Application of the Scicos Simulator in the Investigation of an Adaptive Control of a Mechanical System of Free Internal Degree of Freedom

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Abstract: In this paper the novel modeling and adaptive control technique developed at Budapest Tech is investigated in the case of an approximately and partially modeled cart plus double pendulum system in which one of the pendulums has variable length due to an undamped spring. This spring corresponds to an internal degree of freedom that is not controlled though it is in dynamic coupling with the controlled ones. Its existence also is "unknown" by the controller. The novelty of the approach is that in contrast to the traditional soft computing that tries to build up some "complete" and "permanent" system model it is satisfied with "temporal" and "partial" models that are valid only in the actual dynamic environment of the system. That is, its validity is limited only to some "spatio-temporal vicinity" of the actual observations. The main benefits are the use of small, simple, lucid, and uniform structures, as well as the occurrence of short algebraic operations instead of some obscure tuning in a huge dimensional parameter space. Since no unnecessary effort is exerted to identify any analytical model it is expected that the effects of the dynamically coupled unmodeled degree of freedom on the motion of the controlled and observed ones can also be "learned". For the investigation of the new technique in the past the integration of the equations of motion happened by the simplest 1st order finite element approach in the time domain. At the end of the summer of 2004 INRIA issued its SCILAB 3.0 containing the improved scientific co-simulator called "Scicos". Due to it new prospects were opened for making "professional" and in the same time "convenient" simulations. In the paper the typical tools available in Scicos, and others developed by the authors, as well as the improved simulation results and conclusions are presented. It was found that the method successfully compensates the effect of the coupled, "unknown" internal degree of freedom.

1 Introduction

A new approach for the adaptive control of imprecisely known dynamic systems under unmodeled dynamic interaction with their environment was initiated in [1]. Instead of the supposed analytical model's parameters this controller uses several parameters of some abstract Lie groups serving as flexibly adjustable components of a temporal and environment-dependent, uniform, non-analytical model of the system to be controlled. This „non-analytical modeling" is akin to the Soft Computing philosophy, too. In this approach adaptivity means that instead of the simultaneous tuning of numerous parameters, a fast algorithm finding some linear transformation to map a very primitive initial model based expected system-behavior to the observed one is used. The so obtained „amended model" is step by step updated to trace changes by repeating this corrective mapping in each control cycle. Since no any effort is exerted to identify the possible reasons of the difference between the expected and the observed system response it is referred to as the idea of "Partial and Temporal System Identification". This anticipates the possibility for real-time applications. Regarding the appropriate linear transformations several possibilities were investigated and successfully applied. For instance, the „Generalized Lorentz Group" [2], the „Stretched Orthogonal Group", the "Partially Stretched Orthogonal Transformations" [3], and a special family of the „Symplectic Transformations" [4] can be mentioned.

The key element of the new approach is the formal use of the „Modified Renormalization Transformation". The „original" version was widely used e.g. by Feigenbaum in the seventies to investigate the properties of chaos [5-7]. This (originally scalar) transformation modifies the solution of an $x=f(x)$ fixed-point problem. Since the adaptive control can be formulated as a fixed-point problem, too [8], this transformation was considered a possible candidate for the solution of such a task. The modification of the original transformation was necessary due to phenomenological reasons. Satisfactory conditions of the complete stability of the so obtained control for Multiple Input-Multiple Output (MIMO) systems were also highlighted in [8] by the means of perturbation calculation. This means the most rigorous limitation of the circle of possible application of the new method. To release this restriction to some extent "ancillary" but simple interpolation techniques and application of "dummy parameters" were also introduced in [8]. The applicability of the method was investigated for electro-mechanical and hydrodynamic systems via simulation [9-10]. In this paper a quite simple but lucid typical paradigm, a cart conveying a double pendulum is chosen to be the subject of the adaptive controller. One of the pendulums has variable length due to an undamped spring. This spring corresponds to an internal degree of freedom that is not controlled though it is in dynamic coupling with the controlled ones. Its existence also is "unknown" by the controller.

Typical problems arise when the motion of the system is simulated by the use of its "exact" equations of motion and a finite element method regarding the time-resolution. The selection of the length of the interval between the discrete time-steps considered may seriously concern the numerical results of the calculations. This length has to be decreased till the effect of the decrease cannot be observed in the numerical results. It has to be stressed that in the case of a real time control system the cycle time of the control commands cannot be chosen to be arbitrarily small. The "internal" loop of a complex controller can be realized by fast hardware and simple calculations while the "external adaptive loop" may need more calculations and may have relatively long cycle-time. During these finite "external" time intervals the torque/force values exerted by the drives can be supposed to be constant while the contribution by the Coriolis and gravitational terms of the exact equations of motion must be traced in a finer resolution in the simulations and in the internal loop.

In the sequel at first the paradigm is set mathematically. Following that the basic principles of the adaptive control are described. Following the presentation of the typical simulation results the conclusions are drawn.

2 The Dynamic Model of the Cart and Double Pendulum of Varying Length System

Let the cart consist of a body and wheels of negligible momentum and inertia having the overall mass of M [*kg*]. Let the pendulums be assembled on the cart by parallel shafts of negligible masses. At the end of each arm a ball of negligible size and considerable mass is attached (m_1 and m_2 [*kg*], respectively). Let the first arm be of fixed length L_1 [*m*]. The length of the 2nd arm is limited by a hub of radius r_{min}. This hub acts as a stiff, elastic bumper that is modeled by conservative potential energy term that becomes singular in a thin layer behind its surface. The elastic spring that determines the length of the second arm has the stiffness of k [*N/m*], and length of zero force l [*m*]. In Fig. 1 the generalized coordinates of this system are defined. The potential energy of this term is described the equation as follows:

$$V = m_1 g q_3 \sin q_1 + m_2 g L_2 \sin q_2 + \frac{1}{2} k (q_3 - l)^2 + \frac{A}{q_3 - r_{\min} + \varepsilon} \quad (1)$$

in which A and ε describe the behavior of the bumper at the hub.

Since the kinetic energy of this system can be constructed in a quite standard manner, it is satisfactory to provide the reader only with the Euler-Lagrange equations of motion of this system in which g denotes the gravitational acceleration [m/s^2], Q_1 and Q_2 [$N\times m$] denote the driving torque at the rotational shaft 1 and 2, respectively, Q_4 [*N*] stands for the force moving the cart and pendulum system in the horizontal direction, and $Q_3 \equiv 0$ [*N*], since the internal

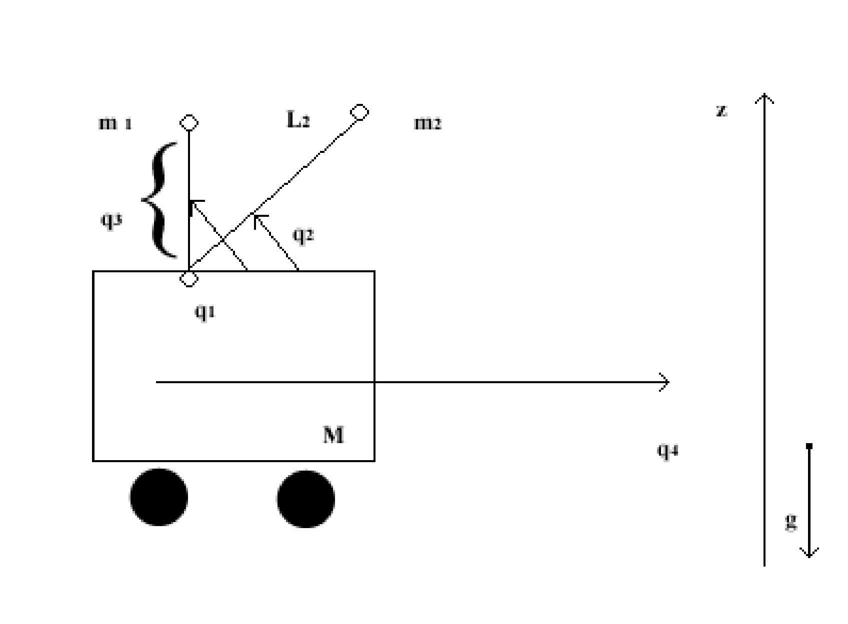

Figure 1
Definition of the generalized coordinates of the mechanical system

degree of freedom does not have any drive. (The appropriate rotational angles are q_1 and q_2 [*rad*], and the linear degrees of freedom belong to q_3, *and* q_4 [*m*], respectively.)

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} m_1 q_3^2 & 0 & 0 & -m_1 q_3 \sin q_1 \\ 0 & m_2 L_2^2 & 0 & -m_2 L_2 \sin q_2 \\ 0 & 0 & m_1 & m_1 \cos q_1 \\ -m_1 q_3 \sin q_1 & -m_2 L_2 \sin q_2 & m_1 \cos q_1 & (M + m_1 + m_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{bmatrix} +
$$

$$
\begin{bmatrix} 2m_1 q_3 \dot{q}_3 \dot{q}_1 + m_1 g q_3 \cos q_1 \\ m_2 g L_2 \cos q_2 \\ -m_1 q_3 \dot{q}_1^2 + m_1 g \sin q_1 \\ + k(q_3 - l) - \dfrac{A}{(q_3 - r_{\min} + \varepsilon)^2} \\ -m_2 L_2 \cos q_2 \dot{q}_2^2 - \\ -2m_1 \sin q_1 \dot{q}_3 \dot{q}_1 \\ -m_1 q_3 \cos q_1 \dot{q}_1^2 \end{bmatrix} \tag{2}
$$

On the basis of (2) it is easy to express the inverse dynamical equations of motion by using numerical matrix inversion for simulation purposes. In the sequel the principles of the adaptive control are detailed.

3 Principles of the Adaptive Control

From purely mathematical point of view the control task can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated to obtain a desired system response $\mathbf{i}^d$ as $\mathbf{e}=\varphi(\mathbf{i}^d)$. The system has its inverse dynamics described by the unknown function $\mathbf{i}^r=\psi(\varphi(\mathbf{i}^d))=f(\mathbf{i}^d)$ and resulting in a realized response $\mathbf{i}^r$ instead of the desired one, $\mathbf{i}^d$. Normally one can obtain information via observation only on the $\mathbf{i}^r$ values. The function $f()$ can considerably vary in time, and no any possibility exists to directly "manipulate" its nature: only $\mathbf{i}^d$ as the input of $f()$ can be “deformed” to $\mathbf{i}^{d*}$ to achieve and maintain the $\mathbf{i}^d=f(\mathbf{i}^{d*})$ state. On the basis of the modification of the method of renormalization transformation widely applied in Physics the following "scaling iteration" was suggested for finding the proper deformation:

$$\begin{gathered}\mathbf{i}_0;\mathbf{S}_1\mathbf{f}(\mathbf{i}_0)=\mathbf{i}_0;\mathbf{i}_1=\mathbf{S}_1\mathbf{i}_0;\ldots;\mathbf{S}_n\mathbf{f}(\mathbf{i}_{n-1})=\mathbf{i}_0;\\ \mathbf{i}_{n+1}=\mathbf{S}_{n+1}\mathbf{i}_n;\mathbf{S}_n\xrightarrow[n\to\infty]{}\mathbf{I}\end{gathered}\tag{3}$$

in which the $\mathbf{S}_n$ matrices denote some linear transformations to be specified later. As it can be seen these matrices map the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller „learns” the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (3) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. The most important points of view are fast and efficient computation, and the ability for remaining as close to the identity transformation as possible.

For making the problem mathematically unambiguous (3) can be transformed into a matrix equation by putting the values of **f** and **i** into well-defined blocks of bigger matrices. Via computing the inverse of the matrix containing **f** in (3) the problem can be made mathematically well-defined. Since the calculation of the inverse of one of the matrices is needed in each control cycle it is expedient to choose special matrices of fast and easy invertibility. Within the block matrices the response arrays may be extended by adding to them a “dummy”, that is a physically not interpreted dimension of constant value, in order to evade the occurrence of the mathematically dubious $0\to 0$, $0\to$finite, finite$\to 0$ transformations. In the present paper the special symplectic matrices announced in

[4] were applied for this purpose. In general, the Lie group of the Symplectic Matrices is defined by the equations

$$\mathbf{S}^T \Im \mathbf{S} = \Im \equiv \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{I} & \mathbf{0} \end{array}\right], \ \det\mathbf{S} = 1 \tag{4}$$

The inverse of such matrices can be calculated in a computationally very cost-efficient manner as $\mathbf{S}^{-1} = \Im^T \mathbf{S}^T \Im$. In our particular case the symplectic matrices are constructed from the desired and the observed joint coordinate accelerations corresponding to the response of the mechanical system to the excitation of torque and force by the use of the matrix

$$[\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_6] = \begin{bmatrix} \ddot{q}_1 & -\ddot{q}_1 & e_1^{(3)} & e_1^{(4)} & e_1^{(5)} & e_1^{(6)} \\ \ddot{q}_2 & -\ddot{q}_2 & e_2^{(3)} & e_2^{(4)} & e_2^{(5)} & e_2^{(6)} \\ \ddot{q}_3 & -\ddot{q}_3 & e_3^{(3)} & e_3^{(4)} & e_3^{(5)} & e_3^{(6)} \\ \ddot{q}_4 & -\ddot{q}_4 & e_4^{(3)} & e_4^{(4)} & e_4^{(5)} & e_4^{(6)} \\ d & -d & e_5^{(3)} & e_5^{(4)} & e_5^{(5)} & e_5^{(6)} \\ D & \dfrac{\ddot{\mathbf{q}}^2 + d^2}{D} & e_6^{(3)} & e_6^{(4)} & e_6^{(5)} & e_6^{(6)} \end{bmatrix} \tag{5}$$

in the blocks of a bigger one as

$$\mathbf{S} = \left[\begin{array}{c|c} \mathbf{0} & \dfrac{-1}{s}\mathbf{m}^{(1)} \quad \dfrac{-1}{s}\mathbf{m}^{(2)} \quad -\mathbf{e}^{(3)} \ldots -\mathbf{e}^{(6)} \\ \hline \mathbf{m}^{(1)} \quad \mathbf{m}^{(2)} \quad \mathbf{e}^{(3)} \ldots \mathbf{e}^{(6)} & \mathbf{0} \end{array}\right] \tag{6}$$

in which the $\mathbf{e}^{(3)},\ldots,\mathbf{e}^{(6)}$ symbols denote unit vectors that lie in the orthogonal sub-space of the first two columns of the matrix, d is the "dummy" parameter used for avoiding singular transformations, and

$$D^2 \equiv \ddot{\mathbf{q}}^T \ddot{\mathbf{q}} + d^2, s = 2D^2 \tag{7}$$

The unit vectors in (6) can be created e.g. by using El Hini's algorithm [3] as detailed e.g. in [4]. With the above completion the appropriate operation in (3) evidently equals to the identity operator if the desired response just is equal to the observed one, and remains in the close vicinity of the unit matrix if the non-zero desired and realized responses are very close to each other. Since amongst the conditions for which the convergence of the method was proven near-identity transformations were supposed in the perturbation theory, a parameter ξ measuring the „*extent of the necessary transformation*", a "*shape factor*" σ, and a „*regulation factor*" λ can be introduced in a linear interpolation with small positive ε_1, ε_2 values as

$$\xi := \frac{|\mathbf{f} - \mathbf{i}^d|}{\max(|\mathbf{f}|, |\mathbf{i}^d|) + 1}, \quad \lambda = 1 + \varepsilon_1 + (\varepsilon_2 - 1 - \varepsilon_1)\frac{\sigma\xi}{1 + \sigma\xi}, \quad \hat{\mathbf{i}}^d = \mathbf{f} + \lambda(\mathbf{i}^d - \mathbf{f}) \tag{8}$$

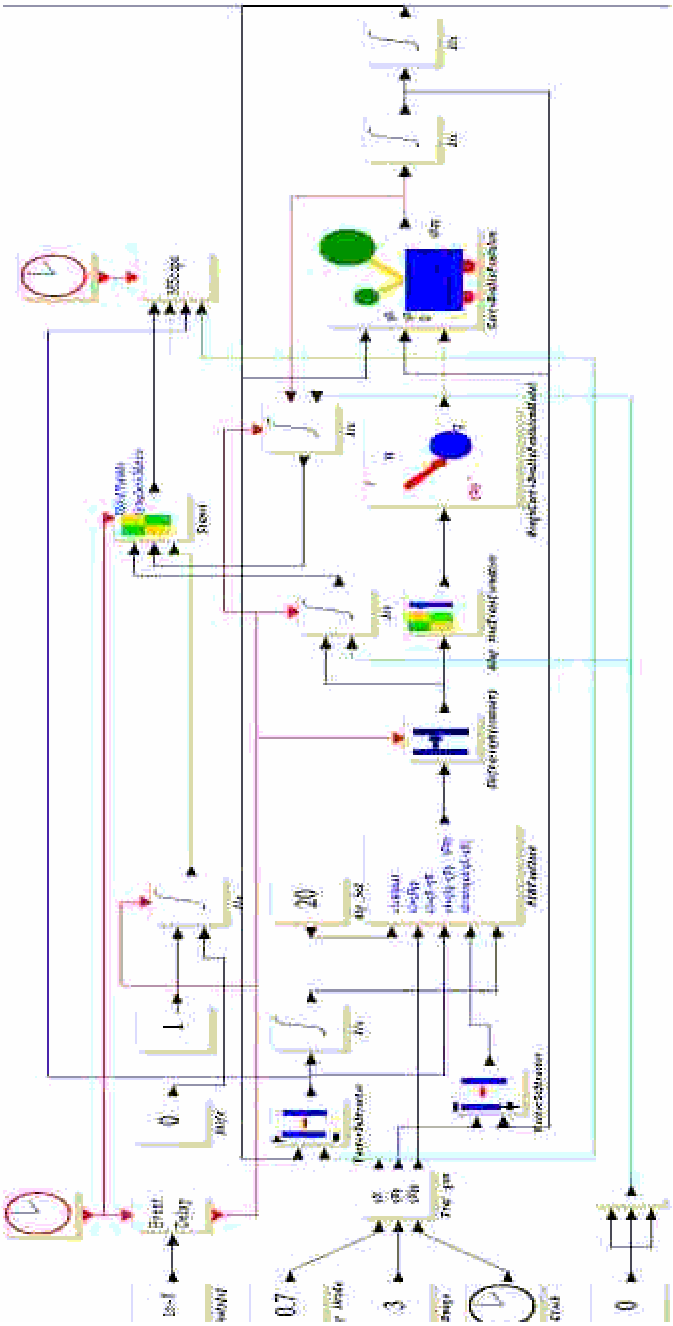

Figure 2
The Scicos scheme of the adaptive control and simulation

This interpolation reduces the task of the adaptive control in the more critical sessions and helps to keep the necessary linear transformation in the vicinity of the identity operator. In the forthcoming simulations the following numerical data were used: d=800, σ=22, ε_1=0.2, ε_2=0.1. They were selected "experimentally". The unobserved degree of freedom was treated by this formalism by simply dropping the row belonging to the coordinate q_3. Consequently, the number of the columns in (6) was also decreased by one.

4 Simulation Results

In Fig. 2 the Scicos model of the simulation scheme of the controller, the "rough" and the "exact" system models is presented. In the simulations for the desired relaxation of the trajectory tracking error a simple PID-type rule was prescribed by the use of purely kinematic terms. This error relaxation could be achieved exactly only in the possession of the exact dynamic model of the physical system to be controlled. Instead of the exact actual dynamic model detailed in (2) the constant 10×**I** (**I**= unit matrix) matrix was used as the inertia matrix, and the Coriolis and inertial terms were modeled by the constant vector $[10, 10, 10]^T$. This evidently corresponds to a very rough approximation of the reality in which m_1=15 kg, m_2=10 *kg*, L_1=2 m, M=3 *kg*, were chosen.

The typical "built in" elements as the integrator, the "source elements" as the constants, the clock, the "*periodic event generators*", and the only "sink", that is the multiple oscilloscope simulator called "*Mscope*" can well be identified in the figure. The other blocks contain "user-developed functions" as the trajectory generator "*Trajgen*", the model of the PID controller, the rough and the exact system models and the "*Vector Subtractors*". These user-developed functions can be given as common SCILAB instructions that are "interpreted" by Scicos. To speed up the operation of the simulator an alternative method is loading and compiling the user functions instead of directly writing them in the user blocks. (In this case the user block contains only a simple call for the compiled function.) The compilation of the necessary user functions at the beginning can be prescribed in the so-called "*Context*" box of the simulator. The here defined variables behave as "*global*" ones from the point of view of the user-defined functions. They can be referred to as "global" variables in the heading (beginning lines) of the user's functions. The "wires" correspond to the traditional function calls via the stack making the use of the simulator similar to data flow programming. (The global variables can directly be modified by the functions without the use of any "wire".)

In the control calculation of the "desired" and measurement of the "realized" joint coordinate accelerations was needed. Within the frames of Scicos this can be done by averaging these signals for finite time-intervals using event driven integrators that reset their initial value to zero when the appropriate event happens. (The length of the time-interval can be obtained by integrating the constant function 1.) In the possession of the averaged joint coordinate accelerations the special symplectic matrices described in (5) and (6) can be updated as a *global variables*. The values of the *desired joint coordinate accelerations* are kept constant due to a "*Vector Shift Register*" during the integration. Therefore the cycle-time of the external adaptive loop approximately corresponds to the duration of this integration plus that of the necessary calculations.

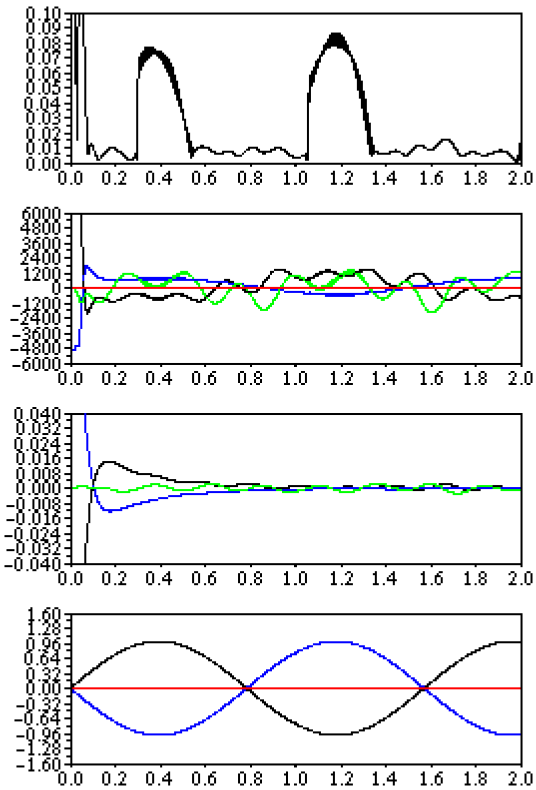

Figure 3

The operation of the adaptive controller: 1st box the norm of the adaptive signal $||\mathbf{S}_n\text{-}\mathbf{I}||$; 2nd box: the generalized forces [in *Nm* for Q_1 (black) and Q_2 (blue), *N* for Q_4 (green) and 0 for q_3 (red)]; 3rd box: the tracking errors [in *rad* for q_1 (black) and q_2 (blue), *m* for q_4 (green)]; 4th box: the nominal trajectory [in *rad* for q_1 (black) and q_2, (blue), *m* for q_4 (red)] vs. time [*s*].

In Fig. 3 the operation of the adaptive controller can be seen. (Each simulation was carried out with the default settings of Scicos prescribing 0.0001 for the *integrator absolute tolerance*, 1D-06 for the *integrator relative tolerance*, and 1D-10 value for the *tolerance on time*.) The considerable improvement in trajectory tracking as time passes by is apparent.

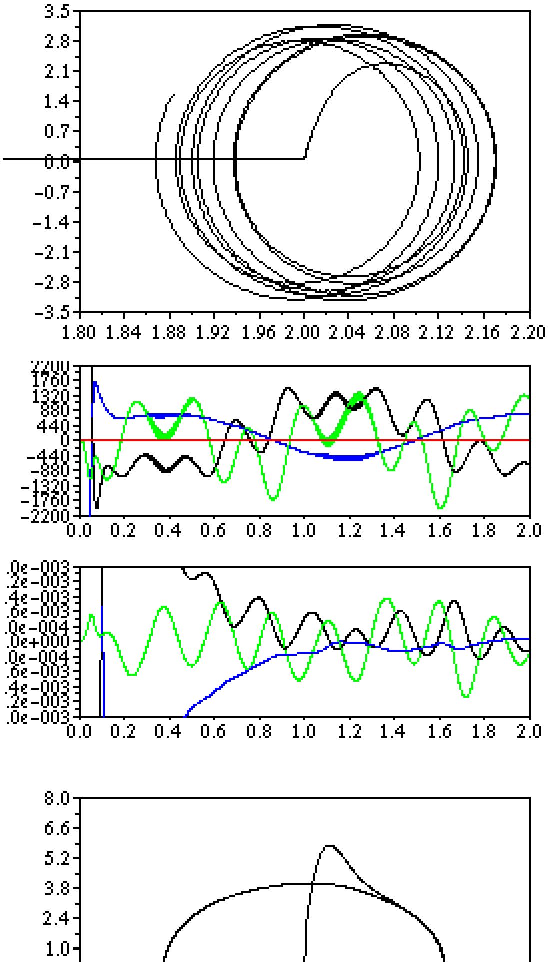

Figure 4

The operation of the adaptive controller: 1st box: the phase-space of the uncontrolled degree of freedom [*m, m/s*]; 2nd box: zoomed excerpt of the generalized forces, units as in Fig. 3; 3rd box: zoomed excerpts of the tracking errors, units as in Fig. 3; 4th box: phase space of q_1 [*rad, rad/s*];

The phase space of the uncontrolled degree of freedom and q_1 is given in Fig. 4 together with the zoomed excerpts of the graphs of the generalized forces. Due to the limited amplitude of the vibration of the unmodeled degree of freedom its presence is rather traceable in the fluctuation of the generalized forces needed for controlling the motion of the 1st arm and the linear displacement of the arm.

Conclusions

At the end of the Summer of 2004 INRIA issued its SCILAB 3.0 containing an advanced numerical simulation tool called "Scicos". Due to it new prospects were opened for making "professional" and in the same time "convenient" simulations for studying the sensitivity of the novel adaptive control developed at the Budapest Tech. A simple but lucid paradigm, a cart conveying an asymmetric double pendulum system also having an uncontrolled and unmodeled internal degree of freedom was chosen to be controlled. The simulator well demonstrated that the novel adaptive approach seems to be applicable, it is able to learn the varying dynamic properties of the subsystem to be controlled by it. The details of the dynamic coupling responsible for the varying dynamic properties were also well revealed. Scicos seems to be a reliable, convenient and accurate simulation tool for similar investigations in the future.

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