

$$y_{\text{par}\,4}(x) = 5\cdot 10^{-4}\cdot (x-135.71429)^2 + 90.79082$$

$$y_{\text{par}\,5}(x) = 5\cdot 10^{-4}\cdot (x-64.28571)^2 + 97.93367$$

Modified parabolas

$$y_{\text{par}\,\psi 1}(x) = y_{\text{par}\,1}(x) = 5\cdot 10^{-4}\cdot (x-350)^2 + 38.75$$

$$y_{\text{par}\,\psi 2}(x) = 5.0127\cdot 10^{-4}\cdot (x-278.75295)^2 + 61.04941$$

$$y_{\text{par}\,\psi 3}(x) = 5.0508\cdot 10^{-4}\cdot (x-208.57864)^2 + 78.02663$$

$$y_{\text{par}\,\psi 4}(x) = 5.1135\cdot 10^{-4}\cdot (x-140.47091)^2 + 89.90999$$

$$y_{\text{par}\,\psi 5}(x) = 5.2001\cdot 10^{-4}\cdot (x-75.27887)^2 + 97.05316$$

Results

TABLE II. MAXIMUM SAGS OF THE PARABOLAS

	D_ψ (m)	D (m)
Case 1	61.25	61.25
Case 2	61.40605	61.25
Case 3	61.87184	61.25
Case 4	62.64047	61.25
Case 5	63.70096	61.25

TABLE III. LENGTHS OF CATENARIES AND THEIR PARABOLIC APPROXIMATIONS

	L_{cat} (m)	$L_{\text{par}\,\psi}$ (m)	L_{par} (m)
Case 1	714.37946	714.03991	714.03991
Case 2	716.12709	715.79359	715.72366
Case 3	721.34459	721.02879	720.75464
Case 4	729.95754	729.66983	729.07293
Case 5	741.84770	741.59654	740.58204

TABLE IV. DIFFERENCES OF LENGTHS FROM TABLE III

	$L_{\text{cat}} - L_{\text{par}\,\psi}$ (m)	$L_{\text{cat}} - L_{\text{par}}$ (m)	$L_{\text{par}\,\psi} - L_{\text{par}}$ (m)
Case 1	0.33955	0.33955	0
Case 2	0.33350	0.40343	0.06993
Case 3	0.31580	0.58995	0.27415
Case 4	0.28771	0.88461	0.59690
Case 5	0.25116	1.26566	1.01450

Conclusion

From the aspect of the length computation the use of the multiplier $1/\cos\psi$ for the parabola in inclined spans is recommended, since it ensures results closer to the catenary length than in the case when $1/\cos\psi$ is not used.

$1/\cos\psi$ multiplier does not have any influence in level spans (then $\psi=0$) because $\cos(0)=1$.

In inclined spans the modified parabola is longer than the basic parabola, but shorter than the catenary. In level spans the modified parabola is as long as the basic parabola, but shorter than the catenary.

When the span inclination or $|h_2-h_1|$ increases, then the difference in lengths of the catenary and its approximation by the modified parabola decreases, whereas the difference in lengths of the catenary and its approximation by the basic parabola increases. It is expressed by adequate mathematical relations.

$$L_{\text{cat}\,2}^{(inc)} - L_{\text{par}\,\psi\,2}^{(inc)} < L_{\text{cat}\,1}^{(inc)} - L_{\text{par}\,\psi\,1}^{(inc)} \quad \forall \quad \left|h_2^{(2)} - h_1^{(2)}\right| > \left|h_2^{(1)} - h_1^{(1)}\right|$$

$$L_{\text{cat}\,2}^{(inc)} - L_{\text{par}\,2}^{(inc)} > L_{\text{cat}\,1}^{(inc)} - L_{\text{par}\,1}^{(inc)} \quad \forall \quad \left|h_2^{(2)} - h_1^{(2)}\right| > \left|h_2^{(1)} - h_1^{(1)}\right|$$

Thank you for your attention !