On Integrity Problem of Inference Systems in Adaptive Fuzzy-Neural Computing

Georgi M. Dimirovski[1 & 2] and Dijana J. Tanevska[3]

[1] Dogus University, Faculty of Engineering, Department of Computer Eng.

Acibadem – Kadikoy, TR-34722 Istanbul, Republic of Turkey

FAX: +90-216- 327-9631; E-mail: gdimirovski@dogus.edu.tr

[2] SS Cyril & Methodius University, Institute of Automation & Systems Eng.

Faculty of Electrical Eng., MK-1000 Skopje, Republic of Macedonia

[3] Makedonski Telekomunikacii AD, MK-1000 Skopje, Rep. of Macedonia

Fax:+389 2 3127 219; E-mail: dijana.tanevska@mt.com.mk

Abstract: *This paper explores the learning fuzzy inference systems implemented as adaptive fuzzy-neural networks. The research into application of learning techniques to fuzzy inference systems (FIS) has matured into a family of adaptive fuzzy inference systems (AFIS). In most cases, the learning FIS and AFIS families can be interpreted as a partially connected multilayer feedforward neural network with Gaussian activation function for the hidden neurons. The connection can be interpreted in terms of rules. Often, these rules are designed a priori implying the connections are a priori fixed, and their strengths can be adapted from input and output data. However, the strengths of the rules and membership-function parameters are adapted in the learning process from an input-output training data set, such that the error function is minimized. The latter as well as information granulation gave rise to integrity problem which must be observed in applications.*

Keywords*: Integrity preservation, hard limit, rough fuzzy sets, soft limit, uniform and non-uniform granulation.*

1. Introduction

Since 1965 when L. A Zadeh has put forward the concepts of fuzzy sets and systems allowing for degrees of truthfulness mimicking the reality of human thinking and of approximate reasoning subsequently (1973, 1975), computational (machine) intelligence has evolved as much as to attained both theoretically and practically the status of ‘computing with words’ (Zadeh, 1994, 1996). These developments of computational intelligence have remarkable evolved into learning fuzzy-neural or neuro-fuzzy systems (e.g. see Brown and Harris, 1994; Cherkassky and Mulier, 1997; Jang et al, 1997; Kosko, 1992; Kruse and Nauk, 1998), and also led to fuzzy-Petri systems (Pedrycz and Gromide, 1994; Dimirovski, 1998). In particular, these have found a number of applications in areas of control and decision (e.g. Lin and Lee, 1991; Jang and Sun, 1995) a general overall model of which is illustrated in Figure 1 (Dimirovski and Jing, 2003).

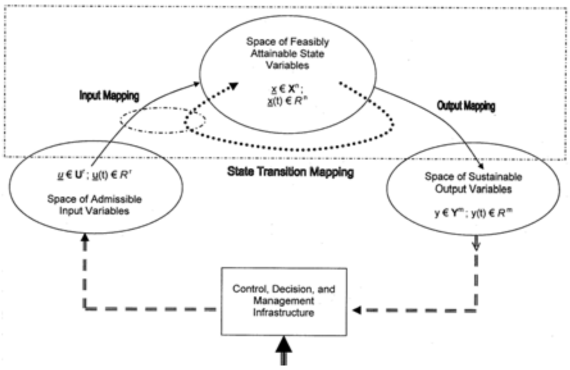

Fig. 1. Integrated control, decision and supervision in engineering and system-theoretic terms.

Fuzzy inference systems (Zadeh, 1973, 1975), FIS for short, lie in its very core regardless of the theory and the technology of implementation on which systems engineering designs are based (Zadeh, 1994). Their subsequent developments has led to the advent of adaptive fuzzy inference systems (AFIS) implemented by means of fuzzy-neural networks (e.g. see Buckley and Hayashi, 1994, 1995; Jang, 1993; Keller and Tahini, 1992; Lotfi and Tsoi, 1996).

In particular, it may well be said that Jang's creation of ANFIS (1993) has generated a new paradigm in fuzzy-neural computation. Its potential towards generalized AFIS was further explored in Jang et al. (1997), see Figure 2, which gave hints to commence our investigation (Dimirovski and Tanevska, 1999) this paper being one of the outcomes.

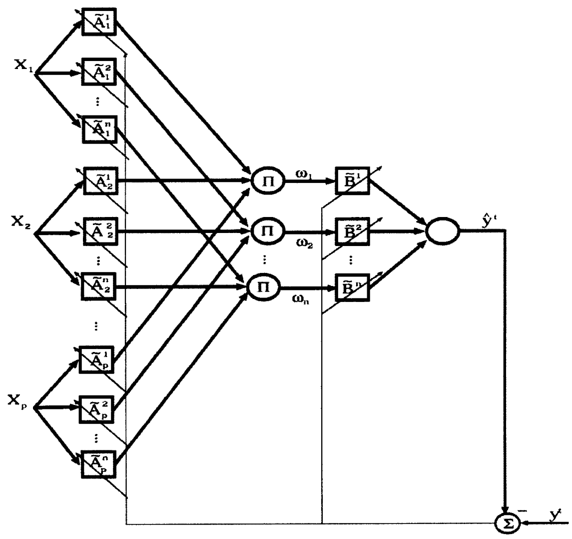

Fig. 2. Schematic of an adaptive fuzzy-neural system

One of the main advantages of AFIS over classical learning systems and neural networks is their ability to utilize intuitive knowledge, which may be presented in a linguistic form such as employed by humans. The knowledge once stored in the membership functions (MFs) and rules of the system, preserving the integrity of this knowledge is as desirable as in continuing human learning and let alone in industrial applications. This means the engineering intelligent system would be able to use the same intuitive understandings, which has been used to create the FIS, to interpret its behavior at all times in the future. For it has become clear (Watkins, 1996) that this ideal cannot be guaranteed for AFIS designs. Yet the purpose of generalized fuzzy-neural inference system models has to exploit evolving antecendents (MFs) and consequents (fuzzy rules), and input space partitions (Jang et al., 1997).

In AFIS designs, the MFs of an AFIS may lose the meaning which is initially assigned to them if allowed to adapt freely. For instance, they may change their relative positions such that "low" may become greater than "high", or the range of their activations may become excessively wide or narrow. This has emphasized the importance of preserving the physical meaning of MFs albeit until recently no close and deeper studies on restrictions to their adaptability during tuning have been carried out (Cherkassky, 1998; Jang et al., 1997; Lotfi and Tsoi, 1996; Tanevska, 2002).

Namely, it has been observed that although the adapted MFs may have attained a new significance, after the original meaning has been lost, it may be very difficult or undesirable to interpret. In some cases, an AFIS may have changed to such a degree that a conventional linguistic interpretation is no longer possible. In such a case, the AFIS may be viewed as a "black box" approximated function similar in function to a neural network. All these possibilities make a conventional AFIS unsuitable for many control and decision applications in which maintainability and reliability are of prime importance, despite their likely superior performance relative to ones which may have restricted range of MFs parameter variations.

The rest of the paper is organized as follows. Next Section 2 indicates how the above can be achieved, gives a brief address to information granulation in fuzzy variables and rules of the knowledge base, and an outline of the adequate learning algorithm. Section 3 is focused on the use of rough fuzzy sets and the limiting bounds of grades of membership functions (MFs) within the learning algorithm operation. Conclusion and references follow thereafter.

2. Granulation of Fuzzy Variables and Fuzzy Rules

It is a general requirement that the MF assigned to a fuzzy value should not exceed certain maximum and minimum limits of fuzziness after adaptation. If the similarity between the initial MF and the MF during training is measured, when this similarity measure exceeds its limit, the linguistic meaning assigned to the MF is said to be lost. In case when a MF becomes too narrow, meaning that it has a smaller similarity, it will be totally deleted. Alternatively, two or more MFs can be merged into a single MF, when they are very similar. In semantic similarity has been used to apply a constraint on the fuzzy values and the fuzzy functional dependency (FFD) for relational data bases.

In order to cope with the above problem, the concept of rough sets has been proved useful in constructing constrained training algorithms that maintain the integrity of AFIS during training (Brown and Harris; 1994; Cherkassky and Mulier, Jang et al, 1997; Tanevska, 2002). This may be viewed as a paradigm that enables the adaptive fuzzy controller to adapt itself in manner so as to remain still conceptually comprehensible to a human expert.

In order to achieve proper self-adaptation of fuzzy controllers (with embedded AFIS) that still remain conceptually comprehensible to a human expert a tradeoff between performance measure cost function is indispensable. The idea is to allow for a certain slight degradation of the performance of the AFIS (reflected in value level of the cost function) in the sense that the error function may attain a higher value than in the case when the MFs are allowed to adapt freely. In most cases in control and decision applications, this tradeoff is acceptable because the ability to interpret the behavior of the AFIS is more important essentially than to achieve a lower minimum in the cost function. For this purpose the concept of constrained training algorithm for AFIS has to be employed, which is directly related to the granulation problem of fuzzy variables and fuzzy rules in the knowledge base

(Dimirovski and Tanevska, 1999), which in turn are closely related to linguistic modeling (Pedrycz and Vasilakos, 1999). For all these reflect on the training algorithms.

Let us recall that adaptive fuzzy systems can essentially be classified into two groups; one having a *uniform* granulation, and the other having a *non-uniform* granulation of universe of discourse of the inputs and outputs. In the uniform granulation AFIS, initially the universe of discourses are divided into uniform partitions with linguistic meaning. In non-uniform granulation system, the linguistic understandability of fuzzy system is not necessary and only nonlinear mapping is of prime concern. The non-uniform granulation can be divided into different subclasses, e.g. tree partition, scatter partition.

Most commonly, for the purpose of control and decision the uniform granulation is used. Assuming a certain number of rules with some initial MFs for antecedent and consequent of each rule, a gradient descent training algorithm (and others too) can be employed. This training algorithm minimizes the output error by tuning the membership function parameters.

In order to clarify the issue of concern that distinguishes these two main groups of adaptive fuzzy systems refer to illustration in Figure 3 below. Figure 3-a shows a uniform granulation FIS for a two input-one output system which is granulated into three individual MFs for the first input, x_1, and two individual MFs for the second input, x_2. A maximum of six rules can be formed to specify the behavior of this FIS. In contrast, Figure 3-b depicts a non-uniform granulation FIS with three rules and three individual MFs for each input.

The concept of integrity preservation becomes significant when the granulations are uniform, that is, a set of meaningful linguistic rules are available. In fact, often this is the starting point in the design of a FIS. On the other hand, it should be noted that non-uniform granulation AFIS are most commonly used for clustering applications. In this case, the concept of rules is not important, thus the notion of integrity after training is not essential. This is because the system does not have any specific physical meaning before training in the sense of rules being attached to any clusters which might have emerged.

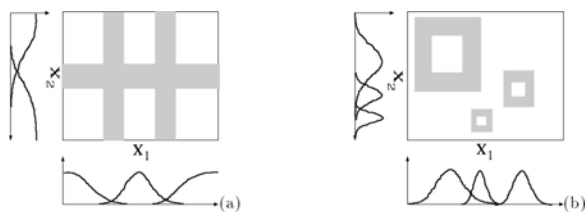

Fig. 3. The issue of granulation in fuzzy if-then rules

In general, the most famous types of AFIS systems can be used with a uniform or non-uniform granulation. In present paper, we focus our attention on so-called Type II adaptive systems (also see Figure 2) having the following fuzzy if-then rules and configuration employed for the modeling of linguistic information:

$$
\begin{aligned}
R^i:\ &If\ \ x_1\ is\ A_1^i\ \ and\ \ldots\ \ x_j\ \ is\ \ A_j^i\ \ldots \\
&\ldots\ and\ \ x_p\ \ is\ \ A_p^i\ \ then\ \ y\ \ is\ \ B^i; \qquad (1) \\
&i = 1, 2, \ldots, n.
\end{aligned}
$$

Here the symbols denote: $x_j \quad (j = 1,2,\ldots p)$ is the label of j^{th} input variable, y the output; $A_j^i \quad (i = 1,2,\ldots,n \quad and \quad j = 1,2,\ldots p)$ are fuzzy labels, and B^i are real numbers; n and p are the numbers of rules and individual inputs respectively. The number of individual MFs for a specific input value $x_j \ (A_j^1, A_j^2, \ldots, A_j^n)$ is given by $\hat{p}_j \ (\sum_{j=1}^{p} \hat{p}_j = m)$.

In the present investigations, including the simulation experiments (Tanevska, 2002), the MFs of the linguistic values, A_j^i are defined by Gaussian functions and are given as follows:

$$A_j^i = \exp\left(\left(\frac{x_j(k) - \sigma_j^i}{\rho_j^i}\right)^2\right) \ , \qquad (2a)$$

$$j = 1, 2, \ldots, p, \quad i = 1, 2, \ldots, n \ , \qquad (2b)$$

where σ_j^i and ρ_j^i are unknown constant parameters. It is these parameters that can be adjusted on-line by making use of a gradient descent algorithm. The decision, $\hat{y}(k)$ at k^{th} instant, as a function of inputs $x_j(k)$ is obtained from following equation:

$$\hat{y}(k) = \frac{\sum_{i=1}^{n} \omega^i B^i}{\sum_{i=1}^{n} \omega^i} \ , \qquad (3)$$

where B^i is the consequent parameter and ω^i is the rule firing strength given by:

$$\omega^i = \prod_{j=1}^{p} A_j^i(x_j(k)) \ \ i = 1,2,\ldots,n \qquad (4)$$

The updating of the parameters in the AFIS can be implemented in two ways: *partial updating* and *full updating.* Partial updating involves changing only the parameters of the consequent *part* of rules, while keeping the parameters in the antecedent part constant: and full updating implies changing all parameters in both the antecedent and consequent parts of the fuzzy if-then rules. Full updating allows the system more freedom in adapting to the circumstances and subsequently results in a better performance for AFIS.

Wang (1997) has carried out wide ranging studies on the theoretical aspects of adaptive fuzzy controllers with partial updating. For example: partial update for nonlinear function approximation is employed in the analysis of the taste of rice. Usually an adaptive fuzzy system with uniform granulation of inputs is considered. The parameters in the antecedent and the consequent premises are defined as $\Theta_{ij} = \left[\sigma^i_j, \rho^i_j, B^i\right]$. To update Θ_{ij} we can use a steepest descent gradient method to minimize the cost function J. The value $\Delta\Theta_{ij}$ at the $(k+1)^{th}$ instant as a function of the $\Delta\Theta_{ij}$ at the k^{th} instant is obtained as follows:

$$\Delta\Theta_{ij}(k+1) = -\eta\nabla J_{ij} + \alpha\,\Delta\Theta_{ij}(k) \qquad (5)$$

where $\eta, \alpha, \nabla J_{ij}$ are the learning rate , momentum and the gradient of the parameters. The proposed constrained tuning method is carried out for N epochs. The parameters will be updated after each iteration using the following update rule

$$\Theta_{ij}(k+1) = \Theta_{ij}(k) + \Delta\Theta_{ij}(k+1)\ x\ \Re(\Theta_{ij})\ . \qquad (6)$$

A restriction function, $\Re$, specifies the constraint on the updating of the parameters. Then the integrity of MFs are not substantial, meaning that there is no constraint on the parameters, the restriction function $\Re = 1$. When $\Re = 1$, there is no limitation on MF parameters and they can be adapted freely. If we apply the restriction on the MF parameters, for each parameter σ^i_j, ρ^i_j , a dedicated restriction function $\Re_{\sigma^i_j}$, $\Re_{\rho^i_j}$ ought to be employed. Its choice is not unique but depends on the application domain (discussed in next section). When partial updating is used, i.e. the parameters of MFs in the antecedent are fixed and only the parameters of consequent are adaptable, restriction function becomes equal to one.

3. On Limits of Membership Function Grades and Concept of Rough Fuzzy Sets

In addition, let consider the class of typical MFs of Gaussian form given in equation (2). This is specified by two parameters: σ, ρ and universe of discourse $[X^-, X^+]$. The grade of MF can be given a linguistic label in a specific universe. For example, it can be defined with the linguistic label, "medium". If absolute maximum and minimum levels are defined, then the membership of the fuzzy label is limited to those bounds. These bounds can be hard or soft, as defined in the following subsections. It is to be noted that these limit bounds are widely applicable and not solely for the bell shaped Gaussian MFs. In principle, limit bounds can be introduced for any form of MFs, e.g. triangular, sigmoid, etc., to ensure these remain within certain bound (hard or soft) specified by an application domain expert.

3.1. Hard Limit of Membership Functions

Consider the MF given in equation (2) with two generic parameters: σ, ρ representing the mean and the spread of Gaussian bell shape. To preserve the linguistic label assigned to this shape, the parameters of the Gaussian function are allowed to move only within a. certain limit. To be specific: $\underline{\rho} \le \rho \le \overline{\rho}$ where $\underline{\rho}$ *and* $\overline{\rho}$ are respectively the lower and upper bounds in which the linguistic variable can move without destroying the interpretation of the "variable". In a similar manner, we have $\underline{\sigma} \le \sigma \le \overline{\sigma}$ where $\underline{\sigma}$ *and* $\overline{\sigma}$ the lower and tipper bounds of σ are respectively.

These restrictions can be applied to the MF tuning algorithm. The restriction function, $\Re_\sigma, \Re_\rho$ are shown in figure (2-a) and governed by the following set of equations:

$$\Re_\sigma = \operatorname{sgn}(\sigma - \underline{\sigma}) - \operatorname{sgn}(\sigma - \overline{\sigma}), \qquad (7)$$

$$\Re_\rho = \operatorname{sgn}(\rho - \underline{\rho}) - \operatorname{sgn}(\rho - \overline{\rho}), \qquad (8)$$

where

$$\operatorname{sgn}(c) = \begin{cases} 1 & \textit{if } c > 0 \\ -1 & \textit{otherwise} \end{cases}.$$

Thus, if $\underline{\sigma} \le \sigma \le \overline{\sigma}$ i.e. σ lies within the permissible range, then $\Re_\sigma = 1$. On the other hand, if $\sigma \le \underline{\sigma}$ then $\Re_\sigma = 0$; if $\sigma > \overline{\sigma}$ again $\Re_\sigma = 0$. Hence, regardless of whether or not a minimum solution for the cost function J is achieved or not, the hard bounds do not permit the MF parameters to move beyond the defined limits. It preserves the integrity at the possible expense of yielding a less optimal solution. The soft limit, which is proposed in the following subsection, is expected to give a better performance.

3.2. Soft Limit of Membership Functions

By using a soft limit bound, we can allow the MF parameters to be updated with a varying degree of scaling. As the parameters approach their predefined maximum and minimum limits, smaller and smaller updates are performed.

The hard bounds introduced in the previous section can be replaced by a soft bound of the following form:

$$\Re_\sigma = \frac{1}{1+e^{-\left(\frac{\sigma-\underline{\sigma}}{v_\sigma}\right)}} - \frac{1}{1+e^{-\left(\frac{\sigma-\overline{\sigma}}{v_\sigma}\right)}} \qquad (9)$$

$$\Re_\rho = \frac{1}{1+e^{-\left(\frac{\rho-\underline{\rho}}{v_\rho}\right)}} - \frac{1}{1+e^{-\left(\frac{\rho-\overline{\rho}}{v_\rho}\right)}} \qquad (10)$$

where $v_\sigma \, and \, v_\rho$ are respectively the dispersion parameters of $\sigma \, and \, \rho$. These are introduced so that a "gentle roll off" be achieved (Figure 3 b) instead of a hard limit occurring at the boundary (Figure 3 a). The above restricting functions for the purpose of obtaining 10 percent distortion around the boundary values, $v_\sigma = 0.045\overline{\varsigma}$ and $v_\rho = 0.045\overline{\rho}$, are depicted in Figure 3-b.

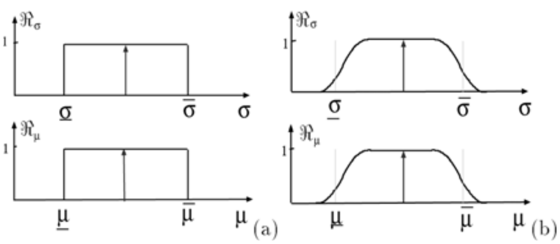

Fig. 3. Limit bounds of membership function parameters

This methodology allows certain penetration of the parameter values beyond their hard limits. However, by controlling the parameter v the extent of the penetration can be adjusted, hence a certain adaptation as most appropriate.

4. Conclusion

The integrity of AFISs can indeed be preserved by using a constrained training algorithm. For this purpose both hard and soft limit bounds for the parameters should be employed. It should be noted, however, a certain tradeoff has to be carefully reached. At design stage, care has to taken about the needed tradeoff between obtaining the minimum of the performance measuring cost function, and the preservation of the integrity in the sense of the interpretability of the converged AFIS.

Should reaching the minimum of the cost function be the primary goal, then it may well be observed that the MFs can be radically altered from their initial definition. This in turn may render the converged AFIS uninterruptible. Should preservation of the integrity of the AFIS, after the adaptation process has converged, be of primary importance, then it may well be found that the cost function can only be attained at a higher value. In this case, the MFs retain their original meaning and can be interpreted properly.

In the case of function approximation for wider range of applications, the AFIS without any constraints may achieve a slightly better fit to the input-output data, than the one with constraints. However, the one with constraints provides MFs which can be interpreted, while the one without constraint may not allow this.

The way in which the constraints have been introduced in this discussion is only one among a many potential possibilities. The one discussed in here has been found rather appropriate for control and decision applications where the integrity is of primary importance. Should it be desirable or necessary to preserve the integrity of AFIS, the techniques presented in here can be applied directly. Ultimately, in AFIS designs, the choice of using an AFIS without or with constraints remains application domain and purpose dependent. Hence the main systems engineering reasons in each particular design must be observed.

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