# Adaptive Control of a Wheel of Unmodeled Internal Degree of Freedom

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#### Abstract:

A new branch of Computational Cybernetics based on principles akin to that of the traditional Soft Computing (SC) was recently developed for the control of inaccurately modeled dynamic systems under external disturbances. In the present paper the operation of this controller is studied in the case of an incompletely modeled dynamic system, that is when the system to be controlled contains internal degree of freedom not modeled by the controller. As starting point the method uses a simple, incomplete dynamic model to predict the propagation of the state of the modeled degrees of freedom also influenced by that of the unmodeled internal ones by nonlinear coupling. The controller is restricted to the observation of the behavior of the generalized coordinates the modesl of which are available for it. By the use of a priori known, uniform, lucid structure of reduced size, simple and short explicit algebraic procedures especially fit to real-time applications the controller is able to learn the behavoir of the observed system. Simulation examples are presented for the control of a wheel to one of the spokes of which a ballast is attached by two strong springs as unmodeled components. Dispalcement of the ballast influences the momentum of the wheel and the springs can obtain potential energy via te inertial and gravitational forces. It is found that the adaptive controller can successfully cope with the problem of imperfect modeling.

#### 1 Introduction

The basic components of Soft Computing were almost completely developed by the sixties. In our days it roughly is a kind of integration of neural networks, fuzzy systems enhanced with high parallelism of operation and supported by several deterministic, stochastic or combined parameter-tuning methods (learning). Its main advantage is evading the development of intricate analytical system models. Instead of that typical problem classes has been identified for the solution of

which typical uniform architectures has been elaborated (e.g. multilayer perceptron, Kohonen-network, Hopfield-network, Cellular Neural Networks, CNN Universal Machine, etc.). Fuzzy systems also use membership functions of typical (e.g. trapezoidal, triangular or step-like, etc.) shapes, and the fuzzy relations can also be utilized in a standardized way by using different, even parametric classes of fuzzy operators. The "first phase" of applying traditional SC that is the identification of the problem class and finding the appropriate structure for dealing with it, normally is easy. The next phase, i.e. determining the necessary size of the structure and fitting its parameters via machine learning is far less easy. For neural networks certain solutions starts from a quite big initial network and apply dynamic pruning for getting rid of the "dead" nodes [1]. An alternative method starts with small network, and the number of nodes is increased step by step (e.g. [2-3]). Due to the possible existence of "local optima", for a pure "backpropagation training" inadequacy of a given number of neurons cannot be concluded simply. To evade this difficulty "learning methods" also including stochastic elements were considerably improved in the last decade (e.g. [4-7]).

In spite of this development it can be stated that for strongly coupled non-linear MIMO systems traditional SC still has several drawbacks. The number of the necessary fuzzy rules strongly increases with the degree of freedom and the intricacy of the problem. To reduce modeling complexity fuzzy interpolation methods were developed and checked [8]. The conventional fuzzy modeling techniques also need further investigation and development [9]. Similar problems arise regarding the necessary number of neurons in a neural network approach. External dynamic interactions on which normally no satisfactory information is available influence the system's behavior in dynamic manner. Both the big size of the necessary structures, the huge number of parameters to be tuned, as well as the "goal" varying in time still are serious problems.

Realizing that "generality" and "uniformity" of the "traditional SC structures" excludes the application of plausible simplifications made the idea rise that by addressing narrower problem classes a novel branch of soft computing could be developed by the use of far simpler and far more lucid uniform structures and procedures than the classical ones.

The first steps in this direction were made in the field of Classical Mechanical Systems (CMSs) [11], based on the Hamiltonian formalism detailed e.g. in [11]. This approach used the internal symmetry of CMSs, the Symplectic Group (SG) of Symplectic Geometry in the tangent space of the physical states of the system. The "result" of the "situation-dependent system identification" was a symplectic matrix compensating the effects of the inaccuracy of the rough dynamic model initially used as well as the external dynamic interactions not modeled by the controller. By the use of perturbation calculus it was proved that under certain restrictions this new approach could be successful in the control of the whole class of classical mechanical systems [12]. It is interesting that the method of Taylor

series extension combined with the Hamiltonian formalism is widely used in our days for problem solution, e.g. [13, 14].

Later the problem was considered from a purely mathematical point of view. It became clear that all the essential steps used in the control could be realized by other mathematical means than the symplectic matrices related to some phenomenological interpretation. Other Lie groups defined in similar manner by some basic quadratic expression like in the case of the Generalized Lorentz Group [15], the Stretched and the Partially Stretched Orthogonal Matrices [16], or symplectic matrices of special structure[17]. In these approaches the Lie group used in the control does not describe any internal physical symmetry of the system to be controlled.

The next essential step was to turn from the inaccurate modeling and unmodeled external perturbations to the control of partially modeled physical systems containing internal degrees of freedom that are not modeled by the controller. The first results belong to a special case in which the unmodeled parts correspond to very stiff but flexible joints in a robot arm [18]. The great stiffness of the unmodeled joints makes this situation special in the sense that the motion belonging to these degrees of freedom is very much restricted. The paradigm investigated in this paper, that is a wheel containing a ballast balanced by two deformable springs that can be excited due the nonlinear coupling corresponds to a less restricted system. The spring+ballast subsystem's vibational eigenfrequency can be estimated very simply. The excitation of this degree of freedom is investigated by the means of the Fast Fourier Transformation.

In the sequel at first the paradigm is set mathematically, and following that the basic principles of the adaptive control is described. Following the presentation of the typical simulation results the conclusions are drawn.

# 2 The dynamic model of the wheel-ballast system

Let the wheel have the rotational generalized coordinate  $q_1$  [rad] and momentum of  $\Theta$  [ $kg \times m^2$ ], the ballast have the linear generalized coordinate along the spoke  $q_2$  [m] and mass of m [kg], and the net spring constant of the springs be equal to k [N/m]. The radius of the wheel is R [m]. The net spring force is zero when the radial location of the ballast is equal to  $q_2$ =R/2. The Let the ballast's motion limited within the interval  $q_2 \in [0, R]$ . To model the buffers at the hub and at the rim a potential energy term is introduced, which is very sharp at the edges of this interval while in the internal points it is very flat. It is described by two parameters, namely by the "strength" A [ $N \times m^2$ ], and a small parameter  $\varepsilon$  [m] determining the "nearness" of the singularity of this potential at the rim and at the hub. The Lagrangian of the system is:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \Theta \dot{q}_{1}^{2} + \frac{1}{2} m (q_{2} \dot{q}_{1})^{2} + \frac{1}{2} m \dot{q}_{2} - mgq_{2} \sin q_{1} - \frac{1}{2} k \left( q_{2} - \frac{R}{2} \right)^{2} - \frac{A}{R - q_{2} + \varepsilon} - \frac{A}{q_{2} + \varepsilon}$$

$$(1)$$

The Euler-Lagrange equations of motion of this system are given as follows:

$$(\Theta + mq_2^2)\ddot{q}_1 + 2mq_2\dot{q}_1\dot{q}_2 + mgq_2\cos q_1 = Q_1$$

$$m\ddot{q}_2 - mq_2\dot{q}_1^2 + mg\sin q_1 + k\left(q_2 - \frac{R}{2}\right) + \frac{A}{(R - q_2 + \varepsilon)^2} - \frac{A}{(q_2 + \varepsilon)^2} = Q_2$$
(2)

in which g denotes the gravitational acceleration  $[m/s^2]$ ,  $Q_1$   $[N \times m]$  denotes the driving torque rotating the whell, and  $Q_2$  [N] stands for the force moving the ballast along the spoke. While  $Q_1$  can be controlled via a computed torque control,  $Q_2 \equiv 0$ , that is the ballast corresponds to an uncontrolled degree of freedom. In the sequel the principles of the adaptive control are detailed.

# 3 Principles of the adaptive control

From purely mathematical point of view the can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated to obtain a desired system response  $\mathbf{i}^d$  as  $\mathbf{e} = \boldsymbol{\varphi}(\mathbf{i}^d)$ . The system has its inverse dynamics described by the unknown function  $\mathbf{i}^r = \boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d)) = f(\mathbf{i}^d)$  and resulting in a realized response  $\mathbf{i}^r$  instead of the desired one,  $\mathbf{i}^d$ . Normally one can obtain information via observation only on the function f() considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only  $\mathbf{i}^d$  as the input of f() can be "deformed" to  $\mathbf{i}^{d*}$  to achieve and maintain the  $\mathbf{i}^d = f(\mathbf{i}^{d*})$  state. [Only the *model function*  $\boldsymbol{\varphi}()$  can directly be manipulated.] On the basis of the modification of the method of renormalization widely applied in Physics the following "scaling iteration" was suggested for finding the proper deformation:

$$\mathbf{i}_{0}; \mathbf{S}_{1}\mathbf{f}(\mathbf{i}_{0}) = \mathbf{i}_{0}; \mathbf{i}_{1} = \mathbf{S}_{1}\mathbf{i}_{0}; \dots; \mathbf{S}_{n}\mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_{0};$$

$$\mathbf{i}_{n+1} = \mathbf{S}_{n+1}\mathbf{i}_{n}; \mathbf{S}_{n} \xrightarrow[n \to \infty]{} \mathbf{I}$$
(3)

in which the  $S_n$  matrices denote some linear transformations to be specified later. As it can be seen these matrices maps the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller "learns" the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (3) does not unambiguously determine the possible applicable quadratic

matrices, we have additional freedom in choosing appropriate ones. The most important points of view are fast and efficient computation, and the ability for remaining as close to the identity transformation as possible. For making the problem mathematically unambiguous (3) can be transformed into a matrix equation by putting the values of  $\mathbf{f}$  and  $\mathbf{i}$  into well-defined blocks of bigger matrices. Via computing the inverse of the matrix containing  $\mathbf{f}$  in (3) the problem can be made mathematically well-defined. Since the calculation of the inverse of one of the matrices is needed in each control cycle it is expedient to choose special matrices of fast and easy invertibility. Within the block matrices the response arrays may be extended by adding to them a "dummy", that is physically not interpreted dimension of constant value, in order to evade the occurrence of the mathematically dubious  $0\rightarrow0$ ,  $0\rightarrow$ finite, finite $\rightarrow0$  transformations. In the present paper the special symplectic matrices announced in [17] were applied for this purpose. In general, the Lie group of the Symplectic Matrices is defined by the equations

$$\mathbf{S}^{T} \mathfrak{I} \mathbf{S} = \mathfrak{I} \equiv \begin{bmatrix} \mathbf{0} & | \mathbf{I} \\ -\mathbf{I} & | \mathbf{0} \end{bmatrix}, \ \det \mathbf{S} = 1$$
 (4)

The inverse of such matrices can be calculated in a computationally very cost-efficient manner as  $\mathbf{S}^{-1} = \mathfrak{I}^T \mathbf{S}^T \mathfrak{I}$ . In our particular case the symplectic matrices are constructed from the desired and the observed joint coordinate accelerations corresponding to the response of the mechanical system to the excitation of torque and force by the use of the block of the matrix

$$\left[ \mathbf{m}^{(1)}, \mathbf{m}^{(2)}, \mathbf{e}^{(3)} \right] = \begin{bmatrix} \ddot{q}_1 & -\ddot{q}_1 & e_1^{(3)} \\ d & -d & e_2^{(3)} \\ D & \frac{\ddot{q}_1^2 + d^2}{D} & e_3^{(3)} \end{bmatrix}$$
 (5)

as

$$\mathbf{S} = \begin{bmatrix} \mathbf{0} & \frac{-1}{s} \mathbf{m}^{(1)} & \frac{-1}{s} \mathbf{m}^{(2)} & -\mathbf{e}^{(3)} \\ \mathbf{m}^{(1)} & \mathbf{m}^{(2)} & \mathbf{e}^{(3)} & \mathbf{0} \end{bmatrix}$$
(6)

in which  $e^{(3)}$  denotes a unit vector, which lies in the orthogonal sub-space of the first two columns of the block matrix, d is the "dummy" parameter used for avoiding singular transformations, and

$$D^2 \equiv \ddot{q}_1^2 + d^2, s = 2D^2 \tag{7}$$

The unit vectors can be created e.g. by using El Hini's algorithm [16], which, while rotates vector  $\mathbf{b}$  to into the direction of vector  $\mathbf{a}$ , leaves the orthogonal subspace of these vectors invariant. So if the operation starts with an orthonormal set

 $\{{\bf e}^{(1)},...,{\bf e}^{(3)}\}$  and at first it is rigidly rotated until  ${\bf e}^{(1)}$  becomes parallel with the  $1^{\rm st}$  column of  ${\bf M}$ , its  $2^{\rm nd}$  column will lie in the orthogonal sub-space of the  $1^{\rm st}$  one spanned by the transformed  $\{{\bf e}^{*(2)},{\bf e}^{*(3)}\}$  set. In the next step this whole set can rigidly be rotated until the new  ${\bf e}^{**(2)}$  becomes parallel with the  $2^{\rm nd}$  column of  ${\bf M}$ . (This operation leaves the previously set  ${\bf e}^{*(1)}$  unchanged because it is orthogonal to the two vectors determining this special rotation.) With the above completion the appropriate operation in (3) evidently equals to the identity operator if the desired response just is equal to the observed one, and remains in the close vicinity of the unit matrix if the non-zero desired and realized responses are very close to each other. Since amongst the conditions for which the convergence of the method was proved in [12] near-identity transformations were supposed in the perturbation theory, a parameter  $\xi$  measuring the "extent of the necessary transformation", a "shape factor"  $\sigma$ , and a "regulation factor"  $\lambda$  can be introduced in a linear interpolation with small positive  $\varepsilon_1$ ,  $\varepsilon_2$  values as

$$\xi := \frac{\left|\mathbf{f} - \mathbf{i}^{d}\right|}{\max(\left|\mathbf{f}\right|, \left|\mathbf{i}^{d}\right|)}, \quad \lambda = 1 + \varepsilon_{1} + \left(\varepsilon_{2} - 1 - \varepsilon_{1}\right) \frac{\sigma \xi}{1 + \sigma \xi}, \quad \hat{\mathbf{i}}^{d} = \mathbf{f} + \lambda \left(\mathbf{i}^{d} - \mathbf{f}\right)$$
(8)

This interpolation reduces the task of the adaptive control in the more critical session and helps to keep the necessary linear transformation in the vicinity of the identity operator. Other important fact concerning the details of the numerical calculations is the ratio of  $\|\ddot{\mathbf{q}}\|$  and d in (5). The controller has a priori information only on the nominal accelerations, but for the appropriate error-relaxation much higher desired accelerations may occur. For this purpose a slowly forgetting integrating filter was introduced to create a weighting factor for  $0 < \beta < 1$  as

$$w(t_i) := \frac{\sum_{j=0}^{\infty} \beta^j \left\| \ddot{\mathbf{q}}^{\text{Des}}(t_{i-j}) \right\|}{\sum_{j=0}^{\infty} \beta^j}$$
(9)

and in (5) instead of the actual values ( $\ddot{\mathbf{q}}$ ) the actual weighted ones  $\ddot{\mathbf{q}}/w$  were taken into account. The numerical realization of such a filter is very easy: the content of a buffer has to be multiplied by  $\beta$  in each control cycle, and the new  $\|\ddot{\mathbf{q}}^{Des}\|$  value has to be added to it. It also is easy to calculate the sum of the weights in the denominator of (9):  $\Sigma=1/(1-\beta)$ . In the forthcoming simulations the following numerical data were used: d=80,  $\beta=0.92$ ,  $\sigma=0.5$ ,  $\varepsilon_I=0.2$ ,  $\varepsilon_2=10^{-5}$  were chosen.

#### 4 Simulation results

In the simulations for the desired relaxation of the trajectory tracking error a simple PID-type rule was prescribed by the use of purely kinematic terms. This error relaxation colud be achieved exactly only in the possession of the exact dynamic model of the system to be controlled. Instead of the exact actual dynamic model of the constant momentum  $\Theta$ =50 [ $kg \times m^2$ ] a model value 2 [ $kg \times m^2$ ] was used. The wheel had the raduis of R=2 [m]. For the Coriolis and inertial forces the constant term 10 [Nm] was applied in the rough initial model. The ballast had the mass of m=10 [kg], the spring constant was  $k=10^4$  [N/m], and the buffer forces had the constant  $A=100 \ [N\times m^2]$  with  $\varepsilon=10^{-3} \ [m]$ . By neglecting the buffering forces the circular eigenfrequency of the vibration of the ballast was approximated as  $(k/m)^{1/2} \cong 31$  [1/s]. For evaluating the controller's operation Fast Fourier Transformation was used to trace the appearance of this coupled frequency peak in the motion of the controlled joint. The cycle-time of the controller was supposed to be 1 [ms], and this interval was divided into 50 sub-intervals of equal length for calculation (simulation) purposes. For the desired trajectory periodic motion for  $q_1$ was prescribed from the value zero to a finite amplitude 1 [rad].

Typical results are presented in Fig. 1 for a relatively fast nominal motion of circular frequency  $\Omega$ =6/s for the adaptive and the simple PID-type controller. It is evident that the using the adaptive law considerably improves the bahavior of the controller, and following a transient phase results in good trajectory tracking. This good tracking results in the excitation of the free degree of freedom which seems to vibrate at its own estimated circular eigenfrequency as it can be guessed from its phase trajectory. (The phase trajectory of the free joint in the non-adaptive case well exemplifies the buffering nature of the additional potential. Increasing excitation of this degree of freedom makes the simple PID controller diverge.)

The low frequency parts of the absolute values of the Fourier spectra of the controlled and the free degrees of freedom are zoomed out in Fig. 2. In the spectrum of the controlled joint the peak belonging to the nominal motion's circular frequency  $\Omega$ =6/s can well be identified.

The definite lack of any peak at  $\omega=31/s$  in the spectrum of the controlled joint reveals that adaptivity quite well supresses the coupling of the vibration of the free degree of freedom (the 31/s peak of which quite significantly can be recognized in its FFT spectrum).

The non-adaptive divergent motion results in more diffuse spectra than the adaptive one. The last row of Fig. 2 of the adaptive motion exemplifies that quite significant variation in the exerted torque was needed for compensating the effect of the free motion of the excited internal degree of freedom. Its effect can also be observed in the diagram desribing the generalized forces in the divergent non-adaptive case, too.

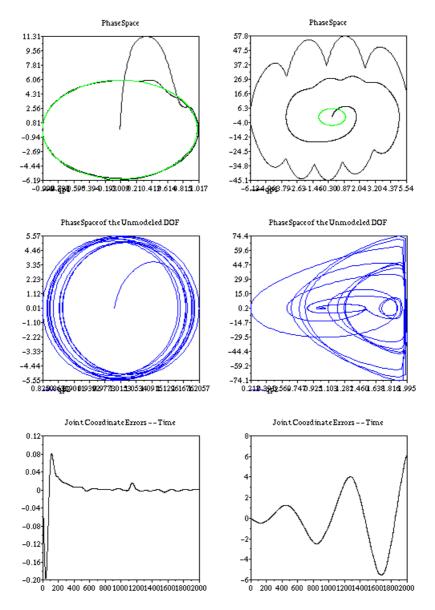


Figure 1. The operation of the adaptive (1<sup>st</sup> column) and the non-adaptive (2<sup>nd</sup> column) controllers: the phase space of the nominal and the simulated motion for the controlled joint  $q_1$  [rad/s vs. rad] (1<sup>st</sup> row), the phase space of the motion of the free joint  $q_2$  [m/s vs. m] (2<sup>nd</sup> row), and the trajectory tracking error for  $q_1$  [rad] vs. time [ms] (3<sup>rd</sup> row) for the nominal circular frequency  $\Omega$ =6/s.

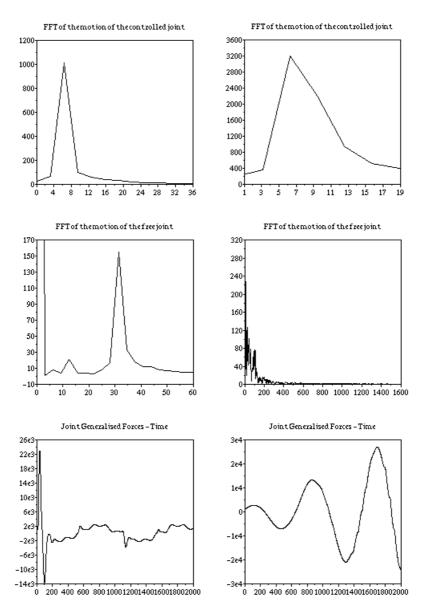


Figure 2. The operation of the adaptive (1<sup>st</sup> column) and the non-adaptive (2<sup>nd</sup> column) controllers: the FFT spectra of the simulated motion for the controlled joint  $q_1$  (1<sup>st</sup> row), and that of the free joint  $q_2$  (2<sup>nd</sup> row) (circular frequency in [Hz] on the horizontal axis), and the torque exerted by the drive  $Q_1$  [ $N \times m$ ] vs. time [ms] (3<sup>rd</sup> row) for the nominal circular frequency  $\Omega$ =6/s.

Fig. 3 reveals similar resulst for a very slow nominal motion of circular frequency circular frequency  $\Omega=1/s$ .

### 5 Conclusions

In this paper the behavior of the conventional PID-type and that of an adaptive controllers based on a novel branch of Computational Cybernetics were compared to each other in the case of controlling an approximately modeled non-linear system having unmodeled and uncontrolled internal degree of freedom. The simulation results made it clear that adaptivity considerably improved the quality of the trajectory reproduction and successfully rejected the frequency coupling

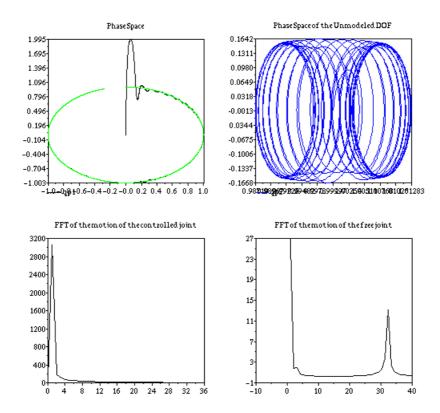


Figure 3. The operation of the adaptive controller for the nominal circular frequency  $\Omega=1/s$ : the phase spece of the controlled and the free joints  $q_1$  and  $q_2$  respectively (1<sup>st</sup> row), the FFT spectra of the simulated motion for the controlled joint  $q_1$  (1<sup>st</sup> row), and that of the free joint  $q_2$  (2<sup>nd</sup> row) (circular frequency in [Hz] on the horizontal axis).

between the controlled and the free, unactuated degrees of freedom. Thie results anicipate that the method can be a useful means for practical applications, e.g. active suspension systems, etc. in which the undesired vibration of a system containing unmodeled and unactuated degrees of freedom must be supressed.

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