Sliding mode control of rigid manipulators using generalized velocity components

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Abstract: *The paper describes a proposition of sliding mode trajectory tracking in joint space of manipulator using generalized velocity component (GVC) vector. Inertial quasi-velocities called GVC were presented by Loduha and Ravani [5]. Their introduction leads to first-order decoupling equations of motion instead of one second-order equation. There are shown some differences between classical sliding mode control according to Slotine and Li [7] and proposition expressed in terms of GVC. Both controls were tested on* $3\ d.o.f.$, $3-D$ *Yasukawa-like robot.*

Keywords: Manipulator, trajectory tracking, sliding mode control, matrix equations, manipulator mass matrix.

1 Introduction

The problem of trajectory control is concerned with the case when manipulator ought to follow a desired trajectory. The second-order nonlinear differential equations of motion are traditionally used in robotics literature [6, 8, 9]. They involve generalized position vector and velocity vector which represent a joint space of manipulator. For tracking purposes the joint space trajectory the nonlinear control law called inverse dynamics control is often proposed [6, 9]. The system under this control is linear and decoupled with respect of the new assumed input. However Slotine and Li introduced [7] so called sliding mode approach for adaptive control of robot manipulators. The idea of this method relies on exploitation the structure of Lagrangian formulation for rigid manipulators without linearization its dynamic equations. Their conception, widely described in [8], is also actual in a case of no parameters uncertainty. Both methods, inverse dynamics control and sliding mode control, are used for tracking of desired, and time dependent trajectory.

For the purpose of control decoupled first-order equations of motion seem more convenient. Dynamics in terms of these equations was described e.g. by Loduha and Ravani [5]. Based on Kane's equations the authors have presented first-order decoupled equations with diagonal mass matrix. Diagonalization of this matrix is realized in velocity space. As a result the obtained diagonal mass matrix $N(\theta)$ is congruent to the matrix $M(\theta)$.

The objective of this paper is a presentation of sliding mode control in terms of generalized velocity components (GVC) vector introduced in [5] and giving some suggestions relating to it.

The rest of the paper is organized as follows. The second Section gives diagonalized equations of motion in terms of GVC. In the third Section the sliding mode control in joint space of manipulator is described. Also some remarks regarding this control are contained there. Simulation results comparing classical controller and the proposed here for 3 $d.o.f.$, $3-D$ Yasukawa-like robot are presented in the fourth Section. The last section contains conclusions.

2 Dynamics in terms of GVC

Recall that the classical equations of motion are written in the following form [7, 8]:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau, \tag{1}$$

where $\mathcal{N}$ - number of degrees of freedom,
$\theta, \dot{\theta}, \ddot{\theta} \in R^{\mathcal{N}}$ - vectors of generalized positions, velocities, and accelerations, respectively,
$M(\theta) \in R^{\mathcal{N}\times\mathcal{N}}$ - system mass matrix,
$C(\theta,\dot{\theta}) \in R^{\mathcal{N}}$ - vector of Coriolis and centrifugal forces in standard equations of motion,
$G(\theta) \in R^{\mathcal{N}}$ - vector of gravitational forces in standard (classical) equations of motion,
$\tau \in R^{\mathcal{N}}$ - vector of generalized forces.
Assuming that there exist some positive constant $\beta_m, \beta_M, \beta_c, \beta_g$, and vector x the following properties can be established [1, 6, 8] (I denotes the identity matrix):
(P1). The inertia matrix $M(\theta)$ satisfies the inequality $\beta_m I \leq M(\theta) \leq \beta_M I$ $\forall\theta \in R^{\mathcal{N}}$.
(P2). Matrix $C(\theta,\dot{\theta})$ satisfies $C(\theta,\dot{\theta}) \leq \beta_c||\dot{\theta}||, \forall\dot{\theta} \in R^{\mathcal{N}}$.
(P3). One can define skew symmetric matrix
$x^T[\frac{1}{2}\dot{M}(\theta) - C(\theta,\dot{\theta})]x = 0, \forall x \in R^{\mathcal{N}}$.

(P4). It can be shown that $\dot{M}(\theta) = C^T(\theta, \dot{\theta}) + C(\theta, \dot{\theta})$.
(P5). The gravity vector $G(\theta)$ is bounded as $||G(\theta)|| \leq \beta_g, \forall \theta \in R^{\mathcal{N}}$.

The method described by Loduha and Ravani is based on searching a congruence matrix to mass matrix of the system (compare for details [5]). The obtained first order equations of motion are modified Kane's equations.

For robot manipulator one can write two first order equations: diagonalized equation of motion and velocity transformation equation, respectively:

$$N\dot{u} + C(\theta, u)u = \pi \tag{2}$$
$$\dot{\theta} = \Upsilon u \tag{3}$$

where matrices and vectors are given as follows ($\dot{\Upsilon}$ denotes time derivative of Υ):

$$N = \Upsilon^T M(\theta)\Upsilon \tag{4}$$
$$C(\theta, u) = \Upsilon^T[M(\theta)\dot{\Upsilon} + C(\theta, \dot{\theta})\Upsilon] \tag{5}$$
$$\pi = \Upsilon^T(\tau - G(\theta)). \tag{6}$$

In equations (2)-(6) N is a diagonal matrix congruent to mass matrix of manipulator $M(\theta)$ (this matrix can be obtained using method described in [5]), $u, \dot{u}$ are vectors of generalized velocity components and their time derivatives, respectively, $C(\theta, u)$ is a new Coriolis force vector and π is a vector of quasi-forces. The invertible matrix Υ [5] transforms joint velocities into generalized velocity components space.

Remark 1. The mass matrix Υ arises from decomposition of matrix $M(\theta)$ therefore (P1) guarantee boundedness of matrix N. From (P2)-(P4) one can conclude that also $C(\theta, u)$ is bounded (matrix $\dot{\Upsilon}$ results from $\dot{M}(\theta)$). Also vector $\Upsilon^T G(\theta)$ is bounded because of property (P1) and (P5).

Referring to modified Kane's equations [5] one can write for a manipulator (the appropriate equivalence was shown in [3]): $M(\theta) = \sum_{k=1}^{\mathcal{N}}[m_k J_k^T J_k + \Omega_k^T I_k \Omega_k]$,
$C(\theta, \dot{\theta} = \sum_{k=1}^{\mathcal{N}}[(m_k J_k^T \dot{J}_k + \Omega_k^T I_k \dot{\Omega}_k)\dot{\theta} + \Omega_k^T W_k I_k \omega_k]$,
$G(\theta) = -\sum_{k=1}^{\mathcal{N}} J_k^T f_k, \quad \tau = \sum_{k=1}^{\mathcal{N}} \Omega_k^T \tau_{Rk}$
where m_k is the mass of k-th body,
J_k is the partial derivative of k-th body's mass center position with respect to the inertial reference frame,
Ω_k is the partial derivative of body k-th angular velocity with respect to the time derivative of the generalized coordinates vector,
I_k is the central inertia matrix,
W_k is the angular velocity matrix associated with the i-th body, and written in terms of body k-th natural frame,
ω_k is the angular velocity of k-th body,
f_k is the resultant active force acting at the mass center of the k-th body,
τ_{Rk} is the k-th resultant moment.

Remark 2. Very important result arising from Eqs.(3) and (4) is that kinetic energy of the manipulator is expressed as:

$$\mathcal{K}(\theta, u) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} = \frac{1}{2}u^T \Upsilon^T M(\theta)\Upsilon u = \frac{1}{2}u^T N u. \tag{7}$$

This result denotes that for $\mathcal{N}$ linked bodies $\mathcal{K}(\theta, u) = \frac{1}{2}\sum_{k=1}^{\mathcal{N}} N_k u_k^2$. Therefore one can consider each inertial quasi-velocity u_k separately in a sense of the kinetic energy. It is because a part of the kinetic energy of each link concerning internal connections is transferred into u_k inertial quasi-velocity.

Remark 3. Because the matrix Υ is invertible, from Eq.(3) arises that $u = \Upsilon^{-1}\dot{\theta}$. Each of components of vector u can be written as $u_k = \sum_{i=1}^{\mathcal{N}} \Upsilon_{ki}^{-1}\dot{\theta}_i$. The presence of coefficients Υ_{ki}^{-1} and joint velocities $\dot{\theta}_i \neq \dot{\theta}_k$ is a consequence of couplings among manipulator links. Calculating the Euclidean norm of vector u_k one can conclude that under condition $||u_k|| > ||\dot{\theta}_k||$ a control which uses this vector may guarantee its faster convergence than using only $\dot{\theta}_k$. Recall also because of the inequality (for Euclidean norm) $||\Upsilon^{-1}\dot{\theta}|| \leq ||\Upsilon^{-1}||\ ||\dot{\theta}||$ the similar condition for the entire vector u is expressed as $||\Upsilon^{-1}|| > 1$.

3 Sliding mode control in joint space using GVC

Firstly recall sliding mode control in joint space of a manipulator according to [6, 8] for classical description this control in joints space is as follows:

$$\tau = M(\theta)\ddot{\theta}_r + C(\theta, \dot{\theta})\dot{\theta}_r + G(\theta) + k_D s. \tag{8}$$

The used symbols denote: $\ddot{\theta}_r = \ddot{\theta}_d + \Lambda\dot{\tilde{\theta}},\quad \dot{\theta}_r = \dot{\theta}_d + \Lambda\tilde{\theta}$ with $\ddot{\theta}_d$ as desired joint acceleration vector and $\tilde{\theta} = \theta_d - \theta$, $\dot{\tilde{\theta}} = \dot{\theta}_d - \dot{\theta}$ the joint velocity error, and joint error between the desired and actual posture, respectively. Matrix Λ is constant and has eigenvalues strictly in the right-half complex plane and k_D is a constant positive definite control gain matrix. The vector s is defined as $s = \dot{\tilde{\theta}} + \Lambda\tilde{\theta}$.

Consider now the problem of sliding mode control in terms of GVC. Because matrix Υ is invertible hence for reference trajectory one can write $u_r = \Upsilon^{-1}\dot{\theta}_r$. Calculating time derivative of $\dot{\theta}_r = \Upsilon u_r$ one can obtain $\ddot{\theta}_r = \dot{\Upsilon}u_r + \Upsilon\dot{u}_r$ and hence $\dot{u}_r = \Upsilon^{-1}(\ddot{\theta}_r - \dot{\Upsilon}u_r)$.

Remark 4. Assumed reference trajectory remembers the case of Cartesian space control [7] (instead of manipulator Jacobian matrix we use transformation matrix Υ^{-1}). But the main difference lies on introduction of sliding surface which involves both kinematic and dynamic parameters of the manipulator. Sliding surface in terms of u can be defined as follows:

$$s_u = u_r - u = \Upsilon^{-1}(\dot{\tilde{\theta}} + \Lambda\tilde{\theta}). \tag{9}$$

The above sliding surface is similar as for Cartesian space control [7].

PROPOSITION. The control in terms of GVC described as

$$\pi = N\dot{u}_r + C(\theta, u)u_r + k_D s_u \tag{10}$$

where k_D is a positive definite control gain matrix enables achieving sliding surface in terms of GVC.

Input moment of manipulator (which arises from (6)) is given as $\tau = \Upsilon^{-T}\pi + G(\theta)$.

Proof of (10). The closed loop system with control (10) using s_u is given as follows:

$$N\dot{u} + C(\theta, u)u = N\dot{u}_r + C(\theta, u)u_r + k_D s_u \tag{11}$$

what leads to equation ($\dot{s}_u = \dot{u}_r - \dot{u}$):

$$N\dot{s}_u + [C(\theta, u) + k_D]s_u = 0. \tag{12}$$

Now as a Lyapunov function candidate consider the following expression (which contains the same quantities as in [8] but decomposed in different way):

$$\mathcal{L} = \frac{1}{2}s_u^T N s_u. \tag{13}$$

The time derivative of N equals (by $M = M(\theta)$):

$$\dot{N} = \frac{d}{dt}(\Upsilon^T M \Upsilon) = \dot{\Upsilon}^T M \Upsilon + \Upsilon^T \dot{M} \Upsilon + \Upsilon^T M \dot{\Upsilon}. \tag{14}$$

Next we calculate the time derivative of the function (13) using equations (2)-(6), Eqs.(12), (14), and property (P3). After transposition of (3) one can obtain ($\dot{\mathcal{L}} = \frac{d\mathcal{L}}{dt}$):

$$\begin{aligned}
\dot{\mathcal{L}} &= s_u^T N \dot{s}_u + \frac{1}{2}s_u^T \dot{N} s_u = s_u^T[-C(\theta, u)s_u - k_D s_u + \frac{1}{2}\dot{N}s_u] = \\
&= s_u^T[-\Upsilon^T M \dot{\Upsilon} s_u - \Upsilon^T C(\theta, \dot{\theta})\Upsilon s_u - k_D s_u + \frac{1}{2}(\dot{\Upsilon}^T M \Upsilon + \Upsilon^T \dot{M} \Upsilon \\
&+ \Upsilon^T M \dot{\Upsilon})s_u] = -s_u^T k_D s_u + s_u^T[\frac{1}{2}\Upsilon^T M \dot{\Upsilon} - \Upsilon^T M \dot{\Upsilon} + \frac{1}{2}\dot{\Upsilon}^T M \Upsilon \\
&+ \Upsilon^T(\frac{1}{2}\dot{M} - C(\theta, \dot{\theta}))\Upsilon]s_u = -s_u^T k_D s_u + \frac{1}{2}s_u^T(\dot{\Upsilon}^T M \Upsilon - \Upsilon^T M \dot{\Upsilon})s_u = \\
&= -s_u^T k_D s_u \leq 0.
\end{aligned} \tag{15}$$

Notice that $\dot{\mathcal{L}} \equiv 0$ only if $s_u \equiv 0$. From expression (15) one can conclude that the output error converges to the sliding surface

$$s_u = \Upsilon^{-1}(\dot{\tilde{\theta}} + \Lambda\tilde{\theta}) = 0. \tag{16}$$

In order to show that $s_u \to 0$ as $t \to \infty$ it is sufficient to show that $\dot{\mathcal{L}} \to 0$ as $t \to \infty$. Since $\mathcal{L}$ is positive, Barbalat's Lemma [8] indicates that $\dot{\mathcal{L}}$ does tend to zero if it is uniformly continuous, and if $\ddot{\mathcal{L}}$ is bounded. Calculating the second time derivative of $\mathcal{L}$ from (15) one can obtain $\ddot{\mathcal{L}} = -2s_u^T k_D \dot{s}_u$. Since $\mathcal{L} \geq 0$ and $\dot{\mathcal{L}} \leq 0$, then $\mathcal{L}$ remains bounded. From expression (13) implies that s_u is bounded. Besides the diagonal matrix N is positive, bounded and invertible (it arises from decomposition of mass matrix M). Similarly matrix Υ^{-1} is bounded and invertible. The closed-loop dynamics (12) shows that also $\dot{s}_u$ is bounded (matrix N is positive definite and its inversion is bounded). Thus, $s_u \to 0$ as $t \to \infty$ and therefore both $\tilde{\theta}$ and $\tilde{\dot{\theta}}$ tend to zero as $t \to \infty$.

Remark 5. The difference between controls (8) and (10) relies on various energy shaping. Both s_u and u_r contain matrix Υ^{-1} which express couplings among each of k-th link and others. If the condition $||\Upsilon^{-1}|| > 1$ given in *Remark 3* is fulfilled, one can expect that the control (10) faster reduces kinetic energy than using controller (8). It results from the fact that $k_{D(GVC)} = (\Upsilon^{-1})^T k_{D(CL)} \Upsilon^{-1}$.

4 Simulation results

In this section simulation results concerning sliding mode control of robot manipulator are presented. The first aim is to point at some differences between two propositions expressed in terms of GVC. The second goal is to show some performances obtained from noninteracting controllers if the equations of motion are given in terms of GVC. As the example $3-D$ Yasukawa-like manipulator was considered. We have used dynamical equations given in [4] and set of manipulator parameters from [2]:
* link masses: $m_1 = 6.04kg$, $m_2 = 17.4kg$, $m_3 = 35kg$;
* link inertias: $J_{xx1} = 0.317kgm^2$, $J_{xx2} = 0.14kgm^2$, $J_{xx3} = 0.862kgm^2$, $J_{xy1} = 0kgm^2$, $J_{xy2} = 0.007kgm^2$, $J_{xy3} = 0.002kgm^2$, $J_{xz1} = 0$, $J_{xz2} = -0.019kgm^2$, $J_{xz3} = 0.001kgm^2$, $J_{yy1} = 0.0169kgm^2$, $J_{yy2} = 0.609kgm^2$, $J_{yy3} = 0.002kgm^2$, $J_{yz1} = -0.012kgm^2$, $J_{yz2} = -0.0017kgm^2$, $J_{yz3} = 0.002kgm^2$, $J_{zz1} = 0.266kgm^2$, $J_{zz2} = 0.626kgm^2$, $J_{zz3} = 0.35kgm^2$;
* distance: axis of rotation - mass center: $p_{x2} = 0.068m, p_{y1} = 0.143m, p_{y2} = 0.006m, p_{y3} = 0.3078m, p_{z1} = 0.014m$;
* length of link: $l_1 = 0.4318m$;
* angle $\alpha : \alpha_1 = \alpha_3 = 0deg, \alpha_2 = -90deg$.

The kinematic scheme is shown in Figure 1.

For tracking we have chosen the fifth-order polynomial trajectory described as follows: initial points $\theta_{i1} = 1/3 * \pi[rad]$, $\theta_{i2} = \pi[rad]$, $\theta_{i3} = -1/2 * \pi[rad]$, and final points $\theta_{f1} = -2/3 * \pi[rad]$, $\theta_{f2} = 0[rad]$, $\theta_{f3} = 1/2 * \pi[rad]$, with time duration $t_f = 1[s]$. Maximal value of joint velocity is $|\dot{\theta}_{kmax}| = 5.89[rad/s]$ for

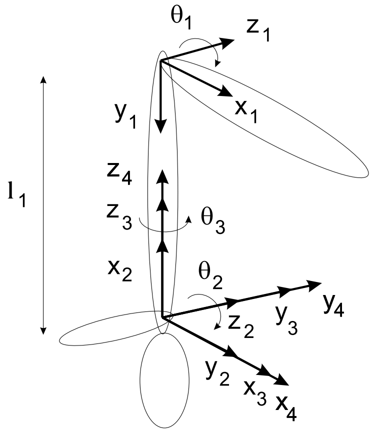

Figure 1: Kinematic scheme of Yasukawa-like manipulator.

each link, and maximal acceleration $|\ddot{\theta}_{kmax}| = 18.14[rad/s^2]$, $k = 1, 2, 3$. Starting points were different from initial points $\Delta = +0.2, +0.2, +0.2$, respectively. All simulations (realized in MATLAB/SIMULINK environment) were performed using the fourth-order Runge-Kutta formula with fixed step size $0.005[s]$. We assumed control coefficients (the same for both controllers):

$$k_D(1, 2, 3) = 9, 9, 9, \quad \Lambda(1, 2, 3) = 12, 12, 12. \tag{17}$$

Profiles of trajectories and joint velocities are given in Figure 2(a). Figures 2(b) and 2(c) show joint position errors for all joints and for GVC and classical (CL) controllers, respectively. One can notice that GVC controller gives slightly bigger error for the first and the second joint. But after some time using this controller all errors tends to zero faster than using CL one. Figure 2(d) confirms this fact because the error norm (in logarithmic scale) has after about $1[s]$ distinctly smaller values for GVC controller than for CL one. Similar conclusion arises from Figure 3(a) which compares joint velocity error norms for both controllers (also in logarithmic scale). In Figure 3(b) joint moments obtained using GVC controller are presented (for CL controller they were very close). Figure 3(c) gives realized GVC velocities u for all joints. They differ themselves which implies that they are decoupled. Besides $u1$ had bigger values than the appropriate joint velocity. Next, in Figure 3(d) elements of matrix N are shown. They represents some inertias along axes arising from the presence of other links of manipulator. Figure 4(a) gives Euclidean norm $||\Upsilon^{-1}||$ of the matrix Υ^{-1} obtained from GVC controller. This norm has all time value bigger than 1 what leads to a conclusion that the performances for GVC controller are better than CL one. In Figure 4(b) phase curves obtained from GVC and CL

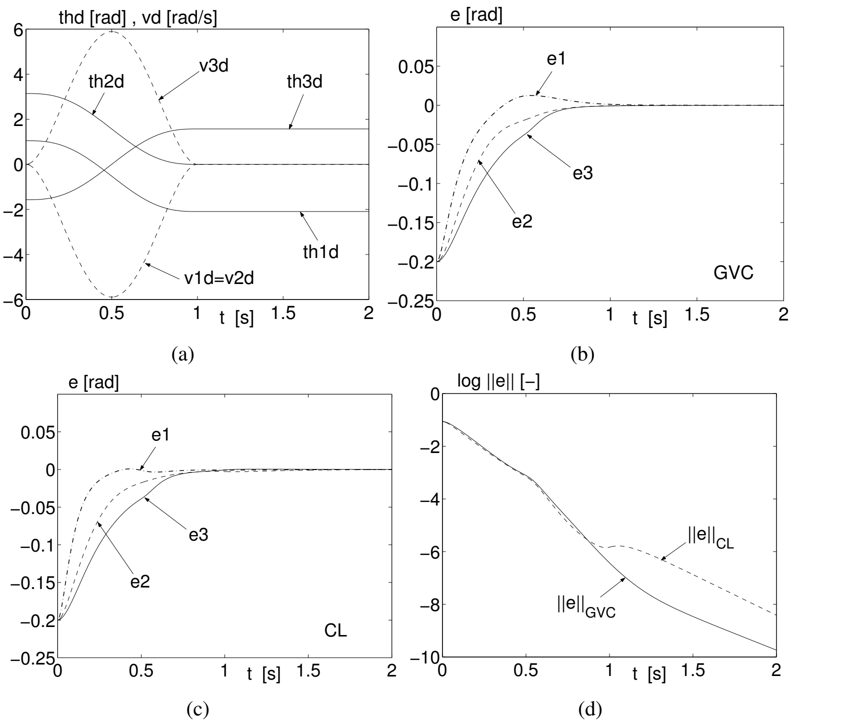

Figure 2: Simulation results: a) profiles of trajectories and joint velocities, b) joint position errors e for GVC controller; c) joint position errors e for classical (CL) controller; d) comparison of joint position error norms $||e||$ (in logarithmic scale) for both controllers.

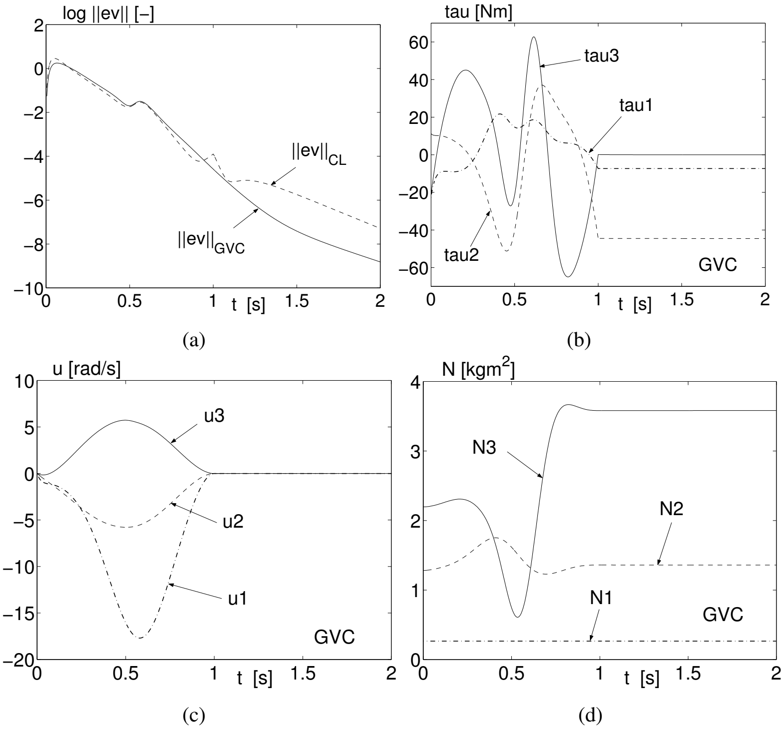

Figure 3: Simulation results: a) comparison of joint velocity error norms $||ev||$ (in logarithmic scale) for both controllers (GVC and CL), b) joint moments tau obtained using GVC controller; c) quasi-velocities u for GVC controller; d) elements of matrix N obtained from GVC controller.

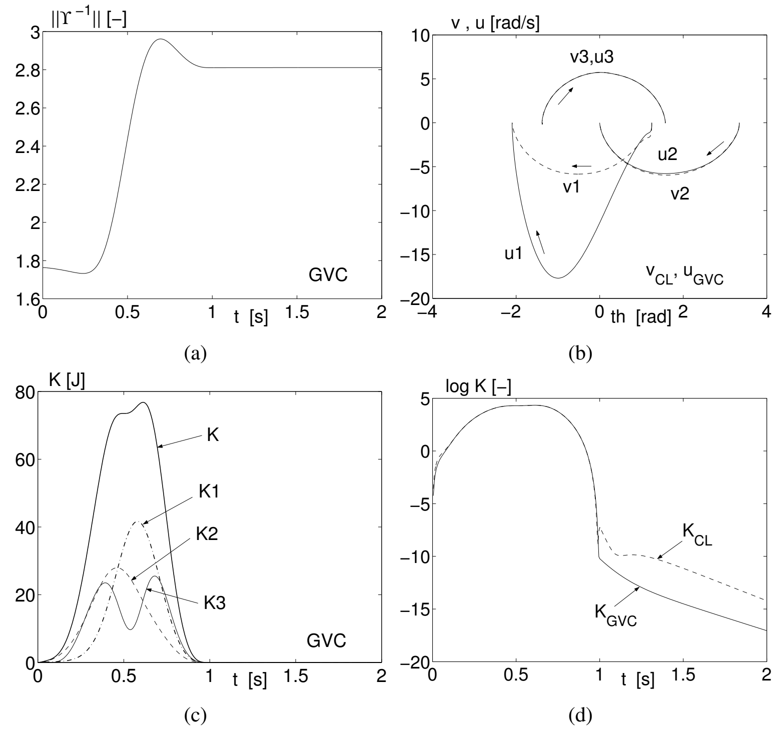

Figure 4: Simulation results: a) Euclidean norm of transformation matrix Υ^{-1} for GVC controller, b) phase curves obtained from CL controller (joint velocity v - dashed line) and for GVC controller (quasi--velocity u - solid line) for all joints; c) kinetic energy in all joints and for the entire manipulator (GVC controller); d) comparison of kinetic energy (in logarithmic scale) for both classical (KE_{CL}), and GVC (KE_{GVC}) controllers, respectively.

controllers are presented. Main difference results from quasi-velocity $u1$ and joint velocity $v1$. Other quasi-velocities are very close the appropriate joint velocities in both cases. However one can conclude that elements of vector u which contain couplings among manipulator links have bigger values than joint velocities. This fact implies faster errors convergence using GVC. Figure 4(c) compares kinetic energy for the entire manipulator K and for all joints. At $0.5[s]$ dominant is energy for the first joint ($K1$). It is related to $u1$ in Figure 3(c). Comparing in Figure 4(d) kinetic energy for manipulator one can see that after duration time ($1[s]$) this energy is reduced faster using GVC controller than using classical controller.

5 Conclusions

In this work some proposition of sliding mode control in terms of the inertial generalized velocity components (GVC) was presented. Based on Barbalat's Lemma convergence of the tracking error was proved. Performances of sliding mode controller in terms of GVC and classical controller, for $3-D$ 3 $d.o.f.$ manipulator, were compared too. It was confirmed that using sliding mode controller in terms of GVC the kinetic energy if shaped in different way than using classical controller. It was stated also that after duration time GVC controllers gives better performances. Additionally information about dynamical influence of links during the motion of the manipulator is also available (on the basis of knowledge of matrix N or kinetic energy in each joint).

Dynamic equations of motion for serial manipulator are decoupled in the sense that the obtained mass matrix is diagonal. From the kinetic energy expressed using vector u arises that its introduction leads also to decoupling of manipulator links in the kinetic energy sense. It is because each 'decoupled link' (i.e. a link which contains also couplings among the k-th link and others) has independent kinetic energy expressed as $(\frac{1}{2}N_k u_k^2)$. Mass of such 'decoupled link' is described by means of k-th element of the matrix N. However one should take into consideration that for some manipulators the initial moments values are quite big. It results from the fact that GVC include both dynamical and kinematical parameters of the system.

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