Sensitivity analysis in some possibilistic linear problems

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Abstract: The goal of this paper is to give a short survey of the results in sensitivity analysis of the possibility distribution of the solution to possibilistic linear systems and possibilistic linear programming problems

1 Introduction

Sensitivity analysis in fuzzy linear programming was first considered by Hamacher et al [18], where a functional relationship between changes of parameters of the right-hand side and those of the optimal value of the primal objective function was derived for almost all conceivable cases. Fullér [8] showed that the solution to fuzzy linear programs with symmetrical triangular fuzzy numbers is stable with respect to small changes of centres of fuzzy numbers. Fedrizzi and Fullér [7] proved that the possibility distribution of the objective function of a possibilistic linear program with continuous fuzzy number parameters is stable under small perturbations of the parameters (in contrast to classical linear programming, where a small error of measurement may produce a large variation in the optimal value of the objective function).

Possibilisitic linear equality systems (PLES) are linear equality systems with fuzzy coefficients, defined by the Zadeh's extension principle. Kovács [13] showed that the fuzzy solution to PLES with symmetric triangular fuzzy numbers is stable with respect to small changes of centres of fuzzy parameters. The goal of this paper is to give a short survey of the results in sensitivity analysis of the possibility distribution of the solution of possibilistic linear systems and possibilistic linear programming problems.

First, we consider PLES with (Lipschitzian) fuzzy numbers and flexible linear programs, and illustrate the sensitivity of the fuzzy solution by several two-dimensional PLES. Then we consider linear possibilistic programs and show that the possibility distribution of their objective function remains stable under small changes in the membership functions of the fuzzy number coefficients.

A fuzzy number $\tilde{a}$ is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by $\mathcal{F}$. To distinguish a fuzzy number from a crisp (non-fuzzy) one, the former will sometimes be denoted with a tilde $\tilde{}$. If $\tilde{a}$, $\tilde{b} \in \mathcal{F}$ and $\lambda \in \mathbb{R}$ then $\tilde{a}+\tilde{b}$, $\tilde{a}-\tilde{b}$, $\lambda\tilde{a}$ are defined by the Zadeh's extension principle in the usual way. An α-level set of a fuzzy number $\tilde{a}$ is a non-fuzzy set denoted by $[\tilde{a}]^\alpha$. The truth value of the assertion "$\tilde{a}$ is equal to $\tilde{b}$", which we write $\tilde{a} = \tilde{b}$ is Poss$(\tilde{a} = \tilde{b})$ defined as

$$\text{Poss}(\tilde{a} = \tilde{b}) = \sup\{\tilde{a}(t) \wedge \tilde{b}(t) \mid t \in \mathbb{R}\}.$$

If $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers with $[a]^\alpha = [a_1(\alpha), a_2(\alpha)]$ and $[b]^\alpha = [b_1(\alpha), b_2(\alpha)]$ then their Hausdorff distance is defined as [12]

$$D(\tilde{a}, \tilde{b}) = \sup_{\alpha \in [0,1]} \max\{|a_1(\alpha) - b_1(\alpha)|, |a_2(\alpha) - b_2(\alpha)|\}.$$

i.e. $D(\tilde{a}, \tilde{b})$ is the maximal distance between the α-level sets of $\tilde{a}$ and $\tilde{b}$.

A fuzzy set of the real line given by the membership function

$$\tilde{a}(t) = \begin{cases} 1 - \dfrac{|a-t|}{\alpha} & \text{if } |a-t| \le \alpha, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

Where $\alpha > 0$ will be called a symmetrical triangular fuzzy number with center $a \in R$ and width 2α and we shall refer to it by the pair (a, α).

2 Possibilistic linear equality systems

Linear equality systems with fuzzy parameters and crisp variables defined by the extension principle are called possibilistic linear equality systems. This section focuses on the problem of stability (with respect to perturbations of fuzzy parameters) of the solution in these systems. A crisp (non-fuzzy) linear equality system can be written as

$$a_{i1}x_1 + \cdots + a_{in}x_n = b_i, \ i = 1, \ldots, m, \tag{2}$$

or shortly, $Ax = b$ where a_{ij}, b_i and x_j are real numbers. It is known that system (2) generally belongs to the class of ill-posed problems, so a small perturbation of the parameters a_{ij} and b_i may cause a large deviation in the solution.

A possibilistic linear equality system is

$$\tilde{a}_{i1}x_1 + \cdots + \tilde{a}_{in}x_n \ = \ \tilde{b}_i, \ i = 1, \ldots, m, \tag{3}$$

or shortly, $\tilde{A}x = \tilde{b}$, where $\tilde{a}_{ij}$, $\tilde{b}_i \in \mathcal{F}(\mathbb{R})$ are fuzzy quantities, $x \in \mathbb{R}^n$, the operations addition and multiplication by a real number of fuzzy quantities are defined by Zadeh's extension principle and the equation is understood in possibilistic sense.

We denote by $\mu_i(x)$ the degree of satisfaction of the i-th equation in (3) at the point $x \in \mathbb{R}^n$, i.e.

$$\mu_i(x) = \mathrm{Pos}(\tilde{a}_{i1}x_1 + \cdots + \tilde{a}_{in}x_n = \tilde{b}_i).$$

Following Bellman and Zadeh [1] the fuzzy solution (or the fuzzy set of feasible solutions) of system (3) can be viewed as the intersection of the μ_i's such that

$$\mu(x) = \min\{\mu_1(x), \dots, \mu_m(x)\}. \tag{4}$$

A measure of consistency for the possibilistic equality system (3) is defined as

$$\mu^* = \sup\{\mu(x) \mid x \in \mathbb{R}^n\}. \tag{5}$$

Let X^* be the set of points $x \in \mathbb{R}^n$ for which $\mu(x)$ attains its maximum, if it exists. That is

$$X^* = \{x^* \in \mathbb{R}^n \mid \mu(x^*) = \mu^*\}$$

If $X^* \neq \emptyset$ and $x^* \in X^*$, then x^* is called a maximizing (or best) solution of (3).

Let $L > 0$ be a real number. By $\mathcal{F}(L)$ we denote the set of all fuzzy numbers $\tilde{a} \in \mathcal{F}$ with membership function satisfying the Lipschitz condition with constant L , i.e.

$$|\tilde{a}(t) - \tilde{a}(t')| \leq L|t - t'|, \ \forall t, t' \in \mathbb{R}.$$

In many important cases the fuzzy parameters $\tilde{a}_{ij}$, $\tilde{b}_i$ of the system (3) are not known exactly and we have to work with their approximations $\tilde{a}^\delta_{ij}$, $\tilde{b}^\delta_i$ such that

$$\max_{i,j} D(\tilde{a}_{ij}, \tilde{a}^\delta_{ij}) \leq \delta, \quad \max_i D(\tilde{b}_i, \tilde{b}^\delta_i) \leq \delta, \tag{6}$$

where $\delta \geq 0$ is a real number. Then we get the following system with perturbed fuzzy parameters

$$\tilde{a}^\delta_{i1}x_1 + \cdots + \tilde{a}^\delta_{in}x_n = \tilde{b}^\delta_i, \ i = 1, \dots, m \tag{7}$$

or shortly, $\tilde{A}^\delta x = \tilde{b}^\delta$. In a similar manner we define the solution

$$\mu^\delta(x) = \min\{\mu^\delta_1(x), \dots \mu^\delta_m(x)\},$$

and the measure of consistency, $\mu^*(\delta) = \sup\{\mu^\delta(x) \mid x \in \mathbb{R}^n\}$, of perturbed system (7), where

$$\mu^\delta_i(x) = \mathrm{Pos}(\tilde{a}^\delta_{i1}x_1 + \cdots + \tilde{a}^\delta_{in}x_n = \tilde{b}^\delta_i)$$

denotes the degree of satisfaction of the i-th equation at $x \in \mathbb{R}^n$. Let $X^*(\delta)$ denote the set of maximizing solutions of the perturbed system (7).

Kovács [13] showed that the fuzzy solution to system (3) with symmetric triangular fuzzy numbers is a stable with respect to small changes of centres of fuzzy parameters. Following Fullér [9] in the next theorem we establish a stability property (with respect to perturbations (6)) of the solution of system (3).

Theorem 2.1. [9] *Let* $L > 0$ *and* $\tilde{a}_{ij}$, $\tilde{a}^{\delta}_{ij}$, $\tilde{b}_i$, $\tilde{b}^{\delta}_i \in \mathcal{F}(L)$. *If (6) holds, then*

$$||\mu - \mu^{\delta}||_{\infty} = \sup\{|\mu(x) - \mu^{\delta}(x)| \leq L\delta \mid x \in \mathbb{R}^n\}, \tag{8}$$

where $\mu(x)$ *and* $\mu^{\delta}(x)$ *are the (fuzzy) solutions to systems (3) and (7), respectively.*

From (8) it follows that $|\mu^* - \mu^*(\delta)| \leq L\delta$, where μ^*, $\mu^*(\delta)$ are the measures of consistency for the systems (3) and (7), respectively.

Consider now the possiblistic equality system (3) with fuzzy numbers of symmetric triangular form

$$(a_{i1}, \alpha)x_1 + \cdots + (a_{in}, \alpha)x_n \;=\; (b_i, \alpha),\; i = 1, \ldots, m,$$

or shortly,

$$(A, \alpha)x = (b, \alpha) \tag{9}$$

Then following Kovács and Fullér [15] the fuzzy solution of (9) can be written in a compact form

$$\mu(x) = \begin{cases} 1 & \text{if } Ax = b \\ 1 - \dfrac{||Ax - b||_{\infty}}{\alpha(|x|_1 + 1)} & \text{if } 0 < ||Ax - b||_{\infty} \leq \alpha(|x|_1 + 1) \\ 0 & \text{if } ||Ax - b||_{\infty} > \alpha(|x|_1 + 1) \end{cases}$$

where

$$||Ax - b||_{\infty} = \max\{|\langle a_1, x\rangle - b_1|, \ldots, |\langle a_m, x\rangle - b_m|\}.$$

If

$$[\mu]^1 = \{x \in \mathbb{R}^n \mid \mu(x) = 1\} \neq \emptyset$$

then the set of maximizing solutions, $X^* = [\mu]^1$, of (9) coincides with the solution set, denoted by X^{**}, of the crisp system $Ax = b$. The stability theorem for system (9) reads

Theorem 2.2. [13] *If the relationships,* $D(\tilde{A}, \tilde{A}^{\delta}) = \max_{i,j} |a_{ij} - a^{\delta}_{ij}| \leq \delta$ *and* $D(\tilde{b}, \tilde{b}^{\delta}) = \max_i |b_i - b^{\delta}_i| \leq \delta$ *hold, then*

$$||\mu - \mu^{\delta}||_{\infty} = \sup |\mu(x) - \mu^{\delta}(x)| \leq \delta/\alpha,$$

where $\mu(x)$ *and* $\mu^{\delta}(x)$ *are the fuzzy solutions to possibilistic equality systems* $(A, \alpha)x = (b, \alpha)$, *and* $(A^{\delta}, \alpha)x = (b^{\delta}, \alpha)$, *respectively.*

Theorem 2.1 can be extended to possibilistic linear equality systems with (continuous) fuzzy numbers.

Theorem 2.3. [10] *Let* $\tilde{a}_{ij}$, $\tilde{a}^{\delta}_{ij}$, $\tilde{b}_i$, $\tilde{b}^{\delta}_i \in \mathcal{F}$ *be fuzzy numbers. If (6) holds, then*

$$||\mu - \mu^{\delta}||_{\infty} \leq \omega(\delta),$$

where $\omega(\delta)$ *denotes the maximum of modulus of continuity of all fuzzy coefficients at* δ *in (3) and (7).*

In 1992 Kovács [16] showed a wide class of fuzzified systems that are well-posed extensions of ill-posed linear equality and inequality systems.

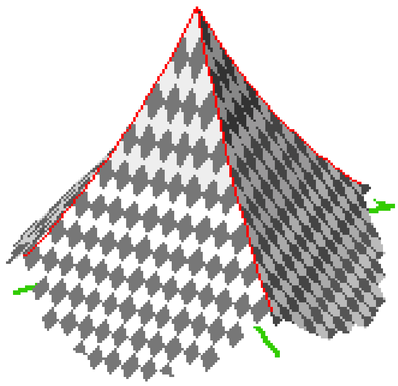

Figure 1: The graph of fuzzy solution of system (10) with $\alpha = 0.4$.

Consider the following two-dimensional possibilistic equality system

$$\begin{aligned}(1,\alpha)x_1 + (1,\alpha)x_2 &= (0,\alpha)\\ (1,\alpha)x_1 - (1,\alpha)x_2 &= (0,\alpha)\end{aligned} \tag{10}$$

Then its fuzzy solution is

$$\mu(x) = \begin{cases} 1 & \text{if } x = 0 \\ \tau_2(x) & \text{if } 0 < \max\{|x_1 - x_2|, |x_1 + x_2|\} \leq \alpha(|x_1| + |x_2| + 1) \\ 0 & \text{if } \max\{|x_1 - x_2|, |x_1 + x_2|\} > \alpha(|x_1| + |x_2| + 1) \end{cases}$$

where

$$\tau_2(x) = 1 - \frac{\max\{|x_1 - x_2|, |x_1 + x_2|\}}{\alpha(|x_1| + |x_2| + 1)},$$

and the only maximizing solution of system (10) is $x^* = (0,0)$. There is no problem with stability of the solution even for the crisp system

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

because $\det(A) \neq 0$.

The fuzzy solution of possibilistic equality system

$$\begin{aligned}(1,\alpha)x_1 + (1,\alpha)x_2 &= (0,\alpha)\\ (1,\alpha)x_1 + (1,\alpha)x_2 &= (0,\alpha)\end{aligned} \tag{11}$$

is

$$\mu(x) = \begin{cases} 1 & \text{if } |x_1 + x_2| = 0 \\ 1 - \dfrac{|x_1 + x_2|}{\alpha(|x_1| + |x_2| + 1)} & \text{if } 0 < |x_1 + x_2| \leq \alpha(|x_1| + |x_2| + 1) \\ 0 & \text{if } |x_1 + x_2| > \alpha(|x_1| + |x_2| + 1) \end{cases}$$

and the set of its maximizing solutions is $X^* = \{x \in \mathbb{R}^2 \mid x_1 + x_2 = 0\}$. In this case we have $X^* = X^{**} = \{x \in \mathbb{R}^2 \mid Ax = b\}$.

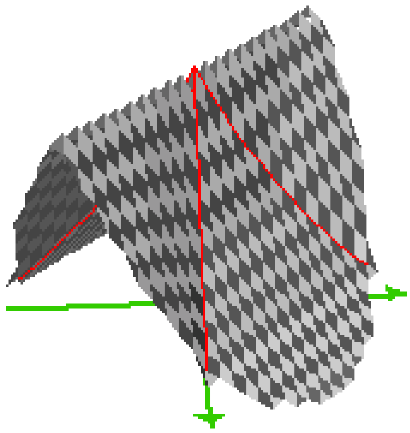

Figure 2: The graph of fuzzy solution of system (11) with $\alpha = 0.4$.

We might experience problems with the stability of the solution of the crisp system

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

because $\det(A) = 0$. Really, the fuzzy solution of possibilistic equality system

$$\begin{aligned} (1, \alpha)x_1 + (1, \alpha)x_2 &= (\delta_1, \alpha) \\ (1, \alpha)x_1 + (1, \alpha)x_2 &= (\delta_2, \alpha) \end{aligned} \qquad (12)$$

where $\delta_1 = 0.3$ and $\delta_2 = -0.3$, is

$$\mu(x) =$$

$$\begin{cases} \tau_1(x) & \text{if } 0 < \max\{|x_1 + x_2 - 0.3|, |x_1 + x_2 + 0.3|\} \leq \alpha(|x_1| + |x_2| + 1) \\ 0 & \text{if } \max\{|x_1 + x_2 - 0.3|, |x_1 + x_2 + 0.3|\} > \alpha(|x_1| + |x_2| + 1) \end{cases}$$

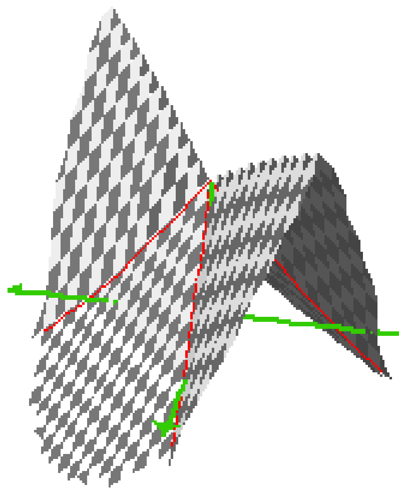

Figure 3: The graph of fuzzy solution of system (12) with $\alpha = 0.4$.

where

$$\tau_1(x) = 1 - \frac{\max\{|x_1 + x_2 - 0.3|, |x_1 + x_2 + 0.3|\}}{\alpha(|x_1| + |x_2| + 1)}$$

and the set of the maximizing solutions of (12) is empty, and X^{**} is also empty. Even though the set of maximizing solution of systems (11) and (12) varies a lot under small changes of the centers of fuzzy numbers of the right-hand side, δ_1 and δ_2, their fuzzy solutions can be made arbitrary close to each other by letting

$$\frac{\max\{\delta_1, \delta_2\}}{\alpha}$$

to tend to zero.

3 Possibilistic linear programming

In this section we will consider certain possibilistic linear programming problems, which have been introduced by Buckley [2] in 1988. In contrast to classical linear programming (where a small error of measurement may produce a large variation in the objective function), Fedrizzi and Fullér [7] showed that the possibility distribution of the objective function of a possibilistic linear program with continuous fuzzy number parameters is stable under small perturbations of the parameters. A possibilistic linear program is

$$\begin{aligned} \text{max/min} \quad & Z = x_1\tilde{c}_1 + \cdots + x_n\tilde{c}_n, \\ \text{subject to } & x_1\tilde{a}_{i1} + \cdots + x_n\tilde{a}_{in} * \tilde{b}_i, \\ & 1 \le i \le m,\ x \ge 0, \end{aligned} \tag{13}$$

where $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{c}_j$ are fuzzy numbers, $x = (x_1, \dots, x_n)$ is a vector of (nonfuzzy) decision variables, and $*$ denotes $<$, $\leq$, $=$, $\geq$ or $>$ for each i.

We will assume that all fuzzy numbers $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ are non-interactive. Non-interactivity means that we can find the joint possibility distribution of all the fuzzy variables by calculating the $\min$-intersection of their possibility distributions. Following Buckley [2], we define $\text{Pos}[Z = z]$, the possibility distribution of the objective function Z. We first specify the possibility that x satisfies the i-th constraints. Let

$$\Pi(a_i, b_i) = \min\{\tilde{a}_{i1}(a_{i1}), \dots, \tilde{a}_{in}(a_{in}), \tilde{b}_i(b_i\},$$

where $a_i = (a_{i1}, \dots, a_{in})$, which is the joint distribution of $\tilde{a}_{ij}$, $j = 1, \dots, n$, and $\tilde{b}_i$. Then

$$\text{Pos}[x \in \mathcal{F}_i] = \sup_{a_i, b_i}\{\, \Pi(a_i, b_i) \mid a_{i1}x_1 + \dots + a_{in}x_n * b_i \,\},$$

which is the possibility that x is feasible with respect to the i-th constraint. Therefore, for $x \geq 0$,

$$\text{Pos}[x \in \mathcal{F}] = \min_{1 \leq i \leq m} \text{Pos}[x \in \mathcal{F}_i],$$

which is the possibility that x is feasible. We next construct $\text{Pos}[Z = z|x]$ which is the conditional possibility that Z equals z given x. The joint distribution of the $\tilde{c}_j$ is

$$\Pi(c) = \min\{\tilde{c}_1(c_1), \dots, \tilde{c}_n(c_n)\}$$

where $c = (c_1, \dots, c_n)$. Therefore,

$$\text{Pos}[Z = z|x] = \sup_c\{\Pi(c)|c_1x_1 + \dots + c_nx_n = z\}.$$

Finally, applying Bellman and Zadeh's method for fuzzy decision making [1], the possibility distribution of the objective function is defined as

$$\text{Pos}[Z = z] = \sup_{x \geq 0} \min\{\text{Pos}[Z = z|x], \text{Pos}[x \in \mathcal{F}]\}.$$

It should be noted that Buckley [3] showed that the solution to an appropriate linear program gives the correct z values in $\text{Pos}[Z = z] = \alpha$ for each $\alpha \in [0, 1]$.

An important question is the influence of the perturbations of the fuzzy parameters to the possibility distribution of the objective function. We will assume that there is a collection of fuzzy parameters $\tilde{a}_{ij}^{\delta}$, $\tilde{b}_i^{\delta}$ and $\tilde{c}_j^{\delta}$ available with the property

$$D(\tilde{A}, \tilde{A}^{\delta}) \leq \delta,\ D(\tilde{b}, \tilde{b}^{\delta}) \leq \delta,\ D(\tilde{c}, \tilde{c}^{\delta}) \leq \delta. \tag{14}$$

Then we have to solve the following perturbed problem:

$$\begin{aligned} \text{max/min } Z^{\delta} &= x_1\tilde{c}_1^{\delta} + \dots + x_n\tilde{c}_n^{\delta} \\ \text{subject to} \quad & x_1\tilde{a}_{i1}^{\delta} + \dots + x_n\tilde{a}_{in}^{\delta} * \tilde{b}_i^{\delta}, \\ & 1 \leq i \leq m,\ x \geq 0. \end{aligned} \tag{15}$$

Let us denote by $\text{Pos}[x \in \mathcal{F}_i^\delta]$ the possibility that x is feasible with respect to the i-th constraint in (15). Then the possibility distribution of the objective function Z^δ is defined as follows:

$$\text{Pos}[Z^\delta = z] = \sup_{x \geq 0}(\min\{\text{Pos}[Z^\delta = z \mid x], \text{Pos}[x \in \mathcal{F}^\delta]\}).$$

The next theorem shows a stability property (with respect to perturbations (14) of the possibility dostribution of the objective function of the possibilistic linear programming problems (13) and (15).

Theorem 3.1. [7] *Let $\delta \geq 0$ be a real number and let $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{a}_{ij}^\delta$, $\tilde{c}_j$, $\tilde{c}_j^\delta$ be (continuous) fuzzy numbers. If (14) hold, then*

$$\sup_{z \in \mathbb{R}} \mid \text{Pos}[Z^\delta = z] - \text{Pos}[Z = z] \mid \leq \omega(\delta) \tag{16}$$

where where $\omega(\delta)$ denotes the maximum of modulus of continuity of all fuzzy coefficients at δ in problems (13) and (15).

From (16) follows that $\sup_z |\text{Pos}[Z^\delta = z] - \text{Pos}[Z = z]| \to 0$ as $\delta \to 0$, which means the stability of the possiibility distribution of the objective function with respect to perturbations (14). It is easy to see that in the case of non-continuous fuzzy parameters the possibility distribution of the objective function may be unstable under small changes of the parameters.

As an example, consider the following possibilistic linear program

$$\begin{aligned} \text{max/min } & \tilde{c}x \\ \text{subject to } & \tilde{a}x \leq \tilde{b}, \\ & x \geq 0. \end{aligned} \tag{17}$$

where $\tilde{a} = (1, 1)$, $\tilde{b} = (2, 1)$ and $\tilde{c} = (3, 1)$ are fuzzy numbers of symmetric triangular form. Here x is one-dimensional $(n = 1)$ and there is only one constraint $(m = 1)$. We find

$$\text{Pos}[x \in \mathcal{F}] = \begin{cases} 1 & \text{if } x \leq 2, \\ \dfrac{3}{x+1} & \text{if } x > 2. \end{cases}$$

and

$$\text{Pos}[Z = z|x] = \text{Pos}[\tilde{c}x = z] = \begin{cases} 4 - z/x & \text{if } z/x \in [3, 4], \\ z/x - 2 & \text{if } z/x \in [2, 3], \\ 0 & \text{otherwise}, \end{cases}$$

for $x \neq 0$, and

$$\text{Pos}[Z = z|0] = \text{Pos}[0 \times \tilde{c} = z] = \begin{cases} 1 & \text{if } z = 0, \\ 0 & \text{otherwise}. \end{cases}$$

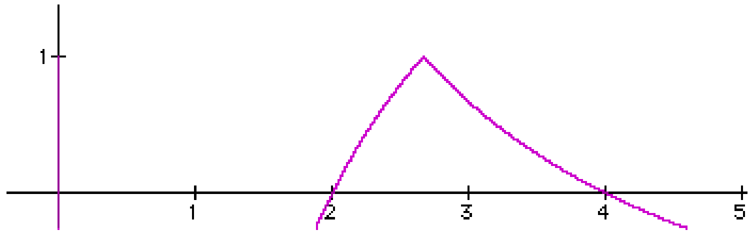

Figure 4: The graph of $\mathrm{Pos}[Z = z|x]$ for $z = 8$.

Therefore,

$$\mathrm{Pos}[Z = z] = \sup_{x \geq 0} \min \left\{ \frac{3}{x+1}, 1 - \left| \frac{z}{x} - 3 \right| \right\}$$

if $z > 6$ and $\mathrm{Pos}[Z = z] = 1$ if $0 \leq z \leq 6$. That is,

$$\mathrm{Pos}[Z = z] = \begin{cases} 1 & \text{if } 0 \leq z \leq 6, \\ v(z) & \text{otherwise.} \end{cases}$$

where

$$v(z) = \frac{24}{z + 7 + \sqrt{z^2 + 14z + 1}}.$$

This result can be understood if we consider the crisp LP problem with the centers of the fuzzy numbers

$$\max / \min 3x; \text{ subject to } x \leq 2, \ x \geq 0.$$

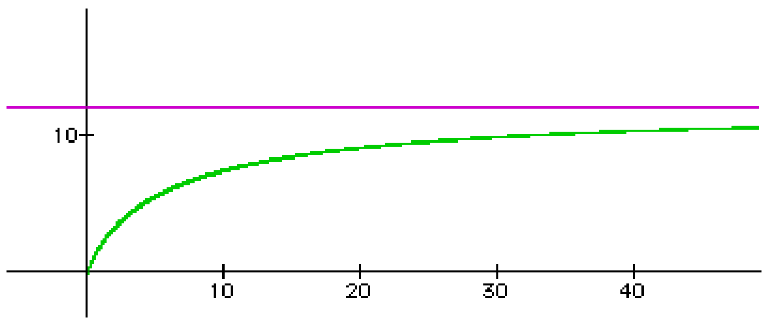

Figure 5: $z \times \mathrm{Pos}[Z = z]$ tends to 12 as $z \to \infty$.

All negative values as possible solutions to the crisp problem are excluded by the constraint $x \geq 0$, and the possible values of the objective function are in the interval $[0, 6]$. However, due to the fuzziness in (17), the objective function can take bigger values than six with a non-zero degrees of possibility. Therefore to find an optimal value of the problem

$$\begin{aligned} (3,1)x &\to \max \\ \text{subject to } (1,1)x &\leq (2,1), \\ x &\geq 0. \end{aligned} \tag{18}$$

requires a determination a trade-off between the increasing value of z and the decreasing value of $\mathrm{Pos}[Z = z]$.

If we take the product operator for modeling the trade-offs then we see that the resulting problem

$$z \times \mathrm{Pos}[Z = z] = \frac{24z}{z + 7 + \sqrt{z^2 + 14z + 1}} \to \max$$
$$\text{subject to } z \geq 0.$$

does not have a finite solution, because the function $z \times \mathrm{Pos}[Z = z]$ is strictly increasing if $z \geq 0$.

In 1994 Fullér and Fedrizzi [11], showed that the possibility distribution of the objectives of an multiobjective possibilistic linear program with (continuous) fuzzy number coefficients is stable under small changes in the membership function of the fuzzy parameters. Finally, in 1996 Canestrelli et al [4] proved that possibilistic quadratic programs with crisp decision variables and continuous fuzzy number coefficients are also well-posed, i.e. small changes of the membership function of the coefficients may cause only a small deviation in the possibility distribution of the objective function.

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