A Contribution to the Hybrid Control

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Abstract: This paper presents two concepts of the hybrid control system. The first one deals with the events of random structural perturbations such as failures, component degradation and structural changes because today complex technological processes must maintain acceptable behaviour in the presence such events. The second approach is quite common in the control practice where are employd several different controllers with switching among them with some type of logical device. On the other hand it is generally difficult to get analytical solutions to mixed difference/differential equations. For some problems it is possible to do qualitative analysis for aggregated models. Because of the lack of good analysis methods, many investigations of hybrid systems have relied heavily on simulation. Unfortunately the general purpose simulation tools today are not so well suited for hybrid systems.

Keywords: hybrid control, jump changes, Markov process, reliability, optimal control, energy control, fuzzy control, PID controller

1 Introduction

The systems which have been traditionally studied by automatic control theory involve mainly familiar dynamical systems that are governed by ordinary differential or difference equations. On the other hand there are many situations in which such models are not appropriate. For example consider hybrid dynamic systems in which the state of systems includes both continuous and discrete variables (robotic systems, rule-based controllers, fault-tolerant systems, systems with many failure modes, etc.). The control of these systems involves discrete sets of alternative interpretations of observed data, models for each of these alternatives and optimal decision. This higher level discrete decision is then incorporated into a feedback control policy. Such a high-level controller should be able to deal with situations which may involve decision processes.

One definition of the hydrid control system is: the hybrid control system is a control system where the plant or the controller contains discrete modes that together with continuous equations govern the behavior of the system [5]. This general definition covers basically every existing control system.

Lately there has been a considerable effort in trying to unify some of the approaches and get a more general theory for hybrid systems. The fundamental problem with hybrid systems is their complex mixture of discrete and continuous variables. Such systems are very general and they have appeared in many different domains. They have, for example, attracted much interest in control as well as in computer science. In automatic control the focus has been on the continuous behavior, while computer science has emphasized the discrete aspects. It is generally difficult to get analytical solutions to mixed difference/differential equations. For some problems it is possible to do qualitative analysis for aggregated models. Because of the lack of good analysis methods, many investigations of hybrid systems have relied heavily on simulation. Unfortunately the general purpose simulation tools today are not so well suited for hybrid systems.

Two approaches to the hybrid control are presented in the contribution. In the first one we introduce the decentralized control method for linear systems with a quadratic performance index in the presence of sensor or actuator failure states or in the presence of structural perturbations in the form of changes in interactions among the subsystems. The resulting control algorithm depends on the system structure, the structural dynamics and the system dynamics.

The second approach represents the application of two different types of controller. Within this approach will be presented two hybrid control systems. The first one will be the hybrid system for single inverted pendulum which consists of a system for energy control and PID controller. The second one will be the hybrid system for inverted pendulum employing a fuzzy controller and PID controller.

2 Decentralized Control with Stochastic Jump Parameter Changes

Our aim is the modelling of structural and system dynamics. Component failures and reconfigurations are modeled by the Markov process. A system structure is a possible mode of operation for a given system, represented by the components, their interconnections and the information flows in the system at a given time. The system configuration is the original design of the system, accounting for all modeled modes of operation and the Markov process governing the configuration or structural dynamics (transitions among the various structures).

The method illustrates the unification of the concepts of reliability and decentralized optimal control. Reliability is defined as the probability that a system will perform within specified constraints for a given period of time. The optimal control is defined as the existence of a state feedback control law for which the closed-loop system is stable and optimal [7], [8].

It will be assumed that the large-scale system to be controlled is described by the linear vector differential equations:

$$\dot{\overline{x}} = \overline{A}_i[r(t)]\,\overline{x}_i + \overline{B}_i[r(t)]\,\overline{u}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \overline{A}_{ij}[r(t)]\,\overline{x}_j \tag{1}$$

where $\overline{x}(t) \in R^n$ is the system state, $\overline{u}(t) \in R^m$ is the control, and $\overline{A}_i, \overline{B}_i, \overline{A}_{ij}$ are matrices of dynamics, control and interactions with dimensions $n\times n$, $n\times m$, $n \times n$, respectively. The stochastic behaviour comes from the dependence of $\overline{A}_i, \overline{B}_i, \overline{A}_{ij}$ on $r(t)$. Denote the event $[\overline{A}_i(t), \overline{B}_i(t), \overline{A}_{ij}(t)] = [\overline{A}_{i,k}, \overline{B}_{i,k}, \overline{A}_{ij,k}]$ when $r(t) = k$ and denote the set $S = \{1, 2, \ldots, s\}$. The stochastic variation of the proces parameter will be described by the Markov process $r(t)$:

$$\Pr\{r(t+\Delta) = h \,|\, r(t) = k\} = \Pr\left\{\overline{A}_i(t+\Delta), \overline{B}_i(t+\Delta), \overline{A}_{ij}(t+\Delta) = \right.$$

$$\left. = \overline{A}_{i,h}, \overline{B}_{i,h}, \overline{A}_{ij,h} \,|\, \overline{A}_i(t), \overline{B}_i(t), \overline{A}_{ij}(t) = \overline{A}_{i,k}, \overline{B}_{i,k}, \overline{A}_{ij,k}\right\} =$$

$$= \begin{cases} p_{kh}\,\Delta + o(\Delta), & k \neq h \\ 1 + p_{kk}\,\Delta + o(\Delta), & k = h \end{cases} \tag{2}$$

where $k, h \in S = \{1, 2, \ldots, s\}$. Here $\overline{P} = [p_{kh}]$ is matrix of dimension $(s \times s)$ with $p_{kh} \geq 0, k \neq h$ and

$$p_{kk} = -\sum_{\substack{h=1\\ l\neq h}}^{s} p_{kh}$$

Denoting the state probability vector as

$$\overline{p}(t) = [p_1(t), \ldots, p_s(t)]^T$$

and the transition matrix as $\overline{P}$ we can consider modes of the system as states of the Markov process with a finite number of states which can be described by Kolmogorov equation:

$$\dot{\overline{p}}(t) = \overline{P}\,\overline{p}(t); \qquad \overline{p}(t_0) = \overline{p}_0 \tag{3}$$

It is assumed that the following sequence of events occurs at time "t":

1. $\overline{x}_i$ and $\overline{x}_j$ are observed exactly,

2. then $\overline{A}_{i,k}$, $\overline{B}_{i,k}$, $\overline{A}_{ij,k}$ switches to $\overline{A}_{i,h}$, $\overline{B}_{i,h}$, $\overline{A}_{ij,h}$,

3. then $\overline{u}_i$ is applied in the sense of some averaged quadratic performance index

$$J = \sum_{i=1}^{N} J_i = \sum_{i=1}^{N} E \left\{ \frac{1}{2} \int_0^T \left(\overline{x}_i^T \, \overline{Q}_i \, \overline{x}_i + \overline{u}_i^T \, \overline{R}_i \, \overline{u}_i \right) dt \right\} \tag{4}$$

The optimal control in the sense of (4) can be expressed as [6]:

$$\overline{u}_i = -\overline{R}_i^{-1} \, \overline{B}_{i,k}^T \, \overline{K}_{i,k} \, \overline{x}_i + \overline{R}_i^{-1} \, \overline{B}_{i,k}^T \, \overline{h}_{i,k} \tag{5}$$

where $i = 1, 2, \ldots, N \, ; \; k = 1, 2, \ldots, s$

The matrices $\overline{K}_{i,k}$ and vectors $\overline{h}_{i,k}$ can be computed from a set of coupled stochastic differential equations [6]:

$$\dot{\overline{K}}_{i,k} = -\overline{A}_{i,k}^T \, \overline{K}_{i,k} - \overline{K}_{i,k} \, \overline{A}_{i,k} + \overline{K}_{i,k} \, \overline{B}_{i,k} \, \overline{R}_i^{-1} \, \overline{B}_{i,k}^T \, \overline{K}_{i,k} - \overline{Q}_i - \sum_{h=1}^{s} p_{kh} \, \overline{K}_{i,h} \tag{6}$$

with boundary condition:

$$\overline{K}_{i,k}(T) = \overline{0}$$

$$\begin{aligned} \dot{\overline{h}}_{i,k} \quad = \quad & - \left[\overline{A}_{i,k} - \overline{B}_{i,k} \, \overline{R}_i^{-1} \, \overline{B}_{i,k}^T \, \overline{K}_{i,k} \right]^T \overline{h}_{i,k} + \overline{K}_{i,k} \sum_{\substack{j=1 \\ j \neq i}}^{N} \overline{A}_{ij,k} \, \overline{x}_j \, + \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{N} \overline{A}_{ji,k}^T \left(\overline{K}_{j,k} \, \overline{x}_j - \overline{h}_{j,k} \right) - \sum_{h=1}^{s} p_{kh} \, \overline{h}_{i,h} \end{aligned} \tag{7}$$

with boundary condition:

$$\overline{h}_{i,k}(T) = \overline{0}; \qquad i, j = 1, 2, \ldots, N; \qquad k, l = 1, 2, \ldots, s$$

Example

Consider the system with two subsystems, each with two parameters b_i and c_i which can change suddenly. So we have 4 possible states of the system:

State	a_1	a_2
1	1	1
2	1	0
3	0	1
4	0	0

where $a_i = 1$ means that $b_i = 1$ and $c_i = 0,325$, what represents normal function and $a_i = 0$ means that $b_i = 0$ and $c_i = 0$, $i = 1, 2$, what represents perturbation. The matrix $\overline{P}$ is in the form:

$$\overline{P} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

The results of simulation (in MATLAB) are shown in Fig. 1.

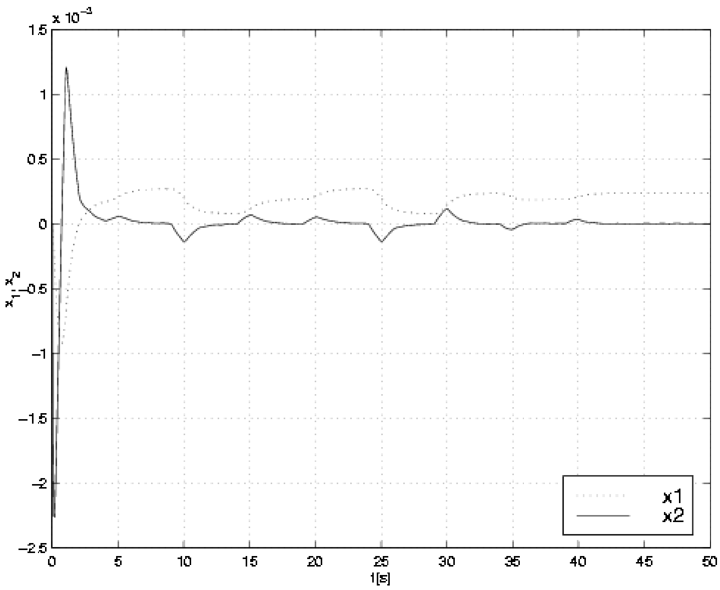

Figure 1: The state variables of the subsystem 1

3 Hybrid Control for Inverted Pendulum

In control practice it is quite common to use several different controllers and to switch between them with some type of logical device. Some examples are systems with selectors, which have been used for constraint control for a long time, systems with gain scheduling, etc. These systems are commonly used for control of chemical processes, power stations, and in flight control. Other examples of systems with mode switching are found in robotics. The controllers based on expert systems represent another type of system with an hierarchical structure where a collection of controllers are juggled by an expert system. All these systems are hybrid control systems. Such a type of hybrid control we have designed for inverted pendulum.

3.1 Energy Control and PID Controller

The first hybrid control system has been designed for the single inverted pendulum. Mass of the pendulum is m, moment of inertia with respect to the pivot point is J_p and l is the distance from the pivot to the center of mass. The angle between the vertical and the pendulum is θ, where θ is positive in the clock-wise direction. The acceleration of gravity is g and the acceleration of the pivot is u. The equation of motion of the pendulum is [1]:

$$J_P\,\ddot{\theta} - m\,g\,l\sin\theta + m\,u\,l\cos\theta = 0 \tag{8}$$

The system has two state variables, the angle θ and the rate of change of the angle - derivative of θ.

The model given by equation (8) has five parameters. It is useful to normalize variables and to introduce the frequency of small oscillations around the downward position in the form [1]:

$$\omega_0 = \sqrt{\frac{m\,g\,l}{J_P}} \tag{9}$$

The equations of motion are then characterized by two parameters only.

Our requirements to the hybrid control system have been determined as follows: minimum energy for swinging up the pendulum, zero deviation from vertical and simplified design of control system. Therefore the hybrid control system has been used which consists of two different controllers. The first controller is an energy controller that will swing-up the pendulum. The second controller is a PID feedback controller that will hold it in the upright position. The energy controller is stabilizing for all initial conditions, whereas the feedback controller is stabilizing only within a certain region which are defined in [5]. The performance of the PID controller is better than the energy controller in the upright position. The switching between controllers occurs when the pendulum achieves the vertical first time.

It is natural to choose the energy so that it is zero in the upright position and normalize it by mgl, which is the energy required to raise the pendulum from the downward to the horizontal position. The normalized energy can then be written as:

$$E = m\,g\,l\left[\frac{1}{2}\left(\frac{\dot{\theta}}{\omega}\right)^2 + \cos\theta - 1\right] \tag{10}$$

The energy E of the uncontrolled pendulum is given by equation (10). To perform energy control it is necessary to understand how the energy is influenced by the acceleration of the pivot. Computing the derivative of E with respect to time we find:

$$\frac{d\,E}{d\,t} = J_P\,\ddot{\theta}\,\dot{\theta} - m\,g\,l\,\dot{\theta}\,\sin\theta = -m\,u\,l\,\dot{\theta}\,\cos\theta \tag{11}$$

It follows from this equation that it is easy to control the energy.

Let the desired energy be E_0. The following control law is a simple strategy for achieving the desired energy

$$u = sat_{ng}\,[k(E - E_0)]\ sign\left(\dot{\theta}\cos\theta\right) \tag{12}$$

where k is a design parameter. The function sat_{ng} denotes a function which saturates at ng. That means the maximum acceleration of the pivot where parameter n is dimension free.

The second controller is standard PID controller with following parameters: $K = 50$; $T_i = 0,01$ and $T_d = 25$ [2].

The hybrid control of inverted pendulum was simulated in MATLAB/Simulink. Fig. 2 shows the trajectory of the angle θ and the trajectory of the energy. The switching between the controllers occurs at time 6,48 seconds.

3.2 Fuzzy Controller and PID Controller

The second hybrid control system has been designed for another model of inverted pendulum. The inverted pendulum is described by equations:

$$\ddot{\theta} = \frac{g\,\sin\theta + \cos\theta\left(\dfrac{-u - m\,l\,\dot{\theta}^2\,\cos\theta}{m_c + m}\right)}{l\left(\dfrac{4}{3} - \dfrac{m\,\cos^2\theta}{m_c + m}\right)} \tag{13}$$

$$\ddot{x} = \frac{u + m\,l\left(\dot{\theta}^2\,\sin\theta - \ddot{\theta}\,\cos\theta\right)}{m_c + m} \tag{14}$$

where l is length to pendulum center of mass, m is mass of the pendulum, m_c is mass of the cart and g is acceleration of gravity. The system has four state variables: the angle between the vertical and the pendulum θ, the rate of change of the angle - derivative of θ, cart position x, and the rate of change of the cart position - derivative of x.

In this case our aims have been to control the angle of the pendulum and also the position of the pendulum and zero deviation from vertical. Therefore we

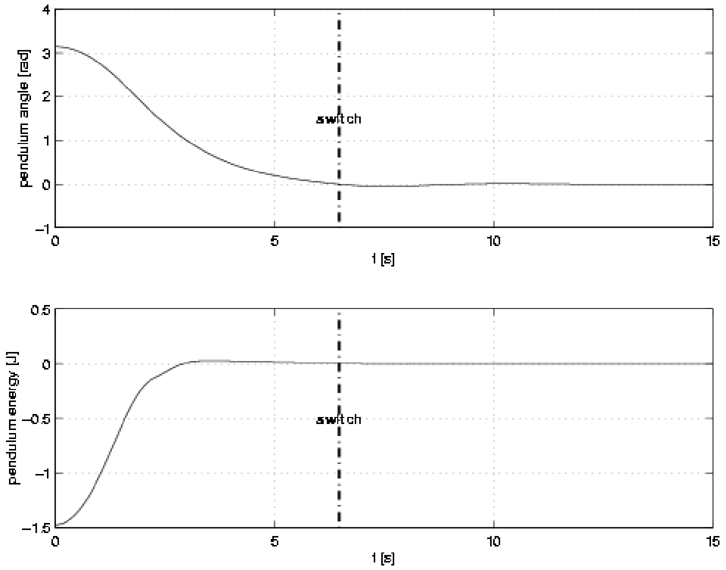

Figure 2: The trajectories of the angle and the energy

have decided to use a hybrid control system consisting of fuzzy controller and PID controller. In this case the control system is designed for perturbation affecting the system in the form of force deviating the pendulum from the vertical. Fuzzy controller controls the angle θ and the position of the cart x. PID feedback controller will control the pendulum near the upright position.

The Sugeno fuzzy controller with four inputs and one output has been used for control of the system. The inputs are all state variables and the output is the control u. The controller has been designed in MATLAB in the Fuzzy Inference System (FIS). This fuzzy controller employs two fuzzy rules only which are in the form [4]:

IF (in1 is in1mf1) AND (in2 is in2mf1) AND (in3 is in3mf1) AND (in4 is in4mf1)
THEN (out is outmf1)

IF (in1 is in1mf2) AND (in2 is in2mf2) AND (in3 is in3mf2) AND (in4 is in4mf2)
THEN (out is outmf1)

Where in1, ..., in4 are the inputs, mf1 and mf2 are the membership functions, out is the output and outmf1 is the membership function of the output.

PID controller is standard PID controller with following parameters: $K = 50$; $T_i = 1$ and $T_d = 25$ [2]. Switch between controllers takes place whenever deviation of the pendulum from vertical is less than one degree.

The hybrid control system was simulated in MATLAB/Simulink. Fig. 3 shows the trajectory of the angle θ and the trajectory of cart position x. The switching between the controllers occurs at time 1,46 seconds.

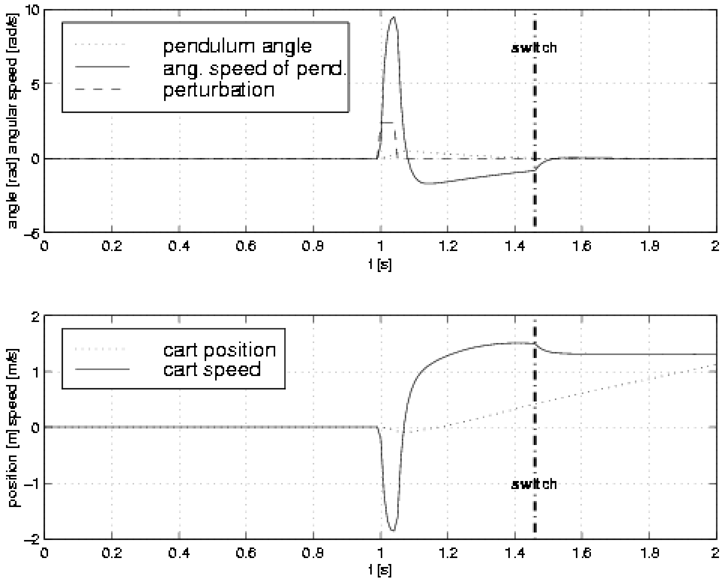

Figure 3: The trajectories of the angle and the cart position

Conclusions

The feedback decentralized hierarchical control of the class of the linear systems with stochastic jump parameters (discrete events) has been studied. The presented approach allows us to increase the system reliability. The resulting optimal decentralized control law depends on the system dynamics, system structure and some parameters (for example transition intensities). The results of simulation of simple examples are presented, which corroborates the validity of the introduced method.

In the second case two hybrid control systems have been presented in the contribution. Both hybrid systems consist of energy controller or fuzzy controller respectively and standard PID controller. The energy controller swings up the pendulum and standard PID controller is used for holding the pendulum in the upright position. In the second case the fuzzy controller controls the pendulum in event of disturbance at the upright position and PID is switched when deviation from the vertical is less than one degree. The behaviour obtained by using hybrid control is better than by using one type of control only. Another benefit is that the design of the single controllers of hybrid system may be simplified opposite to case when one controller of them is designed only.

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