Two Degree of Freedom Fuzzy Controllers: Structure and Development

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Abstract: The paper presents structures and a systematical development method for two degree of freedom fuzzy controllers with non-homogenous dynamics with respect to the two input channels. The proposed controller structures are meant for a low order plant, which is specific to the field of electrical drives. The design relations result because fuzzy controllers can be, in some certain conditions, well approximated by linear controllers and many development methods are applicable for this situation. The analysis points out that the proposed two degree of freedom fuzzy controllers can ensure better control system performance with respect to the reference input in comparison with other structures containing conventional controllers.

Keywords: Development of fuzzy controller, non-homogenous dynamics, Extended Symmetrical Optimum method.

1 Introduction

The number of degrees of freedom (DOF) of a control system (CS) is defined by the number of transfer functions (t.f.s) of which parameters can be independently adjusted [1]. Generally, the design of a CS is a multi-objective problem; in this context, by increasing the number of degrees of freedom from one degree of freedom (1 DOF) to two degrees of freedom (2 DOF) it becomes possible to achieve better CS performance.

The presence of possible variable parameters / nonlinearities in the structure of the controlled plant leads to the idea of introducing fuzzy control. The paper sustains that fuzzy control copes successfully with the above mentioned 2 DOF controllers.

For CS performance enhancement, the fuzzy controllers (FCs) must contain dynamics by employing the knowledge on conventional controllers [2], [3]. Furthermore, in some well stated conditions the approximately equivalence between linear controllers and fuzzy controllers is generally acknowledged and widely accepted and used [4], [5]. In addition, in the design phase the FC can be considered as nonlinear but linearizable near operating points belonging to the control surface [2].

In this context, the paper presents a construction and development method for 2 DOF FCs with non-homogenous dynamics with respect to the two input channels; the method is based on design results from the linear case.

The paper is organized as follows. The structure of a 2 DOF controller and the CS structure with 2 DOF controllers are firstly presented, together with some design conditions and implementation reasons. Then some connection between the 2 DOF and conventional controllers (1 DOF) with input signals filtering are highlighted. Based on these, section 4 presents the structure and development principles of 2 DOF fuzzy controllers. The final part of the paper is focused on the conclusions based on the study.

2 General Approach to a 2 DOF Controller

The structure of a SISO CS with a 2 DOF controller is depicted in Fig.1 and it highlights the presence of two controllers:

- the reference controller C_1 , denoted also C_T, by which the feed-forward control of the system is ensured (r – the reference signal);
- the feedback controller C_2 , denoted also C_S, which deals with the events within the control system (y – the controlled signal/feedback).

The requirements to be fulfilled by a CS structure are the followings:

- zero steady state control error for a well defined variation of the reference input;
- rejection of the effects of constant disturbances;
- robustness: reduced sensibility to parameter changes;
- appropriate transients.

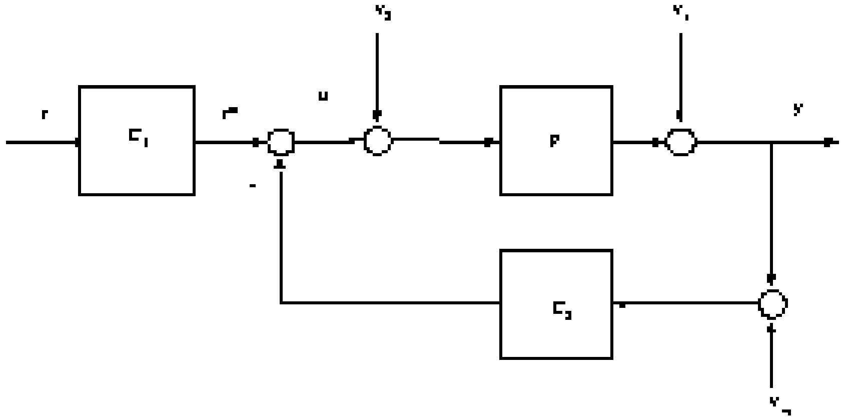

Fig.1. Structure of the 2 DOF controller.

The traditional 1 DOF CSs satisfy only partially these requirements. In case of 2 DOF controllers the attributes enlisted above can be separately adjusted without a strong influence between one and another.

Assuming that the continuous plant is given by its continuous t.f. P(s), then its discrete (pulse) transfer function (t.f.), (the Zero Order Hold (ZOH) is included) can be calculated:

$$\mathrm{P(z)} = (1 - \mathrm{z}^{-1})\mathrm{Z}\{\frac{\mathrm{P(s)}}{\mathrm{s}}\}, \quad \mathrm{P(z)} = \frac{\mathrm{B(z)}}{\mathrm{A(z)}} \ , \tag{1}$$

with:

$$\partial \mathrm{A(z)} = \mathrm{m}, \quad \partial \mathrm{B(z)} = \mathrm{m}, \quad \mathrm{m} < \mathrm{n} \ . \tag{2}$$

The 2 DOF discrete controller is characterized by the relations [6],[7]:

$$u(z) = \frac{T(z)}{R(z)} r(z) - \frac{S(z)}{R(z)} y(z), \ \ C_T(z) = \frac{T(z)}{R(z)}, \ \ C_S(z) = \frac{S(z)}{R(z)} \ , \tag{3}$$

The servo performance can be imposed by a desired continuous reference model, $H_{rm}(s)$, which can be transposed into discrete t.f.:

$$H_{rm}(z) = \frac{B_m(z)}{A_m(z)} \text{ , with } H_{rm}(1) = \frac{B_m(1)}{A_m(1)} = 1 \ . \tag{4}$$

Imposing (4) based on Fig.1, it can be stated:

$$\frac{B(z)T(z)}{A(z)R(z) + B(z)S(z)} = \frac{B_m(z)}{A_m(z)} \ , \tag{5}$$

where $B_m(z)$ and $A_m(z)$ are co-prime and T(z), R(z), S(z) are unknown. This condition represents a possibility of pole placement design method, because $A_m(z)$ determines the poles of the closed loop system, but here a polynomial computation will be performed to set the poles instead of applying other models [6], [7].

Generally, the design steps of 2 DOF controller – the determination of the polynomials T(z), R(z) and S(z) – are presented in literature and usually related to the discreet case; is necessary to remark that the first 2 DOF structures were continuous ones. The computer aided design (CAD) of the 2 DOF control system has specific steps which can be performed – for example – in MATLAB; such a structure of the CAD program is presented in detail and exemplified for a case study (as example) in [8].

The implementation of the 2 DOF controller is based on positioning the R(z) polynomial (that includes the integrators) or just the integral part, $(z-1)^{-1}$, of it:

$$R(z) = (z-1)R'(z) \tag{6}$$

inside the control loop (see Fig.2).

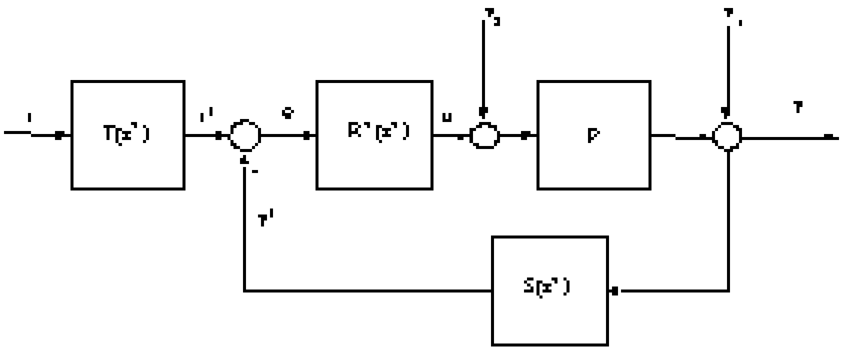

Fig.2. Implementation of the 2 DOF controller.

3 Some Connections Between the 2 DOF Controllers and the Conventional (1 DOF) Controllers

Let the block diagram of a 2-DOF control structure (Fig.1) be considered. By replacing the feedback controller $C_S(z)$ on the input channel and the forward loop, the given CS can be transposed into the structure in Fig.3. Accordingly:

$$C(z) = C_S(z), \quad F(z) = C_S(z)C_T(z) \ . \tag{7}$$

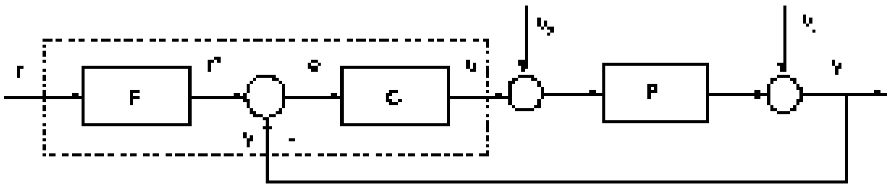

Fig.3. Restructured control system having a conventional controller (C) and a reference filter (F).

For the reference signal tracking, the design relation (5) is valid. Consequently, the controller design can be performed as described in section 3, or according to conventional controller design methods.

Also, for v_2 and v_1 type disturbance rejection the two structures have identical behavior:

$$H_{v2}(z) = \frac{R(z)B(z)}{R(z)A(z)+S(z)B(z)}, \quad H_{v1}(z) = \frac{R(z)A(z)}{R(z)A(z)+S(z)B(z)} \ . \tag{8}$$

The 2DOF controller can be restructured in different ways; for low order plants – from a practical point of view – the presence of a conventional controller (particularly PI or a PID and signal filters) can be highlighted [8], [9], [10]. Two types of structures are detailed in Fig.4. These rearrangements allow:

- to take over design experience from case of PI and PID controllers;
- an easy introduction of supplementary blocks specific to PI and PID controllers (Anti Windup circuit, bumpless switching and others);
- the transformation of PI and PID controllers into 2 DOF structures and vice versa.

Furthermore, the controllers in Fig.4 will be characterized by continuous t.f.s in which the “traditional” tuning parameters are highlighted $\{k_R,\ T_i,\ T_d,\ T_f\}$. Discretizing, the digital control algorithm is obtained.

Taking the basic controller C of PID type, it can be written:

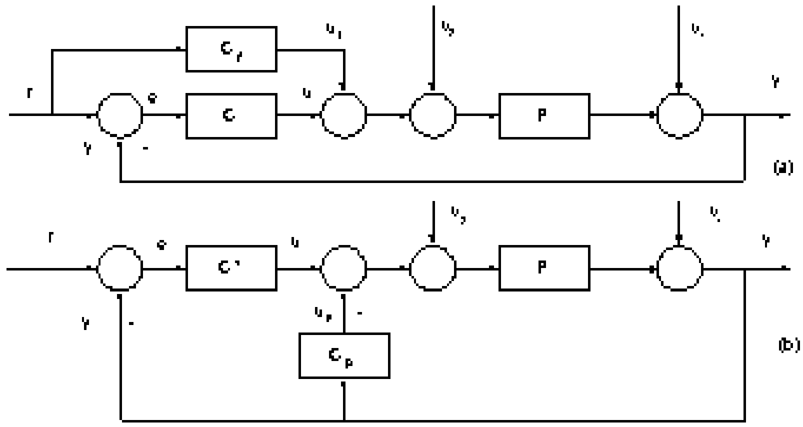

Fig.4. Two alternatives for rearranging a 2 DOF controller.

- for the structure in Fig.3:

$$C(s)=\frac{u(s)}{e(s)}=k_R\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+sT_f}\right),$$
$$F(s)=\frac{r^*(s)}{r(s)}=\frac{1+(1-\alpha)\,T_i s+\dfrac{(1-\beta)T_iT_ds^2}{(1+sT_f)}}{1+T_is+\dfrac{T_iT_ds^2}{(1+sT_f)}}; \tag{9}$$

- for the structure (a) in Fig.4:

$$C(s)=\frac{u(s)}{e(s)}=k_R\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+sT_f}\right),$$
$$C_F(s)=\frac{u_f(s)}{r(s)}=k_R\left(\alpha+\beta\frac{sT_d}{1+sT_f}\right); \tag{10}$$

- for the structure (b) in Fig.4 (with the notation C(s)=C^*(s)):

$$C^*(s)=\frac{u(s)}{e(s)}=k_R\left[(1-\alpha)+\frac{1}{sT_i}+(1-\beta)\frac{sT_d}{1+sT_f}\right],$$
$$C_P(s)=\frac{u_f(s)}{r(s)}=k_R\left(\alpha+\beta\frac{sT_d}{1+sT_f}\right). \tag{11}$$

Depending on the values of α and β parameters, for the presented blocks the behaviors from in Table 1 are obtained. The choice of a certain representation of the controller depends on [8]:

Table 1. Connections between 2 DOF controller and extended 1 DOF controller structure.

Fig.3		F(s)	-	F(s)C(s)	C(s)	Remarks
Fig.4-a		-	C_F	C(s)–C_F(s)	C(s)	-
Fig.4-b		-	C_P	C*(s)	C*(s)+C_P(s)	-
α	β	-	-	(ref. channel)	(feedback)	-
0	0	1	0	PID	PID	1 DOF controller
0	1	PDL2	DL1	PI	PID	1 DOF with non-homogenous behavior
1	0	PD2L2	P	PID-L1	PID	
1	1	PL2	PDL2	I	PID	
α	β	PID controller with pre-filtering (2 DOF controller)				

- the structure of the available controller;
- the adopted algorithmic design method and the result of this design.

4 The 2 DOF Fuzzy Controllers. Construction and Development

The 2 DOF fuzzy controllers can be developed on the basis of two versions:

- by starting with the equivalence between a 2 DOF controller and the conventional 1 DOF controllers extended with filters on the input channels (in this section the reference signal is denoted by w instead r),
- by starting with the discrete model of the 2 DOF controller.

In the case of low order plants the general structure of a 2 DOF fuzzy controller is illustrated in Fig.5, where: FC-T/R' – fuzzy module for the controller T/R' (the reference channel), FC-S/R' – fuzzy module for the controller S/R' (the feedback channel). The integral component brought by the controller and included in the forward channel of the loop is highlighted as follows:

$$R(z) = (z-1)R'(z) \text{ (the discreet case) or } R(s) = sR'(s) \text{ (the continuous)} \quad (12)$$

In steady state, the conditions (13) are fulfilled:

$$e_\infty = \tilde{w}_\infty - \tilde{y}_\infty = 0, \;\rightarrow\; u_\infty = const\ , \quad (13)$$

and the assurance of a desired value of the output y_∞ depends on obtaining an equilibrium of the two variables $\tilde{w}_\infty$ and $\tilde{y}_\infty$:

$$\tilde{w} = k_{FT} w + \Delta u_w, \quad \tilde{y} = k_{FS} y + \Delta u_y \ , \tag{14}$$

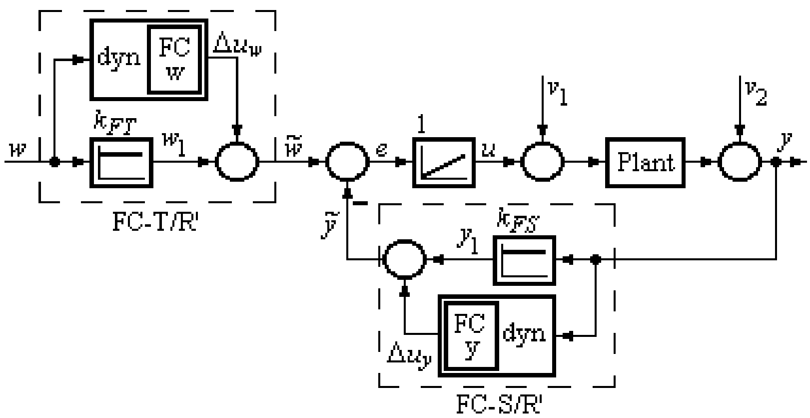

Fig.5. General structure of a 2 DOF fuzzy controller.

where k_{FT} and k_{FR} are parameters that adjust the level of the two signals.

The signals Δu_w and Δu_y represent only the dynamic components processed by the fuzzy controllers with dynamics FCw and Fcy.

From the steady state condition (13) it is obtained (15):

$$k_{FT} w_\infty = k_{FS} y_\infty \ , \tag{15}$$

and it results the necessity for the two components, Δu_w and Δu_y, to have only “transient character”:

$$\Delta u_w(t) \to 0, \ \Delta u_y(t) \to 0 \quad \text{for} \quad t \to \infty \ . \tag{16}$$

It is important to outline that generally the continuous components $k_{FT}w_\infty$ and $k_{FS}y_\infty$ are not allowed to be subject of the fuzzy processing. However, there can be conceivable situations (for example, the case of some reference tracking systems) in which the fuzzy processing can be included in the controller FC-T/R’. For such situations a variable reference input w(t) can be subject to fuzzification, and $\tilde{w}(t)$ will contain corrections as function of the variations of w(t) and of other possible causes.

The structure presented in Fig.5 can be implemented in the two mentioned versions.

In the case of relatively low order continuous plants, in the construction of the 2 DOF fuzzy controller it is possible to take over the design experience specific to the 1 DOF controllers and the extensions with filters on the input channels. In this context, it becomes possible to use the low order informational modules specific to the fuzzy controllers with dynamics with the detailed computation presented in [3].

The structure of a fuzzy module with dynamic processing FC-T/R' or FC-S/R' in this version is presented in Fig.6 with respect to the signal w, for the analog version (Fig.6-a) and for the discretized quasi-continuous digital version (Fig.6-b).

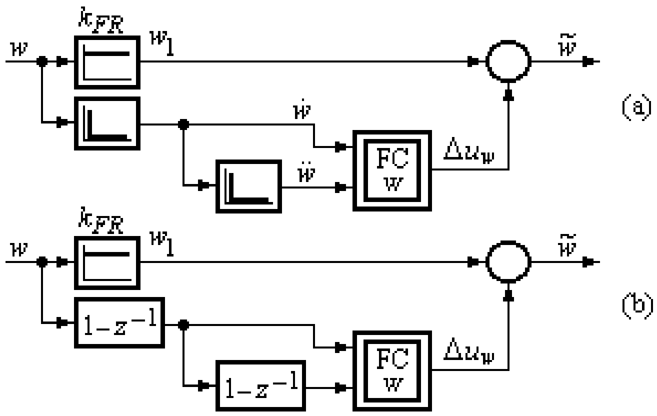

Fig.6. Structure of the module FC-T/R' in analog version (a) and digital version (b).

The digital version is based on the computation of the derivatives (increments):

$$\Delta w_k = w_k - w_{k-1}, \ \Delta^2 w_k = w_k - 2w_{k-1} + w_{k-2} , \tag{17}$$

and the increment of control signal, $\Delta u_{w,k}$, depends on Δw_k and $\Delta^2 w_k$:

$$\Delta u_{w,k} = k_1 \Delta w_k + k_2 \Delta^2 w_k = k_1 (\Delta w_k + \alpha \cdot \Delta^2 w_k) . \tag{18}$$

The parameters k_1, k_2 and α are functions of the parameters of the conventional controller T(s)/R'(s) or S(s)/R'(s) and of the sampling period [3].

On the basis of the relation (18) and of the representation of the increment $\Delta u_{w,k}$ in the phase plane <Δw_k, $\Delta^2 w_k$>, Fig.7, the pseudo-fuzzy features of the conventional control algorithm (18) can be expressed as:

- there exists a "zero control signal line" $\Delta u_{w,k} = 0$, having the equation:

$$\Delta w_k + \alpha \cdot \Delta^2 w_k = 0 ; \tag{19}$$

- with regard to this line it is obtained that in the upper half-plane: $\Delta u_{w,k}>0$, and in the lower half-plane: $\Delta u_{w,k}<0$;
- the distance from any point of the phase plane to the “zero control signal line” corresponds to $|\Delta u_{w,k}|$.

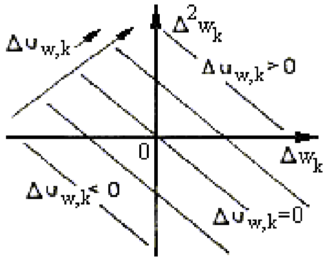

Fig.7. Phase plane representation corresponding to (18).

For the strictly speaking fuzzy controller (the block FCw in Fig.6), the fuzzification can be solved in the initial phase as follows:

- for the input variables Δw_k and $\Delta^2 w_k$, five (or more, but an odd number) linguistic terms with regularly distributed triangular type membership functions having an overlap of 1 are chosen;
- for the output variable $\Delta u_{w,k}$, are chosen 7 linguistic terms with regularly distributed singleton type membership functions are chosen, Fig.8;
- other shapes of membership functions can contribute to CS performance enhancement

These strictly positive parameters of the 2 DOF fuzzy controller, $\{B_w, B_{\Delta w}, B_{\Delta u}\}$, are in direct correlation with the shapes of the membership functions of the linguistic terms corresponding to the input and output linguistic variables.

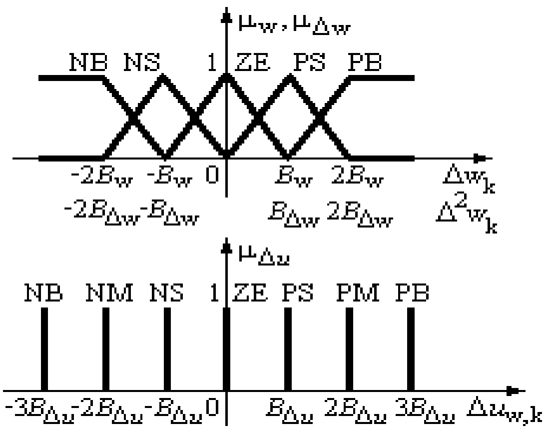

Fig.8. Shapes of membership functions for the block FCw.

The inference engine of the block FCw employs the Mamdani's MAX-MIN compositional rule of inference assisted by a complete rule base. The rule base of the block FCw is expressed as decision table, and it is illustrated in Table 2.

Table 2. Decision table for the block FCw.

$\Delta^2 w_k \backslash \Delta w_k$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

The defuzzification module as part of the block FCw can be done by the center of gravity method, but the choice of the inference method and of the defuzzification method represent the user's options.

The steps for the fuzzy controller development are the classical ones, described in [3]. The step corresponding to the calculus of the 2 DOF controller and to the equivalence with a conventional controller with reference filters have been outlined in sections 2 and 3.

5 Conclusions

By starting from the requirements concerning control applications for some driving and positioning systems, the paper presents a method for the construction and development of 2 DOF fuzzy controllers.

The presented structure and development method is based on starting with the development of the 2 DOF fuzzy controller followed by the transfer to the fuzzy processing of the components with dynamics. The integral element specific to the 2 DOF controllers is included on the forward channel of the control loop.

The method is applicable relatively simple in the case of plants having not extremely large order. If the system order is increasing, there can appear problems in achievement of the fuzzy processing of the dynamic components. In the design of the 2 DOF controller alternative approach methods are possible.

It is to remark that there can appear particular situations in which the continuous component of the reference input can be itself subject to fuzzy processing.

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