Environmental Indices Based on Fuzzy Logic

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Abstract: Traditional environmental indices are based mainly on sets with sharp boundaries. As natural extension we introduce fuzzy sets in this framework. Since fuzzy sets are described by membership functions, it becomes also possible to integrate different kinds of observations, to quantify and classify subjective descriptions of environmental effects and to deal with missing data.

Acceptable Environmental Conditions

Environmental indices rarely have much significance by themselves. Knowing that some variable like pollutant concentration or soil acidity has a specific value is usually meaningful only in the context of knowledge of natural background levels, regulatory policy, and the vulnerability of key environmental components. For this reason it is useful, and probably more practical, to relate these indices to some sort of acceptability measure, which can be interpreted as the membership in a fuzzy set of acceptable environmental conditions.

The concept of ‘acceptability’ is itself seen by some as fuzzy, in the colloquial rather than mathematical sense, but this reflects the reality that we can measure environmental effects far more accurately than we can evaluate their significance. It also reflects a lack of consensus about what different levels mean, and a basic problem within our social structures in defining environmental values and objectives. It may therefore be seen as more honest, although less objective, to transform quantitative measurements of environmental variables into a fuzzy membership, which represents the degree of acceptability of those variable values in the set of acceptable conditions.

Because different segments of society often cannot agree on what is acceptable and what is not, it may be desirable to identify several sets of acceptable conditions, representing the view of these different social groupings.

Individual Memberships and their Combinations

Traditionally the symbol μ has been used to represent fuzzy memberships. If x represents the value of an environmental variable, then $\mu(x)$ is the corresponding membership in the set of acceptable conditions, and takes a value between zero and one. For example, if a lake becomes hypoxic and all the fish die, then μ would presumably be zero, indicating that this situation is totally unacceptable.

In most situations more than one environmental variable is important, and we can define $\mu_i(x_i)$, which is called the 'individual membership', to represent the acceptability of the *i*-th environmental variable. One then needs to develop ways of combining these individual memberships to obtain a general measure of acceptability.

The way in which individual memberships are combined depends on application. In some cases the appropriate combination is a standard fuzzy relationship; for example, if our objective is to have a healthy fish population in a lake, and the environmental variables are oxygen level, water temperature, and nutrients, then since adverse levels of any of these can be lethal to fish, the fuzzy intersection, $\mu = \min(\mu_1, \mu_2, \mu_3)$, is appropriate. This definition states that the combined acceptability is as low as the lowest of the three partial acceptabilities. More commonly, however, there is some compensatory effect and low acceptability of one variable value can be compensated by good values of another. For this reason the choice of combination rule has to be made in the context of a specific application.

Nevertheless, the combination method called 'symmetric summation' (Silvert, 1979) owns advantageous mathematical properties. In addition, there is a political reason for using symmetric summation, which is that it makes it easier to avoid value judgements. One therefore needs to describe effects in such a way that there is no implicit assumption that certain levels are 'good' and others are 'bad'. Symmetric summation can describe effects by generating the same results whether one uses sets or their complements. In mathematical terms, if $\otimes$ represents the symmetric summation operator and ~A is the complement of A (i.e. the membership in ~A is $1 - \mu_A$), then

$$\sim A \otimes \sim B = \sim (A \otimes B)$$

In linguistic terms, symmetric summation offers a compromise between the logical operators AND and OR. Instead of saying that the combined effects are acceptable if A is acceptable AND B is acceptable, or that they are unacceptable if A is unacceptable OR B is unacceptable, one has a more neutral dividing line between what is and what is not acceptable.

Another advantage of using symmetric summation rather than the intersection operator or other more traditional fuzzy relations is that it permits weighting different effects according to their importance and significance, as previously described. The general equation for the weighted symmetric sum of an arbitrary number of fuzzy memberships is

$$\mu / (1 - \mu) = \{[\mu_1 / (1 - \mu_1)]^A [\mu_2 / (1 - \mu_2)]^B [\mu_3 / (1 - \mu_3)]^C \ldots\}^{1/(A+B+C+\ldots)} \qquad (1)$$

where μ is the combined membership, the μ_i are individual memberships for different observations, and the exponents A, B, C, … are the weighting factors. The meaning of this complex looking expression for the symmetric sum is simply that the value of $\mu / (1 - \mu)$ is the weighted geometric mean of the ratios for individual memberships, $\mu_i / (1 - \mu_i)$, and it is clear that one gets the same result by replacing μ by $1 - \mu$ and μ_i by $1 - \mu_i$ all the way through.

One of the advantages of this formalism is that it offers a way of dealing with missing data. Suppose for simplicity that one has just three observations, so that the memberships are given by

$$\mu / (1 - \mu) = \{[\mu_1 / (1 - \mu_1)]^A [\mu_2 / (1 - \mu_2)]^B [\mu_3 / (1 - \mu_3)]^C\}^{1/(A+B+C)}$$

and suppose that for one set of data the second observation is missing (this frequently happens, as a result of equipment failure, bad observing conditions, or many other situations beyond the control of the scientists organising the monitoring program). One can then use

$$\mu / (1 - \mu) = \{[\mu_1 / (1 - \mu_1)]^A [\mu_3 / (1 - \mu_3)]^C\}^{1/(A+C)}$$

to calculate the memberships for this set of data, and although the figures are not as reliable as they would be if one had complete data, this is better than having to discard the complete set of observations because they are not complete.

The relationship between an environmental variable x and its acceptability $\mu(x)$ can be quite complicated. Often values that are too low or too high are unacceptable, and the acceptability is high only for an intermediate range. More important, and more significant in terms of the practical use of fuzzy logic in appraising environmental conditions, one can estimate the acceptability of environmental conditions that are themselves difficult or even impossible to quantify. Odours are a good example of this - there are no commonly accepted techniques for measuring how bad something smells, but it is possible to determine that people find certain smells unacceptable to varying degrees, and so we can assign at least an approximate value to m without having a quantitative measure of x.

This, in fact, is one of the great strengths of fuzzy logic - it lets us deal with subjective and non-quantitative data. Actually, some scientists see this as a great weakness in the use of fuzzy logic, since they feel that only ‘hard’ data should be used in projects where scientists are involved. However, one has to be realistic; environmental effects are of concern not just to scientists, but also to businessmen, politicians, and ordinary citizens, and once one has ventured into the social and political realm where the real decisions are made, it simply is not possible to tell people that their concerns about foul smells and noise-induced headaches will be left out of the equation because they cannot be adequately measured!

Different Types of Environmental Effects

Most activities, whether human or natural, have numerous different effects, and any environmental index must allow for these and provide a means of synthesising a comprehensive measure of all these effects. For example, aquaculture has many potential environmental effects, including (Silvert 1992):

- carbon accumulation under the site, and associated degradation effects
- oxygen consumption by dense populations of farmed organisms
- release of nutrients into the water column

One can quantify the first of these by use of fuzzy logic, and the other two are relatively easy to quantify. However, how does one combine the results of such different types of effects, which not only affect different components of the ecosystem, but operate on very different space and time scales? Is it possible to incorporate the fact that some of these effects, such as the release of nutrients into the water column, may actually be beneficial? These questions reach well beyond the realm of the present study and can involve issues from a wide range of fields. For example, increased productivity due to enrichment of the water column may improve the harvest of other organisms (one often finds lobster and crab fishermen in the vicinity of fish farms), but it may also lead to undesirable algal blooms and has been blamed for the appearance of decaying seaweed along the shoreline.

It is probably a general truth that environmental indices are meaningful only within a restricted context, and that the degree of impact of any project depends on the viewpoint of the observer.

Combining Environmental Fuzzy Indices

One aspect of the problem of applying fuzzy indices to complex situations is the need to combine different indices representing different impacts. Perhaps the strongest positive feature of fuzzy logic in developing environmental indices is the ability to combine such indices much more flexibly than one can combine discrete measures, which are often simply binary indices corresponding to ordinary (‘crisp’) sets, such as ‘acceptable versus unacceptable’. For this reason it is important to discuss how to combine different fuzzy indices. The issues involved are both mathematical and strategic.

The different fuzzy sets used in classifying environmental effects can be classified as complementary or independent. Complementary sets are ones which describe different ranges of the same properties; examples are pristine versus polluted. Independent sets are ones that address different properties. A common example is that in describing humans, the sets of ‘short people’ and ‘tall people’ are complementary, but the sets of ‘short people’ and ‘fat people’ are independent.

Of course with fuzzy sets it is possible to have membership in two or more 'complementary' sets; for example it is possible to belong both to a set and to its complement, but the defining characteristic is that the sum of individual memberships in complementary sets must be one. For example, if all men are divided into the four fuzzy sets 'short', 'tall', 'thin' and 'fat', then someone of medium build might be 40% short, 60% tall, 30% thin and 70% fat; the memberships in 'short' and 'tall' sum to one, as do the memberships in 'thin' and 'fat', but there is no fundamental relationship between memberships in 'short' and 'thin' (although the memberships in independent sets may be statistically correlated).

The manner in which one combines the memberships in two or more sets depends on whether they are complementary or independent. Complementary sets are the easiest to deal with, because they must of necessity be comparable. Independent sets do not have to be comparable in any sense, and they can represent different space and time scales, different areas of effect, and even different sets of objectives.

However, the rules of combination cannot be determined on purely mathematical grounds, which is what we mean by 'strategic' considerations. Two examples have already been given of situations which call for different acceptability criteria. One is the case of environmental conditions which affect the survival of a valued species - all environmental variables must be acceptable, or the species will die out, so the rule of combination is $\mu = \min(\mu_1, \mu_2, \mu_3,\ldots)$. Another case is one where we wish to avoid this kind of value judgement and use the symmetric sum, defined in Eq. (1). It should also be noted that some operators, like min (the minimum value of its arguments), treat all of the individual memberships equally, while the symmetric sum and some other operators permit different observations to have different weights.

The complexity of deciding how to combine individual memberships is dramatically represented by the interplay between scientific, conservation, social, economic, and political issues in assessing environmental effects. It is difficult to reconcile the scientific and quality-of-life issues associated with cutting old-growth forests or actions that might reduce whale populations with concerns about jobs and revenue. It is possible that fuzzy logic could play a role in addressing these conflicts. However, we acknowledge that fuzzy logic has never been applied to such problems (at least so far as we are aware), and there is no solid evidence that it would provide a common framework for resolving conflicts, rather than being seen as a particularly suspect form of mathematical mystification. One can envision fuzzy logic as a valuable tool in quantifying environmental conditions that could have applicability in conflict resolution going well beyond its purely scientific utility, but that remains unproved.

Multi-objective Decision Making

As pointed out earlier, society is not always able to reach consensus on the value of certain components of the environment, so that effects which are acceptable to some segments may be far less acceptable to others. Examples include the abundances of certain birds, which are highly prized for their beauty and entertainment value by recreational users of the environment, but are seen as predators and competitors by fishers and farmers. Many complex issues deal with the marginal (i.e. incremental) value of natural lands, such as the question of how much old-growth forest should remain protected and how much can be exploited. An obvious generalisation of the concept of acceptability, and of the mathematical concept of a fuzzy set of acceptable conditions, is to define several acceptability sets, each representing the viewpoint of a different segment of society.

For each environmental variable i, let μ_{ij} be the membership of the measured value of i in acceptability set j (i.e. the measure of how acceptable i is from viewpoint j, on a scale of 0 - 1). Then the overall degree of satisfaction of objective j is the combined fuzzy memberships over all variables i. We can also define a set of weights w_{ij} which represent the weight of variable i in satisfying objective j, namely its importance in terms of meeting that objective. Although the mathematics tends to become cumbersome, for example we could transform Eq. (1) into a system of even more complicated equations indexed by the different objectives j, the underlying idea is fairly straightforward.

As an example of these considerations, consider indices for marine systems. Possible objectives could be good recreational use, sustainable wild fisheries, and profitable shellfish aquaculture. Relevant variables could include the abundance of birds, and the occurrence of toxic marine algae. From a recreational point of view birds are usually considered very desirable (except for very common species like seagulls), and the more abundant they are, the higher the degree of satisfaction of the recreational objective. Fishermen do not always share this love for birds; fishermen often see cormorants as dangerous competitors, while some of the most attractive birds like oystercatchers and eider ducks are efficient predators on shellfish. Toxic algae are merely a nuisance to swimmers and other recreational users, but can cause the closure and economic ruin of shellfish farms. Thus the memberships are different (the higher the number of eider ducks the greater the membership, i.e. the acceptability, for bird watchers and the lower the acceptability to mussel farmers) and the weights can also vary greatly (toxic algae are a minor irritant to swimmers but a financial disaster for both wild and farmed fisheries).

Consensus building with fuzzy logic

Multi-objective decision making is at the heart of the political process, which involves trying to build a consensus among groups with different values and goals. The formalism described above can be used to identify key areas of disagreement and may possibly contribute to the resolution of conflict in complex situations by providing a language for quantifying these disagreements.

We propose a three-step procedure to deal with these kinds of disagreements:

Identify environmental variables on which agreement can be reached, and reach consensus on the individual memberships and weighting factors. For example, point source air pollution is usually of more concern to nearby residents than to the producers, but everyone agrees that air pollution is undesirable, so it should be possible to arrive at an agreement regarding acceptability levels.

This enables the participants to focus on areas where there is real disagreement, such as birds in the situation described above, without being distracted by issues on which consensus is readily achievable.

Once the basically different objectives of various groups have been clearly delineated, sets of acceptability scores for the different objectives can be calculated for different scenarios, and used to provide a focus for further discussion.

It is of course unrealistic to assume that mathematical calculations will be accepted as a decisive means to solve complex social and political issues, but they offer a quantitative expression of the differing values and needs expressed during negotiations, and as such can help clarify the basic underlying issues.

Conclusion

Fuzzy logic can be applied to the development of environmental indices in a way that resolves many common problems, such as incompatible observations and implicit value judgements. It bridges the gap between scientific measurement and the fulfilment of social objectives and provides a way to translate a wide variety of information - objective data, qualitative information, subjective opinions, and social needs - into a common language for characterising environmental effects.

Because it offers a means for assigning different weights to different types of observation, it can deal with differing perceptions of environmental risk. It can also deal with missing data, a common problem in assessing environmental impacts. Although it has yet to be extensively used in developing environmental indices, it has shown its value in a pilot study, and we believe that it merits considerable further investigation.

References

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