AN EXAMPLE OF THE POSSIBLE INSTABILITY OF LOCAL CONTROLLER NETWORKS/TAKAGI-SUGENO FUZZY CONTROLLERS

D.J.Leith, W.E.Leithead

Department of Electronics & Electrical Engineering,
University of Strathclyde, 50 George St., Glasgow G1 1QE, U.K.
Tel. +44 141 548 2407, Fax. +44 141 548 4203, Email. doug@icu.strath.ac.uk

Abstract: This paper considers a typical Local Model Network/Takagi-Sugeno approach to nonlinear control design. Although approaches of this type are attractive in many respects, the blending procedure involved can introduce many new issues the importance of which is not necessarily immediately obvious. A number of fundamental issues associated with conventional blending approaches are considered and the serious consequences which can arise from their neglect are illustrated by means of an example.

1. INTRODUCTION

It is often attractive to approach a nonlinear design task by first decomposing the problem into a number of linear sub-problems solvable by established methods and then recombining, in some appropriate manner, the resultant collection of linear designs to obtain the required nonlinear design. Divide and conquer methodologies have been proposed in a wide range of fields and, in the control context, these are typically referred to as gain-scheduling design methods. While many approaches to gain-scheduling have been investigated, this paper is concerned with a class of methods which have been the subject of considerable interest in recent years. Briefly, in these methods the dynamics of the nonlinear system to be controlled are formulated as the blended combination of a number of simple local models (typically affine local models are used since these are the simplest type of model for which conventional universal approximation results are available). A linear controller is designed for each local model and the resulting linear controllers are then blended together in a similar manner to the plant local models to obtain a nonlinear controller. Methods in this class have been developed by both the neural network and fuzzy-logic communities and are typically referred to, respectively, as Local Model/Controller Networks and Takagi-Sugeno Fuzzy Models/Controllers (see, for example, Hunt & Johansen 1997, Johansen & Murray-smith 1997, Shorten *et al.* 1999)).

Although blended approaches of this type are attractive in many respects, the blending procedure can introduce many new issues the importance of which is not necessarily immediately obvious. This paper considers a number of fundamental issues associated with conventional blending approaches and illustrates, by means of an example, the serious consequences which can arise from their neglect.

In the context of the present paper, neural network terminology is hereafter adopted arbitrarily to avoid repetition (fuzzy terminology could equally well be used).

2. PLANT MODEL

The dynamics of the plant under consideration are modelled by a discrete-time local model network with affine local models; namely,

$$y(t_{n+1}) = F(u(t_n), y(t_n), y(t_{n-1})) \qquad (1)$$

where y is the output, u is the control input and

$$F(u(t_n), y(t_n), y(t_{n-1})) = \sum_{i=1}^{4} w_i(u(t_n), y(t_n))\, A_i \qquad (2)$$

The sampling interval, T, is 0.05 seconds and the affine local models are

$$A_1 = -0.003267+0.02015u(t_n)+1.147y(t_n)-0.2244y(t_{n-1})$$
$$A_2 = 0.01867+0.05436u(t_n)+1.455y(t_n)-0.496y(t_{n-1})$$
$$A_3 = 0.001666+0.02578u(t_n)+1.512y(t_n)-0.5671y(t_{n-1})$$
$$A_4 = -0.0473+0.0473u(t_n)+1.422y(t_n)-0.51y(t_{n-1})$$

The corresponding weighting functions are

$$w_1(u(n),y(n))=v_1/v,\; w_2(u(n),y(n))=v_2/v \qquad (3)$$
$$w_3(u(n),y(n))=v_3/v,\; w_4(u(n),y(n))=v_4/v$$

with

$$v_1(u,y) = \exp(-0.5\left(\frac{u-\bar{z}_{1,1}}{\Delta z_{1,1}}\right)^2 - 0.5\left(\frac{y-\bar{z}_{1,2}}{\Delta z_{1,2}}\right)^2)$$

$$v_2(u,y) = \exp(-0.5\left(\frac{u-\bar{z}_{2,1}}{\Delta z_{2,1}}\right)^2 - 0.5\left(\frac{y-\bar{z}_{2,2}}{\Delta z_{2,2}}\right)^2)$$

$$v_3(u,y) = \exp(-0.5\left(\frac{u-\bar{z}_{3,1}}{\Delta z_{3,1}}\right)^2 - 0.5\left(\frac{y-\bar{z}_{3,2}}{\Delta z_{3,2}}\right)^2)$$

$$v_4(u,y) = \exp(-0.5\left(\frac{u-\bar{z}_{3,1}}{\Delta z_{3,1}}\right)^2 - 0.5\left(\frac{y-\bar{z}_{3,2}}{\Delta z_{3,2}}\right)^2)$$

$$v = v_1 + v_2 + v_3 + v_4$$

and

$\bar{z}_{1,1} = (-1 + 04.12e-1)/2$, $\quad \Delta z_{1,1} = (4.12e-1 - (-1))/2$
$\bar{z}_{1,2} = (-1 + 1.8928e-1)/2$, $\quad \Delta z_{1,2} = (1.8928e-1 - (-1))/2$
$\bar{z}_{2,1} = (4.03e-1+1)/2$, $\quad \Delta z_{2,1} = (1-4.03e-1)/2$
$\bar{z}_{2,2} = (1.428e-1+1)/2$, $\quad \Delta z_{2,2} = (1 - 1.428e-1)/2$
$\bar{z}_{3,1} = (4.212e-1 + 1)/2$, $\quad \Delta z_{3,1} = (1 - 4.212e-1)/2$
$\bar{z}_{3,2} = (-1 + 1.357e-1)/2$, $\quad \Delta z_{3,2} = (1.357e-1 - (-1))/2$
$\bar{z}_{4,1} = (-1 + 3.909e-1)/2$, $\quad \Delta z_{4,1} = (3.909e-1 - (-1))/2$
$\bar{z}_{4,2} = (1.964e-1 + 1)/2$, $\quad \Delta z_{4,2} = (1.964e-1 - 1)/2$

3. CONTROL DESIGN

The requirement is to design a controller for the plant (1) which achieves a rise time of around 0.2 seconds with no overshoot. (This is, of course, not a complete performance specification but is adequate for the purposes of the present study). Before proceeding, it is useful to confirm that the control problem is well posed in the sense that a solution exists. The closed-loop step responses for a range of magnitudes are shown in Figure 1 with a nonlinear controller designed using the velocity-based inversion approach of Leith & Leithead (1999a). Evidently, a solution to the control task does indeed exist.

3.1 Local Controller Network

A local controller network is obtained by designing a linear controller (no offset term) corresponding to the linear portion of each plant local model (obtained by dropping the offset term in the local model). Each linear controller contains integral action and is designed to achieve a nominal rise time of 0.2 seconds when used with the linear portion of the corresponding local model. The linear controllers are blended to obtain the discrete-time local controller network

$$\begin{aligned} u(t_{n+1}) = {} & w_1(u(t_n),y(t_n))(43.77e(t_n)-41.94e(t_{n-1})+7.773e(t_{n-2}) \\ & \quad +1.037u(t_n)-0.03685u(t_{n-1})) \\ & +w_2(u(t_n),y(t_n))(21.52e(t_n)-25.51e(t_{n-1})+7.547e(t_{n-2}) \\ & \quad +1.017u(t_n)-0.01667u(t_{n-1})) \\ & +w_3(u(t_n),y(t_n))(47.5e(t_n)-58.58e(t_{n-1})+18.593e(t_{n-2}) \\ & \quad +1.015u(t_n)-0.01458u(t_{n-1})) \\ & +w_4(u(t_n),y(t_n))(24.02e(t_n)-28.87e(t_{n-1})+8.943e(t_{n-2}) \\ & \quad +1.016u(t_n)-0.01621u(t_{n-1})) \qquad (4) \end{aligned}$$

where the weighting functions are identical to those in the plant. Bode plots of the closed-loop transfer functions are given in Figure 2.

3.2 Performance with local controller network

It is straightforward to confirm that the performance requirements are satisfied by the linear closed-loop systems formed by the application of the local controllers to the respective plant local models (neglecting the offset terms). Nevertheless, Figure 3 shows typical closed-loop step responses of the nonlinear LMN/LCN system. Evidently, the closed-loop performance of the LCN is rather poor. It is emphasised that the results in Figure 3 are for the situation where there is no measurement noise. The oscillations that can be observed in the output seem to be associated with a limit cycle.

4. ANALYSIS & DISCUSSION

Factors influencing the performance achieved with the LCN include

- the impact of neglecting the offset terms in the local models when designing the corresponding local controllers.
- the nonlinear dynamic effects introduced by the blending of the local models but neglected in the control design (the solution to the LMN is not a weighted combination of the solutions to the local models nor do the linearisations of the LMN correspond to weighted combinations of the local models, see below).
- the nonlinear dynamic effects which are, similarly, associated with the blending of the local controllers in the LCN.

4.1 Importance of Offset Terms in Local Models

Consider modifying the plant local models, (1) to

$$A_1 = 0.003267+0.02015u(t_n)+1.147y(t_n)-0.2244y(t_{n-1})$$
$$A_2 = -0.01867+0.05436u(t_n)+1.455y(t_n)-0.496y(t_{n-1})$$
$$A_3 = -0.001666+0.02578u(t_n)+1.512y(t_n)-0.5671y(t_{n-1})$$
$$A_4 = -0.003562+0.0473u(t_n)+1.422y(t_n)-0.51y(t_{n-1}) \quad (5)$$

Note that the local models are identical to those used previously except for small changes to the offset terms. Since the linear parts of the plant local models are unchanged, the LCN design methodology (Hunt & Johansen 1997) leads to the same controller, (4) as before. Figure 4 shows typical closed-loop step responses of the nonlinear LMN/LCN system Clearly, the system is now stable. In view of the apparently minor differences between the system considered in section 4 and that considered here, this substantial improvement in performance indicates the importance of the offset terms in the local models. (Note that this sensitivity is present despite the inclusion of integral action in the LCN).

4.2 Nonlinear Effects Introduced by Blending

It is straightforward to confirm that the dynamics of a local model network (similarly a local controller network) are indicated, locally, by the first-order series expansion of the dynamics (by trivial extension of the results in Leith & Leithead 1998). The first-order series expansion of the plant dynamics, (1), about an operating point at which $[u(t_n)\ y(t_n)\ y(t_{n-1})]$ equals ρ_1 is

$$\begin{aligned}\delta y_{n+1} &= F(\rho_1)+\nabla_{u_n}F(\rho_1)\delta u_n+\nabla_{y_n}F(\rho_1)\delta y_n+\nabla_{y_{n-1}}F(\rho_1)\delta y_{n-1}\\ &= F(\rho_1)+\sum_{i=1}^{4}\Big(\underline{\underline{\nabla_{u_n}w_i\,A_i}}+w_i\nabla_{u_n}A_i\Big)\delta u_n\\ &\quad+\sum_{i=1}^{4}\Big(\underline{\underline{\nabla_{y_n}w_i\,A_i}}+w_i\nabla_{y_n}A_i\Big)\delta y_n\\ &\quad+\sum_{i=1}^{4}\Big(\underline{\underline{\nabla_{y_{n-1}}w_i\,A_i}}+w_i\nabla_{y_{n-1}}A_i\Big)\delta y_{n-1}\end{aligned} \quad (6)$$

Notice the highlighted terms involving the derivatives of the weighting functions. These cross terms embody nonlinear dynamic effects introduced by the blending of the local models but neglected in the control design. An indication of the impact of these cross-terms is provided in Figure 5 where the transfer functions of the linear part of the first-order expansion, (6) are compared to those obtained when the cross-terms are neglected. It can be seen that the transfer functions differ considerably both in magnitude and phase. Similar effects are also associated with the blending used in the controller itself.

It is, of course, already noted in section 4.1 that the offset term cannot, in general, be neglected. The interpretation of the foregoing comparison can nevertheless be put on a sound basis via the velocity-based linearisation framework. Let Δ denote $\Delta = \frac{1-z^{-1}}{T}$ (i.e. an approximate derivative operator).

On applying Δ to both sides of the first-order series expansion, (6),

$$\begin{aligned}\Delta y_n &= w_n\\ w_{n+1} &= \nabla_{u_n}F(\rho_1)\Delta u_n+\nabla_{y_n}F(\rho_1)w_n+\nabla_{y_{n-1}}F(\rho_1)w_{n-1}\end{aligned} \quad (7)$$

It can be seen that the offset term is no longer present and the transformed system, (7), is genuinely *linear* (rather than affine). Nevertheless, (6) and (7) are dynamically equivalent in the sense that, with appropriate initial conditions, their solutions y_n are the same. The discrete-time Δ transformation is related to the continuous-time velocity-based transformation introduced by Leith & Leithead (1998) and, following the terminology of Leith & Leithead (1998) the linear system (7) is, for simplicity, referred to as the velocity-based linearisation associated with operating point ρ_1 of the nonlinear system, (1)[1]. The solution to the velocity-based linearisation approximates the solution to the nonlinear system (1) locally to relevant operating point. There is a velocity-based linearisation associated with every operating points (not just the equilibrium points as is the case with conventional series expansion linearisations).

By inspection, it can be seen that the linear system obtained by neglecting the offset term in the first-order expansion, (6), is closely related in form to the velocity-based linearisation, (7). In particular, the transfer functions of both are the same. Consequently, control design procedures based on neglecting the offset term in (6) can often lead, albeit inadvertently, to controllers which are similar to those obtained via a velocity-based design procedure. However, it clear from the example in section 4.1 that designs based simply on neglecting the offset term can potentially produce controllers which achieve very poor results indeed. In addition, such approaches provide little insight into the source of these difficulties or into techniques for their resolution. Although superficially similar, it is therefore emphasised that important fundamental differences exist between the velocity-based linearisation, (7), and the linear system obtained by simply neglecting the offset term in (6), namely, the input, state and output of each differ. The input, state and output are perturbation quantities in (6) and thus depend on the operating point considered whereas every velocity-based linearisation, (7), shares the same state, input and output. When operation is confined to the vicinity of single operating point, this difference is unimportant; for example, when operating in the vicinity of a single equilibrium point. Otherwise, the variations in input, state and output of (6) make it difficult to infer the non-local dynamic characteristics of a nonlinear system from those of its first-order expansions (6). The variation in input, state

[1] Note that the discrete time formulation used here does not possess an analogue of every property of the continuous time-velocity-based formulation, but is sufficient for present purposes.

and output introduces a changing frame of reference which can profoundly affect the non-local dynamics in a similar manner to the effects associated with employing a non-interial frame of reference such as the body-centered axes common in flight control applications. Such difficulties are immediately avoided by adopting the velocity-based framework.

The scheduling vector, ρ, which parameterises the velocity-based linearisations of the LMN plant is $[u(t_n)\ y(t_n)\ y(t_{n-1})]^T$ (in comparison, it is noted firstly that the LMN weighting functions only depend on $u(t_n)$ and $y(t_n)$ and so do not distinguish between the equilibrium and off-equilibrium points considered here). Since this is three dimensional, attention is confined to the velocity-based linearisations associated with a number of interesting operating points. Firstly, the equilibrium linearisations are considered. A Bode plot of the transfer function of a typical plant equilibrium linearisation is shown in Figure 5a: Notice, in particular, the phase roll-off from around 0 degrees at low frequencies to around −100 degrees at higher frequencies. Secondly, consider the linearisations at off-equilibrium operating points which might be encountered during transitions between these equilibrium points. In particular, since the closed-loop LCN/LMN system exhibits a high frequency oscillation, consider the linearisations associated with operating points at which subsequent output samples, $y(t_n)$ and $y(t_{n-1})$, are substantially different. A Bode plot of the transfer function of the velocity-based linearisation at such a representative transient operating points ($y(t_n)$=0, $y(t_{n-1})$=0.2) are shown in Figure 5b. Notice that the phase is now around 180 degrees ar low frequencies to around 80 degrees at higher frequencies; that is, a change in sign effectively occurs in comparison with near equilibrium operation. Whilst the LCN local controller associated with the operating region considered achieves a stable closed-loop linearisation at the equilibrium points, the closed-loop linearisation is unstable at the off-equilibrium operating points. This type of linearisation based analysis therefore does indeed seem to successfully capture the observed dynamic characteristics. In comparison, it can be seen that the sign change is not captured by the transfer functions obtained when the cross-terms are neglected (these transfer functions correspond to a straightforward, direct weighted combination of the local models).

6. SUMMARY

This paper considers a typical Local Model Network/Takagi-Sugeno approach to nonlinear control design. Although approaches of this type are attractive in many respects, the blending procedure involved can introduce many new issues the importance of which is not necessarily immediately obvious. A number of fundamental issues associated with conventional blending approaches are considered and the serious consequences which can arise from their neglect are illustrated by means of an example.

ACKNOWLEDGEMENTS

The support provided by K.Hunt, University of Glasgow and T.A.Johansen, Norwegian University of Science & Technology in providing the original LMN and LCN data is gratefully acknowledged. This work was funded in part by the Royal Society.

REFERENCES

HUNT, K.J.,JOHANSEN,T.A., 1997, Design and analysis of gain-scheduled control using local controller networks. *Intl J Control*, **66**, 619-651.

JOHANSEN, T.A., MURRAY-SMITH,R., 1997, The operating regime approach to nonlinear modelling and Control. *Multiple Model Approaches to Modelling & Control* (Ed. Murray-Smith, R., Johansen, T.A.) (Taylor & Francis, London).

LEITH, D.J., LEITHEAD, W.E., 1998, Gain-Scheduled & Nonlinear Systems: Dynamic Analysis by Velocity-Based Linearisation Families. *International Journal of Control*, **70**, 289-317; 1999a, Input-Output Linearisation by Velocity-Based Gain-Scheduling. *International Journal of Control*, **72**, 229-246; 1999b, Analytic Framework for Blended Multiple Model Systems Using Linear Local Models. *International Journal of Control*, **72**, 605-619.

SHORTEN,R., MURRAY-SMITH,R.,BJORGAN,R.,GOLLEE,H.,1999, On the interpretation of local models in blended multiple model structure. *Int J Control*, **72**.

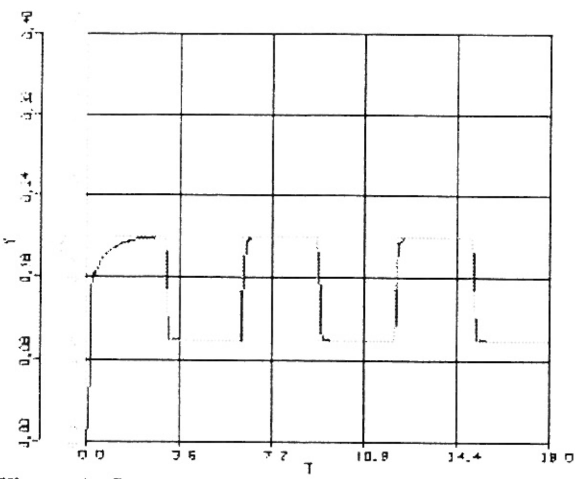

Figure 1a Step responses with velocity-based inverse controller

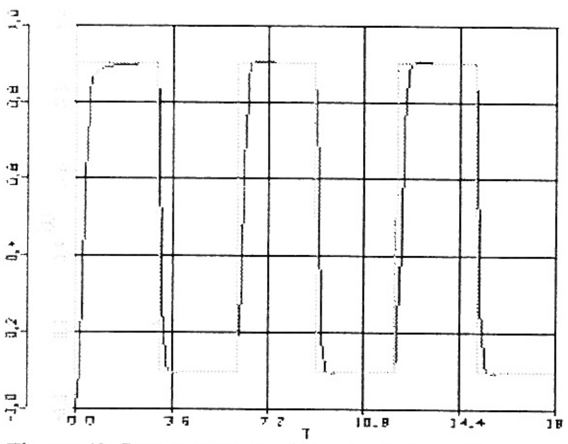

Figure 1b Step responses with velocity-based inverse controller

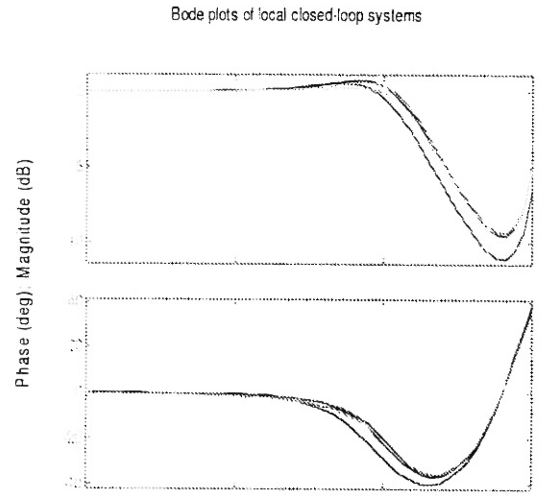

Figure 2 Bode plots of local closed-loop systems (local controlled plus linear part of corresponding local model).

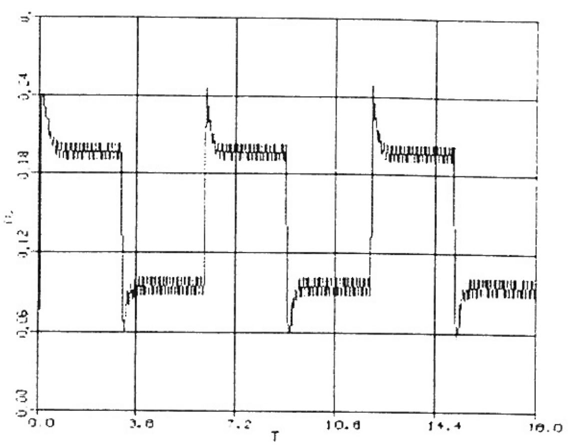

Figure 3 Step response of closed-loop LMN/LCN system (no measurement noise).

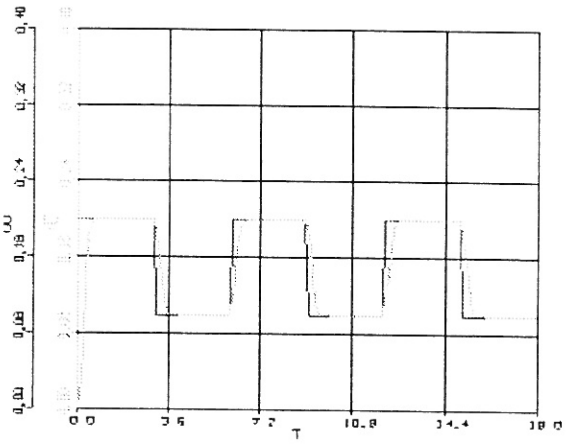

Figure 4 Step response of modified LMN/LCN system.

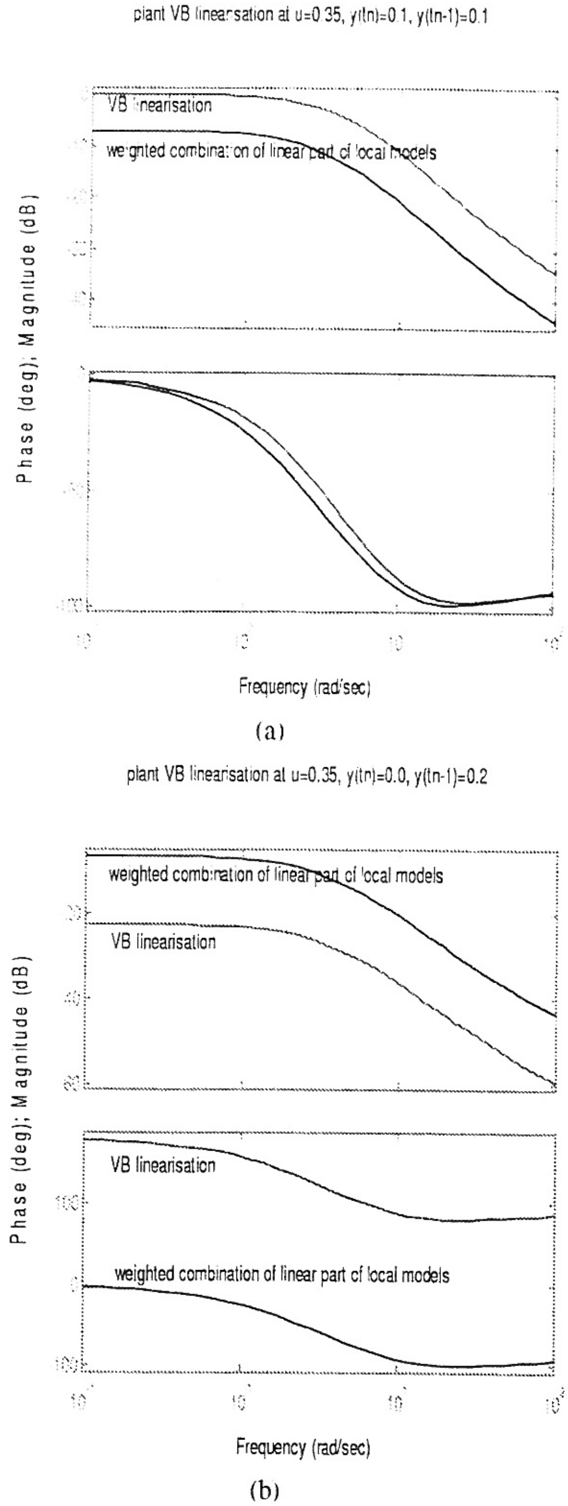

Figure 5 Comparison of typical transfer functions of (a) linearisations at an equilibrium point and (b) linear components of first-order expansions at an off-equilibrium point.