ANALYSIS AND VIEWING THE SURFACE STRUCTURES

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Abstract: In the recent years Geographic information systems become a component of information systems design in last years in consequence of unsubstitutable task of objects and events localization performed by these systems. To relate data and information about activities and resources to its location in space and to monitor or eventually to forecast a trend in time is essential for the modern society. Digital map is a quality data source. This article deals with contour line and fall line maps design facilities. An algorithm of contour line and fall line drawing is presented in the article assuming existence of analytical representation of topographic plane. In case the analytical representation of topographic plane is not available there is possible to come out of a theory of gravitation nodes. The system of gravitation nodes ensures physically correct interpretation of contour line and fall line structure and it provides a simple interface for contour line, fall line and terrain relief construction.

Keywords: GIS – Geographic Information System, contour line, fall line, gravity model

1 Introduction

Data acquisition of important objects and resources location on the earth surface has always been a striking activity of human communities. There were various motivations but a narrow connection to astronomy has always existed, knowledge about the Earth and cosmic orbs aspects has been used. Human activities were bringing newer knowledge about spatial objects on the Earth all the time. The achievement of these activities were maps, for example astronomical, geological, tectonic, ground, demographic, transport, climatic etc. In contrast to the initial need for accurate localization of objects, the storage of grooving amount of information about them had to be solved. Computer systems became the answer for this problem.

Geografic Information System (GIS) as a computer tool for mapping and analysis of objects and events of real world combines common database operations as querying and statistical calculations with unique displaying and spatial analysis facilities provided by a *map*. These features markedly distinguish GIS from the other information systems. GIS finds its application also in the field of cartography [4].

Cartography dealing with displaying and studying of spatial location and interbonding between nature and society phenomena uses graphical and model tools for maps creation. A part of information for cartography is provided by *geodesy,* so the results of many geodetic measurements constitute a basement for creation of various kind of maps with several purposes [3,5].

2 Maps

Apparently it is necessary to scale down the projection of reality when a map is being created [3]. In the course of every scaling down, some generalization of reality occurs – elimination of non-essential facts and details from the map representation (fig. 1). Generalization process must not affect transparency, descriptivity and aesthetics of the map. The map content is created by conventional cartographic means of expression (cartographic symbols), which incorporates *lines* (objects contours, *isolines* – contour lines, etc.), *shape symbols* (geometric, schematic, symbolic, literal, etc.), *line signs, numerical data* in various colors, *planar signs, color overlay of areas, shading, rasterization, point signs, motional signs,* etc.

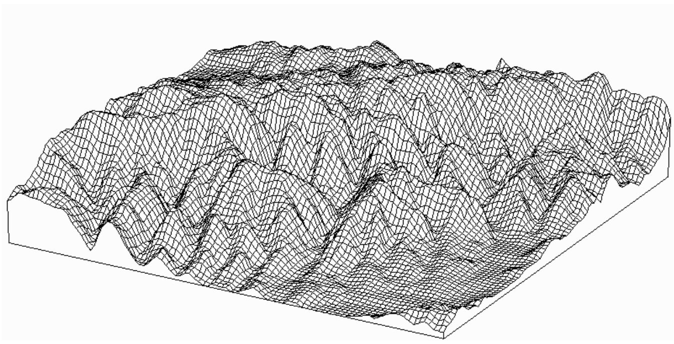

Fig. 1: Axonometric grid terrain mapping

Digital maps are more and more applied in various automated processing routines [8].

2.1 Digital maps

Digital terrain model is a set of selected points – nodal points of a *topographic plane*. The terrain plane is very multiform. It is defined by digital terrain model in its nodal points, while the elevations of the other terrain plane points has to be determined by applicable interpolation. From the approximation point of view the terrain points can be *regular* (there is possible to define tangent planes to topographic plane in them), and *singular* (there is not possible to create tangent planes, the points constitute the "terrain edges", e.g. edge points of the earth fills, excavations, etc.).

If the common equation form of the continuous topographic plane over defined bounded area will be:

$$z = f(x, y) \tag{1}$$

with the unknown analytical formulation and the unknown coefficients, there is possible to define this plane by finite set of *n* nodal points gracefully placed on the area plane, whose N-dimensional coordinate vector:

$$\vec{z}_{n,l} = \vec{f}_{n,l}(x, y) \tag{2}$$

is know from the measuring. Elevations of the other points (*z* - coordinate) are determined by interpolation polynomial of the m-th degree:

$$\begin{aligned} P_m(x, y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + \ldots + \\ + a_{m0}x^m + a_{m-1,1}x^{m-1}y + \ldots + a_{0m}y^2 \end{aligned} \tag{3}$$

which will approximate the unknown formulation of the topographic plane (1).

The minimal number of nodal points for determination of coefficients (3) is:

$$n_{\min} = \frac{(m+1)(m+2)}{2} \tag{4}$$

while *n* is chosen according $n > n_{mjn} > m$.

Topographic plane can also be approximated by other interpolation polynomials, e.g. B-spline polynomial, non-uniform rational B-spline polynomials (NURBS), etc. [3].

3 Constructions on topographic planes

By intersecting of topographic plane with certain set of horizontal contour planes with constant distance between them – the equidistance – *e*, the *contour line image* – contour line plan of topographic plane is obtained [1].

From constructional reasons sometimes the *intercalar* contour lines are placed between existing contour lines by dividing distances of two adjacent contour lines to defined equal number of parts (fig. 2). The terrain between the two adjacent contour lines is assumed to rise or fall continuously.

It is possible to determine the shape of topographic plane from its contour line plan. There is a hatch applied in the direction in which the water would likely flow off the plane to achieve a plastic effect of the plane. This direction is indicated by the *fall lines* of the plane. These lines are orthogonal to all contour lines, what means that the tangenta of the contour line is normal to the fall line (fig. 3).

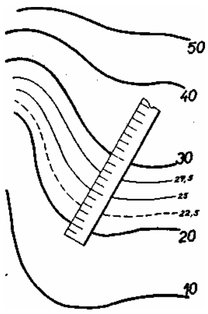

Fig. 2: Interscalar contour lines

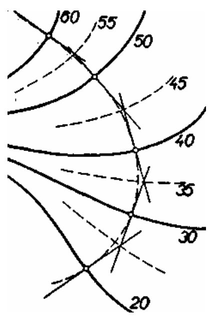

Fig. 3: Fall lines

3.1 Contour lines

A basic requirement for the terrain model is a possibility to derive the terrain elevation for defined place, object, etc. The way of solving this requirement is related to the type of terrain model.

Besides the image approach of representation there is a contour line representation of surface used, created by Euler-Monge [2]. The advantages are apparent in the field of technical applications. This way the geographic and meteorologic maps or pictures representing distribution of the electrical field potential.

The *contour line representation* (fig. 5) can be expanded to functions, which are not defined on the planar area but on the curved planes, for example the distribution of the temperature on the surface of the complicatedly shaped component. The values beside contour lines indicate the function values and the density indicates their slope.

Let the function (1) determining the shape of topographic plane to be bounded on the interval $\Omega = [\alpha,\beta]x[\gamma,\delta]$. The equation of the contour line with nominal value λ will be in form:

$$M(\lambda) = \{(x, y) \in \Omega \,|\, f(x, y) = \lambda\} \tag{5}$$

Then the contour line is a complex of points in which the function reaches the value λ.

If the nominal values of contour lines are not given, extremes (minimum and maximum) on interval Ω are calculated first:

$$f_{\max} = \max_{\Omega}\{f(x,y)\}$$
$$f_{\min} = \min_{\Omega}\{f(x,y)\} \tag{6}$$

Practically it is possible to do it using non-linear optimizing algorithm, which indicates extremes of given function (we will need these points also for displaying contour line structure). As the screen resolution is relatively low the non-linear optimizing algorithm will be in a very simple form, i.e. we will calculate values of function in every point of Ω area, which match the screen pixels. For this reason we will determine minimum and maximum.

For drawing c contour lines with constant distance between them, the nominal values can be calculated according to:

$$\lambda_i = f_{\min} + i \cdot \frac{f_{\max} - f_{\min}}{c+1}; i \in \{0,1,\ldots,c+1\} \tag{7}$$

For contour lines drawing the following algorithm will be used:

INPUT: coordinates list of nodal points of the terrain plane $(x_{0,0}, y_{0,0}, z_{0,0})$, $(x_{0,1}, y_{0,1}, z_{0,1})$, ... , $(x_{n,l}, y_{n,l}, z_{n,l})$

1. Elevations of other points (coordinates z) are determined by interpolation polynomial of mth degree according to (3).
2. Extremes are calculated according to (6).
3. Let the area Ω be represented on display by a rectangle (X_0, Y_0, X_p, Y_q). By dividing the rectangle to pixels there will be p pixels in every row and q pixels in every row:

$$p = X_p - X_0, \quad q = Y_0 - Y_q \tag{8}$$

Connection between points (x_r, y_s) of the Ω area and points (X_r, Y_s) on display can be defined by the following equations:

$$x_r = \alpha + \frac{r}{p}(\beta - \alpha) \quad \rightarrow \quad X_r = X_0 + r; \quad r \in \{0,1,\ldots,p\}$$
$$y_s = \gamma + \frac{s}{q}(\delta - \gamma) \quad \rightarrow \quad Y_s = Y_0 + s; \quad s \in \{0,1,\ldots,q\} \tag{9}$$

4. In ever point where the function changes its sign there is necessary to change

$$g_i(r,s)=f(x_r,y_s)-\lambda_i; \quad i\in\{0,1,...,c\} \tag{10}$$

a color of the raster point.

3.2 Fall lines

An important property of contour lines and fall lines is their perpendicularity (fig. 6). Two fall lines or two contour lines can never intersect in accordance with their definition.

A gradient of the function (1) is vector which coordinates are partial derivations of function *f*, therefore:

$$\nabla f(x,y)=\left(\frac{\partial f}{\partial x}(x,y),\frac{\partial f}{\partial y}(x,y)\right) \tag{11}$$

The gradient vector is always directed to the highest increase.

Let E_x and E_y represent the gradient vector coordinates:

$$E_x=\frac{\partial f}{\partial x}, \quad E_y=\frac{\partial f}{\partial y} \tag{12}$$

Nominal length of the fall line arc is approximated by the vector colinear with the tangent and with projections to the axis of coordinate system O*xy* are *dx* and *dy*. It is true that:

$$\frac{dx}{dy}=\frac{E_x}{E_y} \tag{13}$$

Relation defines the length of the above-mentioned vector:

$$ds=\sqrt{(dx^2+dy^2)} \tag{14}$$

Therefore the equations of the individual projections take the form:

$$dx=\frac{E_x}{E}ds, \quad dy=\frac{E_y}{E}ds \tag{15}$$

where E is the gradient length:

$$E=\sqrt{(E_x{}^2+E_y{}^2)} \tag{16}$$

For the drawing of fall lines the following algorithm is used:

1. The starting point (x^k,y^k) is specified.

2. The individual length of the arc (*ds*) is specified.

3. The coordinates of the gradient vector are calculated in that point.

4. Using relations (15) *dx* and *dy* are calculated. This will be valid for the new fall line point:

$$(x^{k+1}, y^{k+1}) = (x^k + dx, y^k + dx) \tag{17}$$

5. Using dedicated procedure the vector will be outlined, which is an approximation of the unit arc of the fall line.

6. The presented technique is repeated 1 to 5 times also for newly calculated points.

The smaller the unit arc *ds* length is, the highest accuracy it is possible to reach. The fall line ending point is identical with point specifying a global minimum i.e maximum of the function z = f(x,y). It is necessary to define a condition of algorithm termination in these points because the partial derivations in extremes are equal to zero. The condition of algorithm termination is in form $E < \varepsilon$, where ε is a sufficiently small positive number. Further the fall lines can end also on the borders of the given area, therefore there is necessary to define conditions of algorithm termination.

3.3 Gravitational nodes system

In case the analytic formulation of topographic plane is not known there is possible to come out of the theory of **gravitational nodes**. The basis of mentioned plane modelling is the following physical model:

1. The plane points lying in the grid nodes dividing range of definition are considered to be mass points, which can move only in vertical direction.

2. The plane points are connected with the plane $z = 0$ because of the elastic force effect:

$$F_1 = -az\vec{k} \tag{18}$$

where z is a distance of the point from the plane xy, and a is an elasticity coeficient.

3. The gravitational nodes (x_k, y_k, z_k) affect the plane points (x, y, z) by attractive force (of gravitational i.e electric nature) which intensity is inversely proportional to the square of distance between them and there is true for this force that:

$$\vec{F}_2 = \sum_{k=1}^{n} \frac{w_k[(x-x_k)\vec{i} + (y-y_k)\vec{j} + (z-z_k)\vec{k}]}{[(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2]^{3/2}} \tag{19}$$

where w_k is the gravity (weight) of the *k*-th gravitational node.

4. The node position is specified by the following balance of power:

$$F_1\vec{k} + F_2\vec{k} = 0 \tag{20}$$

which for $a = 1$ leads to the following equation:

$$z = \sum_{k=1}^{n} \frac{w_k(z_k - z)}{[(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2]^{3/2}} \tag{21}$$

The above-mentioned equation is a contraction; therefore z can be calculated by iteration method (starting from the point z = 0 the equation (21) is applied while the difference of two consequent values is not smaller then the number ε, where number ε is sufficiently small positive number representing the calculation error).

On the basis of presented algorithm for n = 7, (x_1, y_1, z_1, w_1) = (-3.1, -1.4, 3.0, 1.0), (x_2, y_2, z_2, w_2) = (-1.3, 2.6, -2.4), (x_3, y_3, z_3, w_3) = (2.8, 1.8, -2.9), (x_4, y_4, z_4, w_4) = (1.6, -0.6, 1.9), (x_5, y_5, z_5, w_5) = (-0.4, -2.7, - 2.3), (x_6, y_6, z_6, w_6) = (2.2, 1.5, -2.5), (x_7, y_7, z_7, w_7) = (2.0, 0.0, -3.0), for range of definition [-5, 5]x[-5, 5], in 15-times vertical zoom in the AVSS (Analyzing and Viewing the Surface Structures) program environment there was created an image (fig. 4) of the specified object:

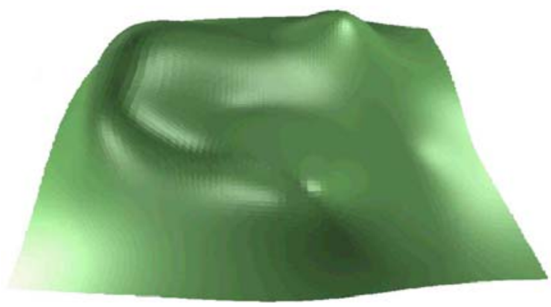

Fig. 4: Interactively created plane using AVSS system

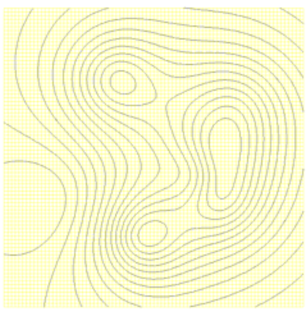

Fig. 5: Contour lines of the plane on fig. 4

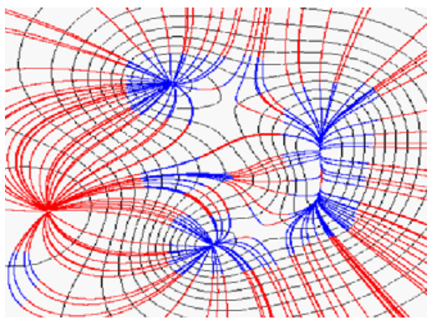

Fig. 6: Contour lines of the plane on fig. 4

Conclusion

Today the new technologies are able to capture data with higher accuracy, speed and regularity then in the past. Software tools as the core of system technologies make it possible to process raw data from various systems performing their acquisition [6,7]. In the various types of geographical operations the result is best visualized as a map or a graph. The maps are very effective tool for storing and transferring spatial information.

This article deals with creation of contour line and fall line maps. In the article an algorithm of drawing contour lines and fall lines is presented assuming existence of the analytical representation of topographic plane. In case the analytic formulation of topographic plane is not available, there is possible to come out of the theory of gravitational nodes. The system of gravitational nodes provides physically correct interpretation of contour line and fall line structure. It presents a simple interface for contour line, fall line and terrain relief construction in the form of gravitational nodes. A disadvantage of this system is the increase of mentioned structures parameters computing time with the increase of gravitational nodes number.

At the present time there is a development of interactive program system AVSS (Analyzing and Viewing the Surface Structures) running. This system is developed on the Department of Computers and Informatics, Faculty of Electrical Engineering and Informatics, Technical University in Košice in Borland Builder C++ 5.0 environment. It uses the above-mentioned theory of gravitational nodes and knowledge achieved in the VEGA project researches. The program system AVSS enables creation of contour line and fall line structures on the basis of gravitational nodes. For the surface structures modelling the following operations have been implemented until now: insertion, removal of gravitational nodes and modifying of their parameters. The system provides information about attributes of individual fall lines, particular information about convexity or concavity of the fall line. It displays graphically the contour line, fall line and terrain relief structure.

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Supported by VEGA project No. 1/1064/04