Adaptive Control of the Double Inverted Pendulum Based on Novel Principles of Soft Computing

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Abstract:
As a possible new modeling and control technique the application of a novel branch of soft computing developed at Budapest Polytechnic was investigated in the adaptive control of an approximately and partially modeled double inverted pendulum via simulation. Such a dynamic system has the curiosity of badly conditioned inertia matrix in certain positions. The control starts from a non-singular rough estimated model step-by-step adjusted on the basis of the observed behavior of the controlled system using the elements of a series of linear transformations computed on the basis of the modified renormalization transformation. It was found that in spite of the almost singular behavior of the system to be controlled the adaptive loop considerably improves the quality of the control, though the consequences of tha bad conditioning cannot be completely eliminated by it.

1 Introduction

A new approach for the adaptive control of imprecisely known dynamic systems under unmodeled dynamic interaction with their environment was initiated in [1]. In the family of the adaptive control methods this new one lies between the linear PID/ST and the parameter identification approaches.

Instead of the supposed analytical model's parameters the control is tuned as in the PID/ST, but it offers the possibility of using several parameters of some abstract Lie groups fit to the needs of the “non-linear control”. In the same time these parameters may be considered as that of the system model's, though they do not belong to certain detailed analytical system-description. This „non-analytical modeling” is akin to the Soft Computing philosophy.

In this approach adaptivity means that instead of the simultaneous tuning of numerous parameters, a fast algorithm finding some linear transformation to map a very primitive initial model based expected system-behavior to the observed one is used. The so obtained „amended model” is step by step updated to trace changes by repeating this corrective mapping in each control cycle. Since no any effort is exerted to identify the possible reasons of the difference between the expected and the observed system response, it is referred to as the idea of "Partial System Identification". This anticipates the possibility for real-time applications. Regarding the appropriate linear transformations several possibilities were investigated and successfully applied. For instance, the „Generalized Lorentz Group” [2], the „Stretched Orthogonal Group”, the “Partially Stretched Orthogonal Transformations” [3], and a special family of the „Symplectic Transformations” [4] can be mentioned.

The key element of the new approach is the formal use of the „Modified Renormalization Transformation”. The „original” Renormalization Transformation was widely used e.g. by Feigenbaum in the seventies to investigate the properties of chaos [5-7]. This (originally scalar) transformation modifies the solution of an $x=f(x)$ fixed-point problem. Since the adaptive control can be formulated as a fixed-point problem, too [8], this transformation was considered a possible candidate for the solution of the task of the adaptive control. The modification of the original transformation was necessary due to phenomenological reasons. Satisfactory conditions of the complete stability of the so obtained control for Multiple Input-Multiple Output (MIMO) systems were also highlighted in [8] by the means of perturbation calculation. This means the most rigorous limitation regarding the circle of possible application of the new method. To release this restriction to some extent “ancillary” but simple interpolation techniques and application of “dummy parameters” were also introduced in [8]. The applicability of the method was investigated for electro-mechanical and hydrodynamic systems and in connection with chemical reactions via simulation [9-11]. These systems were exempt of any kind of controllability problem or singularity. In this paper a quite simple but lucid typical paradigm of controllability problems, a cart conveying a double pendulum is chosen to be the subject of the new type adaptive controller. If the mass of the cart is negligible in comparison with that of the pendulums’ the inertia matrix of the system becomes badly conditioned in certain critical configurations. As a consequence the presence of even small joint coordinate forces / torques can cause considerable joint coordinate accelerations in the vicinity of the singular positions. In the case of an actual concrete physical system controlled by a Computed Torque Control using inaccurate system model this feature may mean a significant problem. Further problems arise when the motion of the system is simulated by the use of its “exact” equations of motion and a finite element method regarding the time-resolution. The selection of the length of the interval between the discrte time steps considered seriously concerns the numerical results of the calculations. This length has to be decreased till the effect of the decrease cannot be observed in the numerical results. It has to be stressed that in the case of a real time control system the cycle time of the control commands cannot be chosen to be arbitrarily small. During these finite time interval the torque/force values exerted by the drives can be considered constant while the contribution by the Coriolis and gravitational terms of the exact equations of motion can be traced in a finer resolution in the simulations.

In the sequel at first the paradigm is set mathematically, and following that the basic principles of the adaptive control is described. Following the presentation of the typical simulation results the conclusions are drawn.

2 The dynamic model of the cart - double pendulum system

Let the cart consist of a body and wheels of negligible momentum and inertia having the overall mass of M [kg]. Let the pendulums be assembled on the cart by parallel shafts and arms of negligible masses and lengths L_1 and L_2 [m], respectively. At the end of each arm a ball of negligible size and considerable masses of m_1 and m_2 [kg] are attached, respectively. The Euler-Lagrange equations of motion of this system are given as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} m_1 L_1^2 & 0 & -m_1 L_1 \sin q_1 \\ 0 & m_2 L_2^2 & -m_2 L_2 \sin q_2 \\ -m_1 L_1 \sin q_1 & -m_2 L_2 \sin q_2 & (M + m_1 + m_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ + \begin{bmatrix} -m_1 L_1 \cos q_1 \dot{q}_1 \dot{q}_3 - m_1 g L_1 \cos q_1 \\ -m_2 L_2 \cos q_2 \dot{q}_2 \dot{q}_3 - m_2 g L_2 \cos q_2 \\ -m_1 L_1 \cos q_1 \dot{q}_1^2 - m_2 L_2 \cos q_2 \dot{q}_2^2 \end{bmatrix} \tag{1}$$

in which g denotes the gravitational acceleration [m/s^2], Q_1 and Q_2 [$N\times m$] denote the driving torque at shaft 1 and 2, respectively, and Q_3 [N] stands for the force moving the cart in the horizontal direction. The appropriate rotational angles are q_1 and q_2 [rad], and the linear degree of freedom belongs to q_3 [m]. The determinant of the inertia matrix in (1) has the form of

$$\det \mathbf{M} = m_1 L_1^2 m_2 L_2^2 \left(M + m_1 + m_2 - m_1 \sin^2 q_1 - m_2 \sin^2 q_2 \right) \tag{2}$$

It can well be seen from (2) that the minimum value of this determinant is equal to

$$\min(\det \mathbf{M}) = m_1 L_1^2 m_2 L_2^2 M \tag{3}$$

and this situation happens whenever $q_1,\ q_2 = \pm\pi/2$ simultaneously. If $M << m_1,\ m_2$ these points correspond to near singular or badly conditioned inertia matrix causing the already mentioned problems in the control and simulation. On the basis of (1) it is easy to express the inverse dynamical equations of motion in closed analytical form used for simulation purposes. In the sequel the principles of the adaptive control are detailed.

3 Principles of the adaptive control

From purely mathematical point of view the can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated to obtain a desired system response $\mathbf{i}^d$ as $\mathbf{e}=\boldsymbol{\varphi}(\mathbf{i}^d)$. The system has its inverse dynamics described by the unknown function $\mathbf{i}^r=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d))=\boldsymbol{f}(\mathbf{i}^d)$ and resulting in a realized response $\mathbf{i}^r$ instead of the desired one, $\mathbf{i}^d$. Normally one can obtain information via observation only on the function $\boldsymbol{f}()$ considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only $\mathbf{i}^d$ as the input of $\boldsymbol{f}()$ can be “deformed” to $\mathbf{i}^{d*}$ to achieve and maintain the $\mathbf{i}^d=\boldsymbol{f}(\mathbf{i}^{d*})$ state. [Only the *model function* $\boldsymbol{\varphi}()$ can directly be manipulated.] On the basis of the modification of the method of renormalization widely applied in Physics the following "scaling iteration" was suggested for finding the proper deformation:

$$\begin{gathered}\mathbf{i}_0;\, \mathbf{S}_1\mathbf{f}\left(\mathbf{i}_0\right)=\mathbf{i}_0;\, \mathbf{i}_1=\mathbf{S}_1\mathbf{i}_0;\ldots;\, \mathbf{S}_n\mathbf{f}\left(\mathbf{i}_{n-1}\right)=\mathbf{i}_0;\\ \mathbf{i}_{n+1}=\mathbf{S}_{n+1}\mathbf{i}_n;\, \mathbf{S}_n \xrightarrow[n\to\infty]{} \mathbf{I}\end{gathered} \tag{4}$$

in which the $\mathbf{S}_n$ matrices denote some linear transformations to be specified later. As it can be seen these matrices maps the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller „learns” the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (4) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. The most important points of view are fast and efficient computation, and the ability for remaining as close to the identity transformation as possible.

For making the problem mathematicaly unambiguous (4) can be transformed into a matrix equation by putting the values of $\mathbf{f}$ and $\mathbf{i}$ into well-defined blocks of bigger matrices. Via computing the inverse of the matrix containing $\mathbf{f}$ in (4) the problem can be made mathematically well-defined. Since the calculation of the inverse of one of the matrices is neeeded in each control cycle it is expedient to choose special matrices of fast and easy invertibility. Within the block matrices the response arrays may be extended by adding to them a “dummy”, that is physically not interpreted dimension of constant value, in order to evade the occurrence of the mathematically dubious $0\to 0$, $0\to$finite, finite$\to 0$

transformations. In the present paper the special symplectic matrices announced in [4] were applied for this purpose. In general, the Lie group of the Symplectic Matrices is defined by the equations

$$\mathbf{S}^T \Im \mathbf{S} = \Im \equiv \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{I} & \mathbf{0} \end{array}\right],\ \det\mathbf{S} = 1 \tag{5}$$

The inverse of such matrices can be calculated in a computationally very cost-efficient manner as $\mathbf{S}^{-1} = \Im^T \mathbf{S}^T \Im$. In our partucular case the symplectic matrices are constructed from the desired and the observed joint coordinate accelerations corresponding to the response of the mechanical system to the excitation of torque and force by the use of the block of the matrix

$$[\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5] = \begin{bmatrix} \ddot{q}_1 & -\ddot{q}_1 & e_1^{(3)} & e_1^{(4)} & e_1^{(5)} \\ \ddot{q}_2 & -\ddot{q}_2 & e_2^{(3)} & e_2^{(4)} & e_2^{(5)} \\ \ddot{q}_3 & -\ddot{q}_3 & e_3^{(3)} & e_3^{(4)} & e_3^{(5)} \\ d & -d & e_4^{(3)} & e_4^{(3)} & e_4^{(5)} \\ D & \dfrac{\ddot{\mathbf{q}}^2 + d^2}{D} & e_5^{(3)} & e_5^{(4)} & e_5^{(5)} \end{bmatrix} \tag{6}$$

as

$$\mathbf{S} = \left[\begin{array}{c|c} \mathbf{0} & \dfrac{-1}{s}\mathbf{m}^{(1)} \quad \dfrac{-1}{s}\mathbf{m}^{(2)} \quad -\mathbf{e}^{(3)} \ldots -\mathbf{e}^{(5)} \\ \hline \mathbf{m}^{(1)} \quad \mathbf{m}^{(2)} \quad \mathbf{e}^{(3)} \ldots \mathbf{e}^{(5)} & \mathbf{0} \end{array}\right] \tag{7}$$

in which the symbols denote unit vectors $\mathbf{e}^{(3)},\ldots,\mathbf{e}^{(5)}$ which lie in the orthogonal sub-space of the first two columns of the block matrix, d is the "dummy" parameter used for avoiding singular transformations, and

$$D^2 \equiv \ddot{\mathbf{q}}^T \ddot{\mathbf{q}} + d^2,\ s = 2D^2 \tag{8}$$

The unit vectors can be created e.g. by using El Hini's algorithm [3], which, while rotates vector **b** to into the direction of vector **a**, leaves the orthogonal sub-space of these vectors invariant. So if the operation starts with an orthonormal set $\{\mathbf{e}^{(1)},\ldots,\mathbf{e}^{(5)}\}$ and at first it is rigidly rotated until $\mathbf{e}^{(1)}$ becomes parallel with the 1st column of **M**, its 2nd column will lie in the orthogonal sub-space of the 1st one spanned by the transformed $\{\mathbf{e*}^{(2)},\ldots,\mathbf{e*}^{(5)}\}$ set. In the next step this whole set can rigidly be rotated until the new $\mathbf{e**}^{(2)}$ becomes parallel with the 2nd column of **M**. (This operation leaves the previously set $\mathbf{e*}^{(1)}$ unchanged because it is orthogonal to the two vectors determining this special rotation.)

With the above completion the appropriate operation in (4) evidently equals to the identity operator if the desired response just is equal to the observed one, and remains in the close vicinity of the unit matrix if the non-zero desired and realized responses are very close to each other.

Since amongst the conditions for which the convergence of the method was proved near-identity transformations were supposed in the perturbation theory, a parameter ξ measuring the „extent of the necessary transformation”, a “shape factor” σ, and a „regulation factor” λ can be introduced in a linear interpolation with small positive ε_1, ε_2 values as

$$\xi := \frac{\left|\mathbf{f} - \mathbf{i}^d\right|}{\max(\left|\mathbf{f}\right|, \left|\mathbf{i}^d\right|)}, \quad \lambda = 1 + \varepsilon_1 + \left(\varepsilon_2 - 1 - \varepsilon_1\right)\frac{\sigma\xi}{1 + \sigma\xi}, \quad \hat{\mathbf{i}}^{\mathrm{d}} = \mathbf{f} + \lambda\left(\mathbf{i}^{\mathrm{d}} - \mathbf{f}\right) \tag{9}$$

This interpolation reduces the task of the adaptive control in the more critical session and helps to keep the necessary linear transformation in the vicinity of the identity operator. Other important fact concerning the details of the numerical calculations is the ratio of $\left\|\ddot{\mathbf{q}}\right\|$ and d in (6). The controller has *a priori* information only on thze *nominal* accelerations, but for the appropriate error-relaxation much higher *desired* accelerations may occur. For this purpose a slowly forgetting integrating filter was introduced to create a weighting factor for 0<β<1 as

$$w(t_i) := \frac{\sum_{j=0}^{\infty} \beta^j \left\|\ddot{\mathbf{q}}^{\mathbf{Des}}\left(t_{i-j}\right)\right\|}{\sum_{s=0}^{\infty} \beta^s} \tag{10}$$

an in (6) instead ofthe actual values ($\ddot{\mathbf{q}}$) the actual weighted ones $\ddot{\mathbf{q}}/w$ were taken into account. The numerical realization of such a filter is very easy: the content of a buffer has to be multiplied by β in each control cycle, and the new $\left\|\ddot{\mathbf{q}}^{Des}\right\|$ value has to be added to it. It also is easy to calculate the sum of the weights in the denominator of (10): Σ=1/(1-β). In the forthcoming simulations the following numerical data were used: d=80, β=0.92, σ=0.5, ε_1=0.2, $\varepsilon_2=10^{-5}$ were chosen.

4 Simulation results

In the simulations for the desired relaxation of the trajectory tracking error a simple PID-type rule is prescribed by the use of purely kinematic terms. This error relaxation colud be achieved exactly only in the possession of the exact dynamic model of the physical system to be controlled. Instead of the exact actual dynamic model detailed in (1) the constant 10×**I** (**I**= unit matrix) matrix was used as the inertia matrix, and the Coriolis and inertial terms were modeled by the constant vector $[10, 10, 10]^T$. This evidently corresponds to a very rough approximation of the reality in which m_1=m_2=10 *kg*, L_1=L_2=2 *m*, and various values for the cart mass M were chosen. The cycle-time of the controller was supposed to be 1 *ms*,

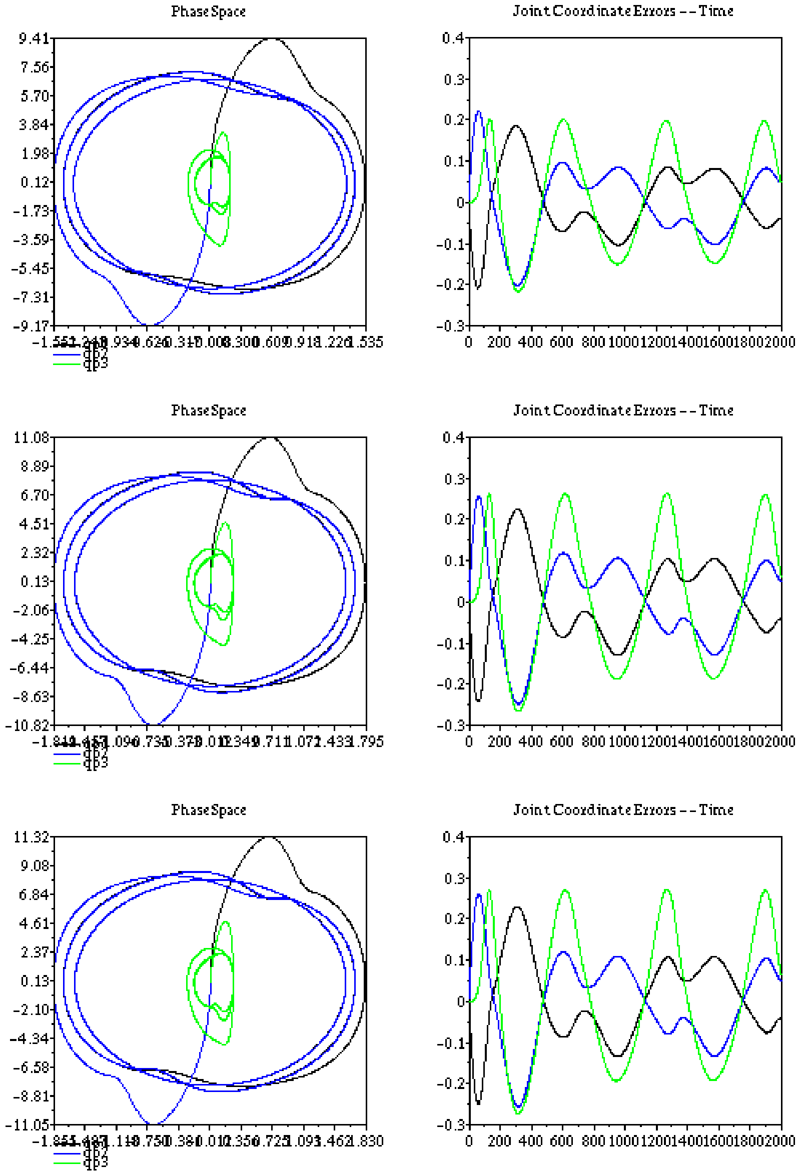

Figure 1. The operation of the non-adaptive controller in the “far from singularity” (1st row), “just touching the singularity” (2nd row), and “crossing the singularity” (3rd row) cases: the phase space and the joint coordinate errors [*rad* and *m*] vs. time [*ms*]. (q_1=*black*, q_2=*blue*, q_3=*green*)

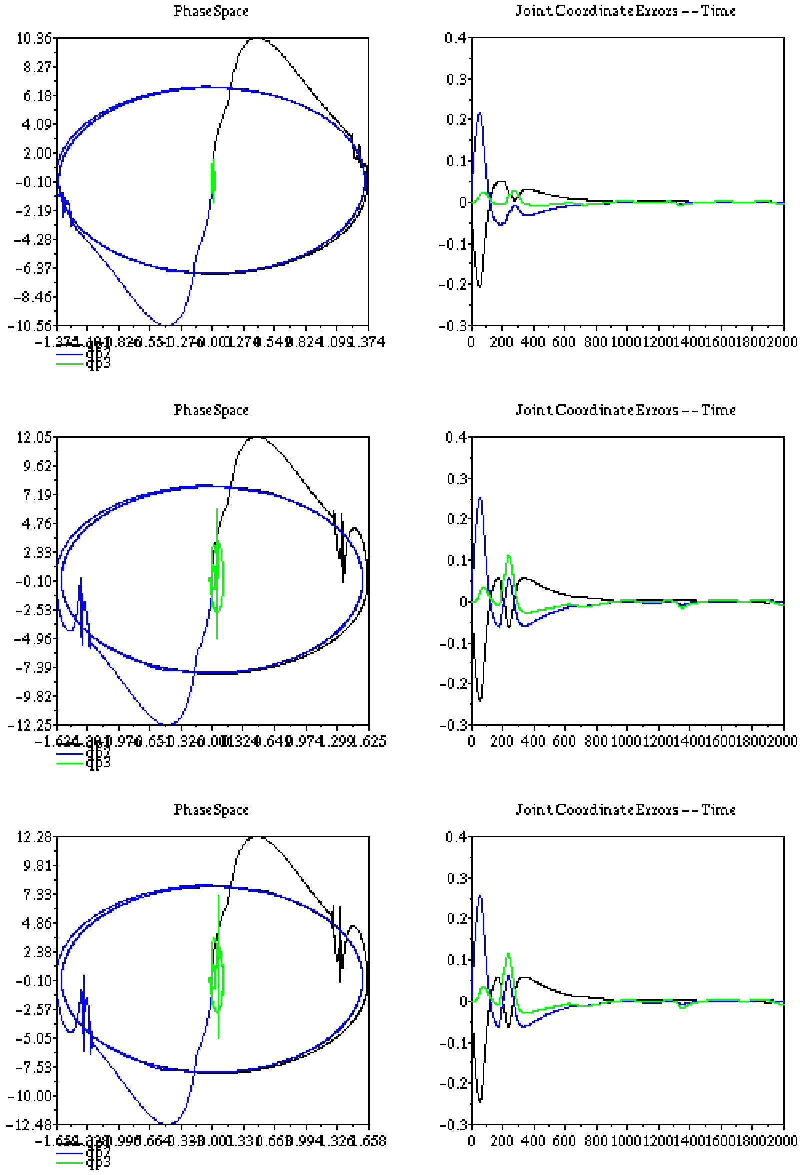

Figure 2. The operation of the adaptive controller in the "far from singularity" (1st row), "just touching the singularity" (2nd row), and "crossing the singularity" (3rd row) cases: the phase space and the joint coordinate errors [*rad* and *m*] vs. time [*ms*]. (q_1=*black*, q_2=*blue*, q_3=*green*)

and this interval was divided into 50 sub-intervals of equal length for calculation (simulation) purposes. For the desired trajectory simultaneous symmetric movement for q_1 and q_2 in the opposite direction is prescribed from the value zero to a finite amplitude. If this amplitude does not approach the "critical" ±π/2 value no controllability problems occur.

In Fig. 1 the trajectory tracking obtained from the non-adaptive controller are presented for M=2.5 kg. Since for approaching the "singularity" the two joint coordinates have to achieve the critical ±π/2 value simultaneously, the really singular position is not approached in this case. It is clear that in spite of that in both the phase space and in the space of the joint coordinates the error is significant. The "adaptive counterparts" of the results of Fig. 1 are presented in Fig. 2. It is evident that adaptivity considerably improves the precision in spite of the fact that due to it the singular positions are well approached in tis latter case. The disturbances are rather observable in the critical point in the graphs describing the phase space.

In Fig. 3. the generalized forces to be exerted by the drives are described in the non-adaptive and in the adaptive case for the "just touching the singularity" trajectory. It can well be seen that at the beginning the adaptive control requires considerable torques and forces to compensate the effects of the modeling errors. It also is evident that the "regulating factor", the "weighting facor" and the norm of the appropriate corrective symplectic matrix minus the unit matrix vary systematically as it is required for "learning" the behavior of the controlled system in real-time. (For comparison, the norm of the unit matrix of size 10×10 is $\sqrt{10}\cong 3$.)

It has to be noted that the minimal cart mass for which acceptable results were obtained is the above considered 2.5 kg. For higher cat mass the inertia matrix is better conditioned so no control problems arise. However, smaller cart mass make the system difficult to control even in the adaptive case.

5 Conclusions

In this paper the behavior of the conventional PID-type and that of an adaptive control based on a novel branch of Computational Cybernetics were compared to each other in the case of controlling an approximately modeled non-linear system having controllability problems in certain points in its joint coordinate space. The simulation results made it clear that adaptivity considerably can improve the quality of the trajectory reproduction though for very small cart masses the controllability problems cannot be completely evaded by it.

The here presented approach evades the sizing problem of the traditional soft computing by applying simple uniform operations in finite number of algebraic steps. The size of the vectors and matrices used by it is simply determined b y the

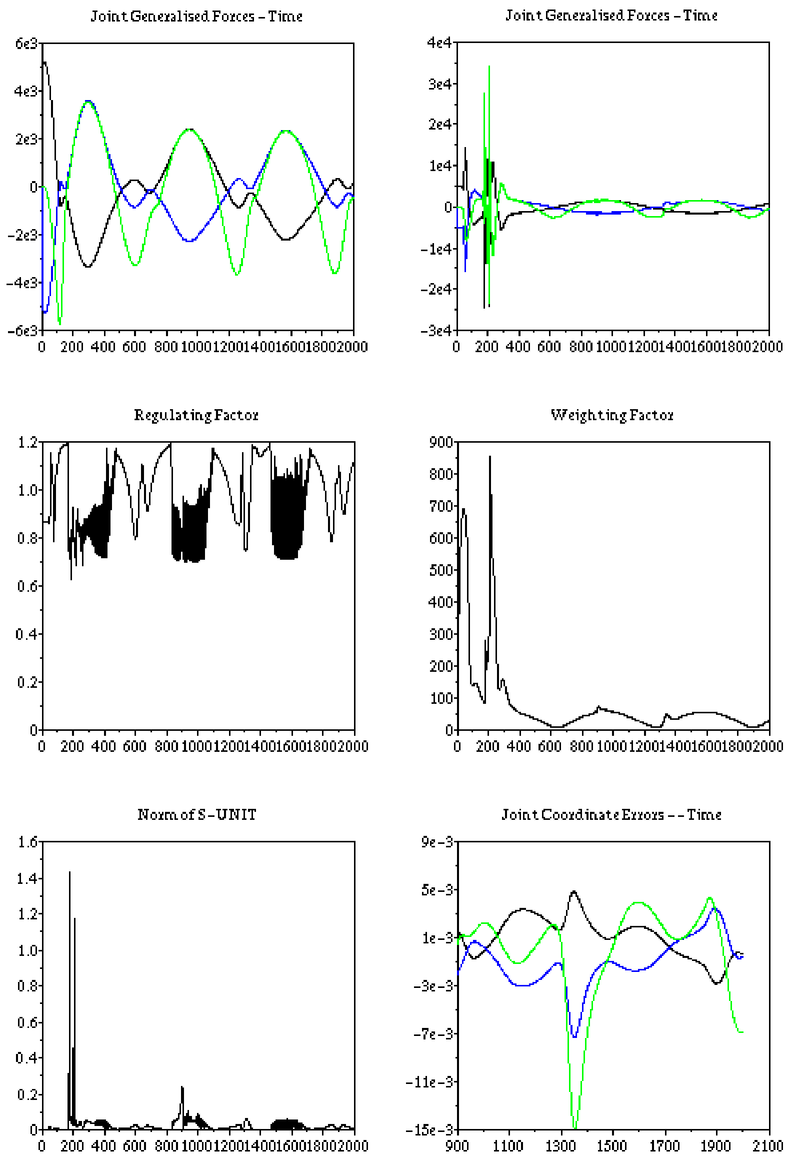

Figure 3. The generalized forces Q_1, Q_2 [*Nm*] and Q_3 [*N*] for the non-adaptive and the adaptive case in the "just touching the singularity" trajectory (1st row), and the internal parameters of the adaptive control: the regulating (interpolation) factor, the weighting factor and the quadratic nonrm of the correction symplectic matrices minus the unit matrix (2nd row and the 1st col. of the 3rd row, and zoomed excerpts of the trajectory tracking error vs. time [*ms*]. (q_1=*black*, q_2=*blue*, q_3=*green*)

modeled number of the degree of freedom of the system to be controlled. The "costs" of these advantages appear in the relatively limited class of problems for which the novel method can be applied.

The critical point is the proper convergence of the series of the linear transformations.

However the here-investigated paradigm suggests that from practical point of view the class of problems for which the new approach can be applied may be quite wide and may have drastic non-linearities, and time lag, too.

5 Acknowledgment

The authors gratefully acknowledge the support by the Hungarian-Greek S&T Project No. GR-39/2001 for 2002-2003, and that of the Hungarian National Research Fund (project: OTKA T034651).

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