# Analyse of adaptive control and design of reference model

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Abstract: Analysis of adaptive control and model design in adaptive control is discussed in this paper. In the first and second part of the paper, basic tendency in adaptive control is analysed. The main aim of the paper is to design the controlled system model or reference model of the controlled system behaviour. With regards to knowledge in adaptive control fuzzy theory can be used to design the model. The WEFUZ method was used in the case. Next, the implementation of the WEFUZ method for model design and some simulation results are presented in the paper.

Keywords: Adaptive control system, system model, WEFUZ method

# **1** Introduction

High level of development of the technical devices for controlled systems is attained, therefore there is a need for developing control algorithms that demand no a priori information about system, that are able to choose the optimal structure and parameters of the adaptive algorithms based on the continuous measurement system's values. Adaptive control systems change the system control form so that its behaviour responds to requirements. It is needed to solve problems that emerge in design of control systems for optimal sources usage. Today, adaptive control is an important part of the control theory. In adaptive control, the way of the control is changed depending on the changing parameters. In the time, it enables the new methods of artificial intelligence for adaptive control systems design to be implemented.

Adaptive control systems can be used in two ways of control:

- Continuous adaptation to dynamics changes of controlled system. Parameters of regulator change dependent on system parameters changing due to non – linearitys of the system in more working points.
- Adaptation to a system with constant but unknown parameters. If adaptation process attains required behaviour it is stopped.

# 2 Methods of adaptive control

In development of adaptive control systems, several different methods and approaches were created.

Most frequent ways of adaptive control are:

- Model Reference Adaptive Control (MRAC);
- Auto-tuning adaptive control systems;
- Adaptive control with measurement of information parameters;
- Heuristic approaches in adaptive control.

#### 2.1 Model Reference adaptive control (MRAC).

One of the basic approaches of adaptive control is model reference adaptive control (MRAC). In this

approach, the output of controlled system Y is compared to output of reference model **Ym**. The structure of MRAC consists of controlled system, which is controlled by a regulator with variable parameters. An adaptive algorithm in "Adaptation block" sets the parameters.The adaptation block minimizes the distinction between output of



Figure 1. Osburn and Whitaker's scheme of Model Reference Adaptive Control

controlled system and output of reference model. Parameters and structure of reference model are specificities on base of requirements of control quality. On Figure 1 scheme MRAC is presented. In 1958, Osburn a Whitaker from Massachusetts Institute of Technology designed this scheme [1].

### 2.2 Auto-tuning adaptive control systems

Despite the previously mentioned approach, auto-tuning adaptive control system is based on indirect adaptation process. This approach indirect adapts parameters regulator and work in discrete time. Closed loop consist linear feedback regulator with adaptive parameter. Parameters of system are identification continuous and

some known methods can be used for calculate parameters of regulator. For every period is calculate new parameters of system. For every identification method can be design other autotuning regulator. Structure auto-tuning adaptive control system is on Figure 2.



Figure 2. Structure auto-tuning adaptive control system

identification parameters of system. F is vector parameters of regulator.

#### 2.3 Adaptive control with measurement informatics parameters

In many processes can be identification parameters of regulator on base merasured

some outputs values. Optimal set parameter are made for only one operating point. For other operating points this setting is not optimal. Goal this adaptive system is adaptation parameters of regulator by change dynamics of system. W In controlled system can measure other as output values. These values are called information values. The values be can information about changes of operation condition and



Figure 3. Gain-Scheduling system with measure informatics parameters of system.

can be using on setting parameters of regulator. On Figure 3 is block scheme Gain-Scheduling regulator.

### 2.4 Heuristic approaches adaptive control

Very simple adaptive algorithms are design by experimental acquired heuristic criteria. These algorithms require minimum priory information about controlled system. Heuristic algorithms have low demands for mathematics operations. These algorithms don't need information about controlled system as order of system or structure. These parameters aren't detected continuous. Marsik adaptive PSD regulator is best-know, because set parameters of regulator upon control error. From control error and criteria of oscillatory is calculated adaptation of parameters. Controlled system can be optional order, linear or nonlinear. Response optimal regulator has only one maximal amplitude. Adaptation of parameters is destined on this behaviour.

### 3. Design of system with method WEFUZ

Advisable decision on design model of system is theory of fuzzy sets. Large development in this area occurred after presentation of paper "Fuzzy Sets" by professor L. A. Zadeh, in 1965. General advantage of this method is in information representation with rules. Rules, with theorems and inferences, describe system in linguistic form. Basic equipment on definition linguistic expression is fuzzy set that uses membership function .Design of model is

established on generation of all kinds of rules, for all membership choices functions of inputs and outputs. Next, weights are adapted based on actual and previous signals from system. Membership function must be chosen according to the range of signals and dynamics of system. Number of membership functions is influenced by precision of model. For model with high dynamic we maybe propose membership more functions. It influences a



Figure 4. Priciple of adaptation mechanism

number of all generated rules in first knowledge base. If we can design precise model, we must for adaptation use signals, which describe full dynamic of system [3].

After all rules are generated, problem with rules occur, which have identical theorems, but different inference. These rules are conflicted, therefore their using in model is not advisable. In model, only these rules will by used, which represent real behaviour of model.

Second knowledge base contains rules, which theorems and inference are identical. Rules are type:

IF x1 is Output l & ... & xN is Output N THEN y1 is Output l & ... & yN is Output N (1)

where

x1, ..., xN, y1, ..., yN – fuzzy variables
Output1, ..., OutputN – membership functions of outputs.

From first and second knowledge bases dependent on degree of firing of individual rules, individual weights of final knowledge base will be calculated. Degree of firing of individual rules depends on signals transported to input of adaptive mechanism. On Figure 4 show adaptive mechanism for adaptation weight of rules.

Description of final rules

$$\mathbf{B} \circ \mathbf{Y} = \mathbf{W} \bullet \mathbf{A} \circ \mathbf{X} \tag{2}$$

where

- **B** matrix of membership function for outputs variables
- Y matrix of outputs variables,
- W weights of rules,
- A Matrix of membership functions for input variables,
- X Matrix of inputs variable
- ∘ T**-**norma
- - Multiplication weights with individual rules

# 3.1 Weight adaptation individual rules with constant coefficient of forgetting $\lambda$

To get the final model weight it's possible to use weight adaptation with constant coefficient of forgetting  $\lambda$ . In this method of all over generated rules are the start weights set to value 1. Coefficient of forgetting  $\lambda$  changes the weights at dependent on signals incoming from system and also considers the weight acquired at adaptation from antecedent signals.

To weight adaptation of a single rule formula is used

$$Wnew(k) = \lambda W(k) + (1 - \lambda) Wold(k)$$
(3)

where

Wnew(k) - New weight acquired after adaptation by all samples of signal

W(k) - Weight of rule for current sample of signal

Wold(k) - Weight of rule acquired after adaptation by all samples but last.

 $\lambda$  - Coefficient of forgetting.

This relation is used for all the rules from knowledge base 1.

# 3.2 Weight adaptation individual rules with adapting coefficient of forgetting $\lambda$

Weight adaptation with adaptation coefficient of forgetting  $\lambda$  is based on similar axiom as weight adaptation with constant coefficient of forgetting  $\lambda$  but coefficient  $\lambda$  is changing during adaptation following the formula:

$$\lambda = \frac{l}{K+i} \tag{4}$$

where

K - is constant chosen empirically at the start of the adaptation

*i* - number weight adaptation, which already overrun.

# 3.3 Adaptation of weight individual rules by method of averaging weight

This method is based on final averaging weight rules for signals connected at input adapting mechanism. For individual rules, rule weights are set for final model as an average of individual weight for all the input signal [2].

#### 3.4 Modification of final weight of system model

After we realize weights of final model it's needed to normalize weights to make the values at range <0,1>. To normalize weights of final model formula was used:

$$w_{vys}(i) = (w_{vyp}(i) - min)/max$$
<sup>(5)</sup>

where

 $W_{vvs}(i)$  - Final value of weight for rule

 $W_{vvp}(i)$  - Calculated value of weight for rule before normalization,

min - Minimal value of weight calculated weight rules

*max* - Maximal value of weight calculated weight rules

It's also recommended to ignore some rules that do not exceed some limit seeing that effect on final model is negligible.

# 4. Realisation of model

Based on method described in chapter 4 design of system model was realized in MATLAB environment.

Model was created for order three system described by transfer function:

$$F(s) = \frac{6}{s^3 + 6s^2 + 11s + 6}$$
(6)

For this system model was designed by method WEFUZ with all three methods of adaptation of weights. Input into adaptation mechanism was signals from system input at the moment t, t-T, t-2T and signal from output at the moment t+T.

By using adaptation of weight with constant coefficient of forgetting  $\lambda$  model of chosen system was designed with different coefficient of forgetting  $\lambda$ . This value was chosen as inverted value of eligible signals count transported into input of adapting mechanism. Final model that was made under this condition was tested and its output was compared to system whose model was described by function of transfer. Testing signal had different run from the like signal, that was the base for this model. Using the other weigth adaptation method, a model of the some system was designed. Final model weight was assigned like arithmetic average of weight acquired for simple combinations of input signals becoming to adapting mechanism.

Count of firing rules	Limit of abandoned	Percent error of design
	weights	model
5	0,01	4,57
5	0,02	4,53
5	0,1	3,68
5	0,2	5,11
10	0,01	14,62
10	0,02	14,58
10	0,1	14,11

Table 1 Error of designed models by adaptive coefficient of forgetting.

Unlike the case before the precision of final model affected no coefficient only the

process signals. To final model is therefore appropriate to include only those rules whose weight has bigger in the than choices limit and also that was more times burned. In final model there are only those rules that were fired more than five-times let us say ten-times. In the table 1 is relation of model percent error and minimal count of firing. Precision of final model depends also on marginal value below which we abandon the weights.



Figure 5. Compare outputs of design model and system

It's interesting that if we include into final model also the rules with lower weights the precision of this model is lower. Therefore it's not recommended at this methods choosing limit of abandoned rules too low. Model precision depends also from count of single rules firings. If we don't use this restriction the final model will be unusable, because it's built of almost all generated rules. On Figure 5 is graph of model output near the abandon weight limit 0,1 and minimal count of burnings 5 and signals feeding at input are the same like signals based of which the model was created.

The last of tested methods is method of adaptation based on adapting forgetting coefficient  $\lambda$ . Adaptation of coefficient of forgetting is defined by formula:

$$\lambda = \frac{1}{5+i} \tag{7}$$

Where

i – count of samples which tread into adaptation.

Final models acquired by this adaptation method indicate similar facilities like models gained at adaptation with constant coefficient  $\lambda$ . Percent error of model for the same abandon weight limits like on first method was also comparable. For adjust error of all models created by method WEFUZ with different weight adaptation methods was applied different input signals like that which models was based on.

#### Conclusions

On making adapting models, many interesting questions are not answered yet. This paper described one of possible system model designing methods with using the theory of fuzzy sets. All methods were appreciated and based on this results't is possible to choose optimal adapting mechanism method. It seems that the optimal is averaging weight method. In this method it was succeeded to obtain high accuracy and error less than 4%. The lower precision achieved methods using to weight adaptation of designed model coefficient of forgetting  $\lambda$ . If we consider that coefficient of forgetting  $\lambda$  and limit whereat the rules with lower weight are abandoned was chosen empirical at optimal choosing this coefficient its possible to achieve the higher precision. Of course it's possible to use this method at mode "off-line" and also at mode "on-line" where it is possible after new combination of input signals to become immediate model rules change and thereby achieve higher precision. Of course it is possible to make rule changes within computing possibilities only.

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