Mamdani-type Implication Inference with Degree of Coincidence

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Abstract: The Mamadani GMP with Mamdani implication inference rule says, that the membership function of the rule output is given with a fuzzy set, which is derived from rule consequence, as a cut of them. This cut is the generalized degree of firing level of the rule, considering actual rule base input, and usually depends on the covering over of the rule base input and rule premisse. But first of all it depends on the sup of that covered memebership function. Because of the non-continuos property of distance-based operators, it was unreasonable to use the classical degree of firing, to give expression to the coincidence of the rule premise, and system input, therefore a Degree of Coincidence (Doc) for those fuzzy sets has been initiated.

Keywords:Mamdani type implication inference, distance based fuzzy operators

1 Mamdani type implication inference

In control theory and also in theory of approximate reasoning introduced by Zadeh in 1979, [14] much of the knowledge of system behavior and system control can be stated in the form of if-then rules. The Fuzzy Logic Control, FLC has been carried out searching for different mathematical models in order to supply these rules.

In most sources it was suggested to represent an

if x is A then x is B

rule in the form of fuzzy implication (shortly $Imp(A,B)$, relation (shortly $R(A,B)$), or simply as a connection (for example as a t-norm, $T(A,B)$) between the so called rule premise: x is A and rule consequence: y is B. Let x be from universe X, y from universe Y, and let x and y be linguistic variables. Fuzzy set A in X is characterized by its membership function μ_A: $x\rightarrow[0,1]$. The most significant differences between the models of FLC-s lie in the definition of this connection, relation or implication.

The other important part of the FLC is the inference mechanism. One of the widely used methods is the Generalized Modus Ponens (GMP), in which the main point is, that the inference y is B' is obtained when the propositions are:

- the i^{th} rule from the rule system of n rules: if x is A_i then y is B_i
- and the system input x is A'.

GMP sees the real influences of the implication or connection choice on the inference mechanisms in fuzzy systems ([4], [13]). Usually the general rule consequence for one rule from a rule system is obtained by

$$B'(y)=\sup_{x\in X}(T(A'(x),Imp(A(x),B(y)))$$

In this topic we can find the new results for left continuos t-norms in [1]. The connection *Imp*(A,B) is generally defined, and it can be some type of t-norm, too.

In engineering applications the Mamdani implication is widely used. The Mamadani GMP with Mamdani implication inference rule says, that the membership function of the consequence B' is defined by

$$B'(y)=\sup_{x\in X}(min(A'(x),min(A(x),B(y)))$$

or generally

$$B'(y)=\sup_{x\in X}(T(A'(x),T(A(x),B(y))) \qquad (1)$$

where T is a t-norm.

Using the t-norm properties, from (1)

$$B'(y)=T(\sup_{x\in X}(T(A'(x),A(x))),B(y))$$

Generally speaking, the consequence (rule output) is given with a fuzzy set $B'(y)$, which is derived from rule consequence $B(y)$, as a cut of the $B(y)$. This cut, $\sup_{x\in X}(T(A'(x),A(x)))$, is the generalized degree of firing level of the rule [13], considering actual rule base input $A'(x)$, and usually depends on the covering over $A(x)$ and $A'(x)$. But first of all it depends on the *sup* of the membership function of $T(A'(x),A(x))$.

The FLC rule base output is constructed as a crisp value calculated with a defuzzification model, from rule base output. Rule base output is an aggregation of all rule consequences *B'*(*y*) in rule base. As aggregation operator, t-conorm is usually used.

$$y_{out} = S(B'_n, S(B'_{n-1}, S(...., S(B'_2, B'_1)))).$$

2 Inference with distance based operators

2.1 Distance based operators

The maximum distance minimum operator with respect to parameter $e \in [0,1]$ is defined as

$$T_e^{\max} = \begin{cases} \max(x,y) & if \quad y-e > e-x \\ \min(x,y) & if \quad y-e < e-x \\ \min(x,y) & if \quad y = x \text{ or } y-e = e-x \end{cases}$$

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The minimum distance minimum operator with respect to $e \in [0,1]$ is defined as

$$T_e^{\min} = \begin{cases} \min(x,y) & if \quad y-e > e-x \\ \max(x,y) & if \quad y-e < e-x \\ \min(x,y) & if \quad y = x \text{ or } y-e = e-x \end{cases}$$

The minimum distance maximum operator with respect to $e \in [0,1]$ is defined as

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2.2 Degree of coincidence in inference mechanism

In system control intuitively one would expect: let's make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker.

The evolutionary operators group, ([9]), and distance-based operators group, ([8]) satisfy that properties, but the covering over $A(x)$ and $A'(x)$ are not really reflect by the *sup* of the membership function of the $T_e^{max}(A(x),A'(x))$.

Hence, and because of the non-continuos property of distance-based operators, it was unreasonable to use the classical degree of firing, to give expression to the coincidence of the rule premise (fuzzy set *A*), and system input (fuzzy set *A'*), therefore a Degree of Coincidence (*Doc*) for those fuzzy sets has been initiated. It is nothing else, but the proportion of area under membership function of the distance-based intersection of those fuzzy sets, and the area under membership function of the their union (using *max* as the fuzzy union).

$$Doc = \frac{\int_X T_e(A, A')dx}{\int_X \max(A, A')dx}$$

This defnition has two advantages:

- it consider the width of coincidence of *A* and *A'*, and not only the "height", the *sup*, and

- the rule output is weighted with a measure of coincidence of *A* and *A'* in each rule ([10]).

The rule output fuzzy set *B'* is achieved as a cut of rule consequence *B* with *Doc*.

$$B'(y)=T_e^{min}(B(y), Doc) \quad \text{or}$$

$$B'(y)=T_e^{max}(B(y), Doc)$$

It is easy to prove that $Doc\in[0,1]$, and $Doc=1$ if *A* and *A'* cover each other, and then $B'(y)=B(y)$, and $Doc=0$ if *A* and *A'* have no point of contact, and then $B'(y)=0$.

The FLC rule base output is constructed as above explained. The output is constructed as a crisp value calculated from rule base output, which is an aggregation of all rule consequences $B_i'(y)$ in rule base. For aggregation, distance based operators S_e^{min} or S_e^{max} can be used.

An additional possibility is if the cut $B'(y)$ of the rule concequence $B_i(y)$ is calculated from the

$$Doc = \frac{\int_Y B'(y)dy}{\int_Y B(y)dy}$$

expression ([11]).

Based on this, for triangular membership functions $A(x)$, $A(x)$, $B(y)$, we have

$$B'(y) = \max\left(B(y), 1 - \sqrt{1 - Doc}\right).$$

$B'(y)$ is obtained as a weighted fuzzy set, and the weight parameter Doc depends on $\int T_e^{max}(A(x),A'(x))dx$. It is a measure number related to the area under membership function $T_e^{max}(A(x),A'(x))$, and it is a fuzzy measure in the same sense, as it can be found in [2] and [5],[6],[7]. Taking this fact into consideration, a connection between Doc type of inference mechanism and generalized fuzzy measures and integrals is being researched.

2.3 Example

Let we have a rule base system, with n=5 rules. Figure 1. shows the membership funktions of the rule premisses ($A_i(x)$), rule input ($A'(x)$), rule concequences ($B_i(y)$), rule outputs ($B'_i(y)$), and the aggregated rule output ($B(y)$).

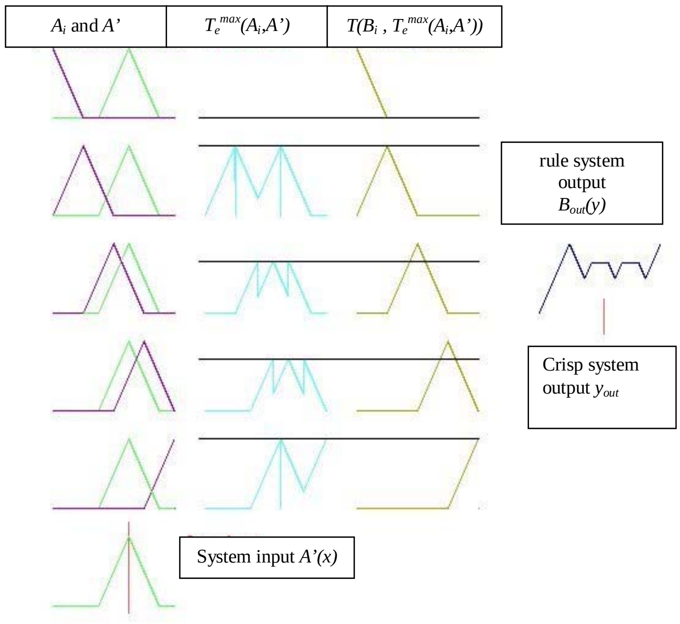

figure 1.

We can see the justification for this line of reasoning in the simulations in a simple dynamic system, using distance based operator-pairs (T_e^{max},S_e^{max}) and ((T_e^{min},S_e^{min})).

3 Conclusion

In a FLC system, where we use distance based fuzzy operators in inference mechanism, the rule base output is obtained as a weighted fuzzy set of the rule concequence, and the weight parameter *Doc*, depends on a measure number related to the area under membership function $T_e^{max\ or\ min}(A(x),A'(x))$, and it is a fuzzy measure in the same sense, as it can be found in [2] and [5],[6],[7]. Taking this fact into consideration, a connection between *Doc* type of inference mechanism and generalized fuzzy measures and integrals is being researched.

The further steps are the investigations of measure-properties of different degree of firing types used by FLC, and the use of the other types of fuzzy integrals in decision-making by FLC, above all integrals introduced in pseudo analysis.

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