MULTIPLE BLENDED CONTROLLER DESIGN FOR BILINEAR SYSTEMS

Juš Kocijan *,** Damir Vrančić **

* *Faculty of Electrical Engineering*
University of Ljubljana
Tržaška 25, 1000 Ljubljana
Slovenia
Tel. +386 61 1768 417, Fax. +386 61 1264 631
Email. jus.kocijan@fe.uni-lj.si
** *Jozef Stefan Institute*
Jamova 39, 1000 Ljubljana
Slovenia

Abstract: In this paper a possible approach for designing multiple blended controller for bilinear systems is presented. Velocity-based linearisation framework for analysis of non-linear systems is used to assist control design. Due to this analysis tool the design of relatively simple controller is possible. The design procedure enables use of knowledge from linear systems control design and achieving desired performance not just in equilibrium points where system can be linearised but in whole operating region. The approach is illustrated by an example of combustion process control design.

Keywords: Bilinear systems, velocity-based linearisation, local area networks, combustion proces

1. INTRODUCTION

Many process systems, for example, heat exchangers, distillation columns, chemical reactors, waste water treatment plants, etc., can be better modelled and controlled if they are treated as bilinear systems. Bilinear systems control is in theory and practice usually pursued with adaptive techniques (e.g. (Cho and Marcus, 1987; Jin *et al.*, 1996)), though other approaches can be found in literature (e.g. (Kocijan, 1997)).

The purpose of this paper is to show a multiple blended control of bilinear systems as an alternative approach which is easy to understand and apply. The mentioned system can be analysed within velocity based framework. The multiple blended control approach is closely related to Takagi-Sugeno fuzzy controllers (Takagi and Sugeno, 1985). This paper is by no means intended to show superiority of one control approach over another, but more an efficient use of analysis tool which can facilitate control design for bilinear systems.

This paper is structured as follows. Next section summarises velocity-based linearisation framework and its application for multiple blended systems. Bilinear systems are analysed in Section 3. An example of simplified combustion process control to illustrate described approach is given in Section 4. Finally, Section 5 concludes this paper.

2. VELOCITY-BASED LINEARISATION

Multi model approach represents a possible solution to control of bilinear systems. Multi-model systems (Murray-Smith and Johansen, 1997; *Multiple model approaches to Modelling and Control*, 1999), as described within the framework of operating regimes and local models, are an efficient way to develop non-linear models which can be used for control design. The concept is based on a divide and conquer strategy. The operating range of interest is divided into a set of operating regimes, each with a local model associated with it. Non-linear systems described by local models are basis for multi-model control systems. Blended multi-model control systems (Leith and Leithead, 1999*a*) as commonly used type of multi-model control systems are closely related to the concept of gain scheduling. The concept of gain scheduling has long been used successfully and some theoretical justifications have also been reported (e.g. (Shamma and Athans, 1992)). However, classical gain-scheduling involves inherent restriction either to near equilibrium operation or to slowly-varying systems (Leith and Leithead, 1999*b*). A detailed description of this problem and a solution to it is reported in (Leith and Leithead, 1999*a*) where this kind of control is analysed within velocity-based linearisation framework.

The velocity-based linearisation framework (Leith and Leithead, 1998*a*; Leith and Leithead, 1998*b*) is proposed for the analysis and design of gain-scheduled and nonlinear systems. The framework associates a family of velocity-based linearisations with a nonlinear system. Each operating point of the nonlinear system, including operating points far from equilibrium, has an associated member of the velocity-based linearisation family which describes the dynamic characteristics in the vicinity of that operating point. In contrast to the conventional series expansion linearisation around an equilibrium operating point, the velocity-based linearisation family indicates the plant dynamics not only in the vicinity of a single equilibrium operating point but also during transitions between equilibrium operating points and when operating far from equilibrium.

In contrast to the previous approaches the velocity-based linearisation analysis offers several advantages. The family of velocity-based linearisations can be pieced together to approximate the solution to a nonlinear system so that stability, as well as the transient behaviour of the nonlinear system, can be investigated.

Given the direct relationship between the velocity-based form of the nonlinear systems and their associated velocity-based linearisation families (set of linearised systems where scheduling variable is replaced by the operating point) and the strong correspondence in their dynamic behaviour, the velocity-based linearisation families constitute a much more appropriate framework for the analysis and design of gain-scheduled controllers than conventional approaches (Leith and Leithead, 1998*a*; Kocijan and Murray-Smith, 1999).

The framework is thoroughly described in (Leith and Leithead, 1998*a*; Leith and Leithead, 1998*b*). Some formulations necessary for reader are summarised below.

Consider nonlinear single input single output plants with dynamics

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{f}(\rho) \\ y &= \mathbf{C}\mathbf{x} + Du + g(\rho) \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^n, u \in \mathcal{R}^m, y \in \mathcal{R}^p$ and $\mathbf{A}, \mathbf{B}, \mathbf{C}, D$ are appropriately dimensioned constant matrices, $\mathbf{f}(\cdot)$ and $g(\cdot)$ are nonlinear functions and $\rho(\mathbf{x}, u) \in \mathcal{R}^q, q \leq m+n$, embodies the nonlinear dependence of the dynamics on the state and input with $\nabla_{\mathbf{x}}\rho, \nabla_u\rho$ functions of ρ alone. This reformulation can always be achieved by letting $\rho = [\mathbf{x}^T u]^T$, in which case $q = m + n$. However, the nonlinearity of the system is frequently dependent on only a subset of the states and inputs, in which case the dimension, q of ρ is less than $m + n$.

The relationship between the nonlinear system and its velocity-based linearisation is direct. Differentiating (1), an alternative representation of the nonlinear system is (Leith and Leithead, 1998*a*)

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{w} \\ \dot{\mathbf{w}} &= (\mathbf{A} + \nabla\mathbf{f}(\rho)\nabla_{\mathbf{x}}\rho)\mathbf{w} + (\mathbf{B} + \nabla\mathbf{f}(\rho)\nabla_u\rho)\dot{u} \\ \dot{y} &= (\mathbf{C} + \nabla g(\rho)\nabla_{\mathbf{x}}\rho)\mathbf{w} + (D + \nabla g(\rho)\nabla_u\rho)\dot{u} \end{aligned} \tag{2}$$

Dynamically, expressions (1) and (2), with appropriate initial conditions, are equivalent (have the same solution $\mathbf{x}$)(Leith and Leithead, 1999*a*).

Instead of using the direct forms of the local models, consider constructing a blended multiple model system using the velocity-based forms; that is, consider the velocity-based blended multiple model system

$$\begin{aligned} \dot{\tilde{\mathbf{x}}} &= \tilde{\mathbf{w}} \\ \dot{\tilde{\mathbf{w}}} &= \left(\sum_i \phi_x^i \mu_i(\tilde{\rho})\right) \tilde{\mathbf{w}} + \left(\sum_i \phi_u^i \mu_i(\tilde{\rho})\right) \dot{u} \\ \dot{y} &= \left(\sum_i \gamma_x^i \mu_i(\tilde{\rho})\right) \tilde{\mathbf{w}} + \left(\sum_i \gamma_u^i \mu_i(\tilde{\rho})\right) \dot{u} \end{aligned} \tag{3}$$

where $\phi_x^i, \phi_u^i, \gamma_x^i$ and γ_u^i are constant matrices and $\mu_i(\tilde{\rho})$ represents weighting functions, called also blending, validity, interpolation or membership functions depending in which contexts they appear. This system is closely related to fuzzy system representation of Takagi and Sugeno (Takagi and Sugeno, 1985).

The corresponding local models, from which the blended multiple model system is constituted, are

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_i &= \bar{\mathbf{w}}_i \\ \dot{\hat{\mathbf{w}}}_i &= \phi_x^i \bar{\mathbf{w}}_i + \phi_u^i \dot{u} \\ \dot{\hat{y}}_i &= \gamma_x^i \bar{\mathbf{w}}_i + \gamma_u^i \dot{u} \end{aligned} \quad (4)$$

It should be noted that the local models are *linear*. Further details about velocity-based blended multiple model system can be found in (Leith and Leithead, 1999*a*).

The solution to a velocity-based blended multiple model system, locally to a specific operating point, is described by the solution to the linear system obtained by "freezing" the blended multiple model system at the relevant operating point. The resulting frozen system is simply a weighted linear combination of the local models. Direct relationship between the dynamics of a blended multiple model system and the local models suits very well for control design requirements.

An overview of available and also previously cited literature on multi model based control reveals that one meets two common approaches to control design. The first is when sub controllers' output signals are combined together (by blending or switching) and the second is when subcontrollers' parameters are combined within a single controller structure. A transparent proof in (Leith and Leithead, 1999*a*) shows that the second possibility is the one where the dynamics of overall blended system is directly related to the one of local models. The design procedure can be divided in the following steps (Leith and Leithead, 1999*a*).

- An appropriate velocity-based blended multiple model representation of the plant is determined.
- A velocity-based blended controller can be designed by first determining a linear controller for each of the local models in the approximate blended multiple model of the plant. Each of the controllers is designed to achieve required performance of the closed-loop system. These linear controllers are then interpolated with the same weighting functions as in the plant model to obtain a continuous controller velocity-based linearisation family. The velocity-based linearisations of the controller are simply the "frozen" forms of blended multiple controller.
- The controller is reformulated in the realisable form and implemented.

The closed-loop system properties, like stability and robustness, achieved in subsequent controller design, are valid throughout the operating envelope of the whole non-linear closed-loop system.

3. CONTROL DESIGN FOR SINGLE INPUT BILINEAR SYSTEMS

For the sake of simplicity, only the single input bilinear systems are considered in the current work.

Bilinear systems can be described by following equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_1\mathbf{x} + \mathbf{A}_2\mathbf{x}u + \mathbf{b}u \\ y &= \mathbf{c}\mathbf{x} \end{aligned} \quad (5)$$

which can be written as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_1\mathbf{x} + \mathbf{b}u + f(\rho) \\ y &= \mathbf{c}\mathbf{x} \end{aligned} \quad (6)$$

where

$$f(\rho) = \mathbf{A}_2\mathbf{x}u$$

and

$$\rho = \begin{bmatrix} \mathbf{x} \\ u \end{bmatrix}$$

Velocity-based equivalent to equation (6) is

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{w} \\ \dot{\mathbf{w}} &= \mathbf{A}'(\rho)\mathbf{w} + \mathbf{B}'(\rho)\dot{u} \\ \dot{y} &= \mathbf{C}\mathbf{w} \end{aligned} \quad (7)$$

where

$$\mathbf{A}' = \mathbf{A}_1 + \mathbf{A}_2 u$$

and

$$\mathbf{B}' = \mathbf{b} + \mathbf{A}_2\mathbf{x}$$

It can be seen from equation (6) that matrices $\mathbf{A}'$ and $\mathbf{B}'$ are linearly dependent on u and $\mathbf{x}$ respectively. This means that multiple blended model can be realised with minimal number of very basic triangularly shaped weighting functions.

If the worst case situation is taken, which means that none of the states is approximated with other, all linearly independent states and input are included in scheduling variable vector $\boldsymbol{\rho}$. If each element of weighting functions vector $\boldsymbol{\mu}(\rho)$ is represented with two symmetrical triangular functions then product $\boldsymbol{\mu}(\rho)$ multiplied with inputs or states of local models (in "frozen" operating point) can be written

$$\begin{bmatrix} u_i & \mathbf{x}_i \end{bmatrix} \boldsymbol{\mu} = \begin{bmatrix} u_i\mu_u \\ \mathbf{x}_i\boldsymbol{\mu}_x \end{bmatrix} \Rightarrow \begin{bmatrix} \mu_{10}u_{min} + \mu_{20}u_{max} \\ \mu_{11}x_{1min} + \mu_{21}x_{1max} \\ \mu_{12}x_{2min} + \mu_{22}x_{2max} \\ \vdots \\ \mu_{1n}x_{nmin} + \mu_{2n}x_{nmax} \end{bmatrix} \quad (8)$$

where $u_{min}, u_{max}, x_{imin}, x_{imax}$ denote minimal and maximal values of input and state values respectively. Since it holds that

$$\mu_{1i} + \mu_{2i} = 1 \tag{9}$$

relation (8) can be written as

$$[\, u_i \ \mathbf{x}_i \,]\, \boldsymbol{\mu}(\rho) \Rightarrow \begin{bmatrix} \mu_{10}u_{min} + (1-\mu_{10})u_{max} \\ \mu_{11}x_{1min} + (1-\mu_{11})x_{1max} \\ \mu_{12}x_{2min} + (1-\mu_{12})x_{2max} \\ \vdots \\ \mu_{1n}x_{nmin} + (1-\mu_{1n})x_{nmax} \end{bmatrix} \tag{10}$$

The state matrices are therefore

$$\begin{aligned} \mathbf{A}' &= \sum_{i=1,2} \phi_u^i \mu_i(\rho) \\ &= \sum_{i=1,2} (\mathbf{A}_1 + \mathbf{A}_2 u_i)\mu_i(\rho) \\ &= \mathbf{A}_1 + \mathbf{A}_2(\mu_{10}u_{min} + (1-\mu_{10})u_{max}) \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mathbf{B}' &= \sum_{i=1,2} \phi_u^i \mu_i(\rho) \\ &= \sum_{i=1,2} (\mathbf{b} + \mathbf{A}_2\mathbf{x}_i)\mu_i(\rho) \\ &= \mathbf{b} + \mathbf{A}_2 \begin{bmatrix} \mu_{11}x_{1min} + (1-\mu_{11})x_{1max} \\ \mu_{21}x_{2min} + (1-\mu_{21})x_{2max} \\ \vdots \\ \mu_{n1}x_{nmin} + (1-\mu_{n1})x_{nmax} \end{bmatrix} \end{aligned} \tag{12}$$

This is all together $n+1$ blending functions if no approximation is taken into account.

Linear compensatory controllers can be designed for the 'frozen' plant (local linear models) with blended parameter. This linear controller is realised with blended parameters and constitute a single non-linear controller.

Some care is required when realising the blended multiple model controller since the input is $\dot{e}$ rather than $e = r - y$. When the local models contain integral action the blended multiple model system can be realised in velocity-based form. The realisation of blended velocity-based systems, including situations where integral action is not present, is discussed in (Leith and Leithead, 1999*a*).

The closed-loop dynamics of the blended multiple model system are described simply by the weighted combination of the closed-loop local models. Results of closed-loop local models analyses extend to the blended closed-loop also in non-equilibrium points.

A question that arises is what happens if scheduling variable vector does not contain all the necessary variables due to intended simplification of system. The model that uses such scheduling variable vector does not describe the process well. Nevertheless, it could be used for a limited operating area. When this simplified scheduling variable vector can be used it should be determined on case to case basis.

The same is true for control design. If simplified scheduling variable vector is used, then adequately robust controller should be designed to successfully operate in non-equilibrium points.

4. EXAMPLE

For the illustration of described methodology a combustion process is used. Several methods have been proposed for the control of combustion processes. These methods range from robust control to fuzzy adaptive control. If the control algorithm is simple (e.g. some robust control algorithms) closed-loop performance depends strongly on operating point. If, on the other hand, equal performance is required in whole operating range, then more complex algorithms are usually used (e.g. various adaptive and predictive algorithms).

According to (Jiya *et al.*, 1999; Kocijan, 1997) a simplified combustion process model with one input (flow of air) and one output (oxygen concentration) can be described by the following equation:

$$\frac{\mathrm{d}x_{O_2}}{\mathrm{d}t} = \frac{1}{V_c}\{-x_{O_2}[\Phi_a + \Phi_f(V_g - V_0)] + 21\Phi_a - 100V_0\Phi_f\} \tag{13}$$

where x_{O_2} is the percentage of oxygen concentration (vol.%), V_c is the volume of the combustion chamber (m^3), Φ_f is the normalised total flow of fuel (kg/s), Φ_a is the normalised total flow of air (Nm^3/kg), V_0 is the theoretically required air volume for the combustion of one unit of fuel (Nm^3/kg), V_g is the theoretically obtained gas volume from one unit of fuel (Nm^3/kg).

Equation (13) can also be expressed in the following form:

$$\dot{x}(t) = A_1u(t)x(t) + A_2x(t) + Bu(t) + E \tag{14}$$

where

$x = x_{O_2}$,
$u = \Phi_a$,
$A_1 = -\frac{1}{V_c}$,
$A_2 = \frac{V_0 - V_g}{V_c}$
$B = \frac{21}{V_c}$,
$E = \frac{-100V_0}{V_c}\Phi_f$.

Equation (14) thus represents a first-order nonlinear model of the process. Parameters of the chosen process (Jiya *et al.*, 1999) are:

$A_1 = -0.02,$
$A_2 = 0.045,$
$B = 0.42,$
$E = -2$

It can be seen from equation (14) that one state and one input signal are necessary for controller design as described in the previous section.

Equation (14) in velocity-based representation is:

$$\begin{aligned} \dot{x} &= w \\ \dot{w} &= (A_2 + A_1 u)w + (b + A_1 x)\dot{u} \\ \dot{y} &= w \end{aligned} \qquad (15)$$

Appropriate controller for the first order system that contains integral action would be of proportional integral (PI) type. Local controllers, at points where μ_x and μ_u are 0 or 1, are designed so that the closed-loop performances are equal. When blended, the controllers should produce equal response in all operating region (in equilibrium points and out of them). Equations describing corresponding controller are:

$$\begin{aligned} \dot{\mathbf{x}}_c &= \mathbf{w}_c \\ \dot{\mathbf{w}}_c &= \begin{bmatrix} -100 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{w}_c + \begin{bmatrix} 100 \\ 0 \end{bmatrix} \dot{e} \\ \dot{u} &= \frac{K_P}{\beta} \begin{bmatrix} 1 & \alpha \end{bmatrix} \mathbf{w}_c \end{aligned} \qquad (16)$$

where

$$\alpha = -\mu_u(A_1 u_{min} + A_2) - (1 - \mu_u)(A_1 u_{max} + A_2)$$

$$\beta = \mu_x(A_1 x_{min} + b) + (1 - \mu_x)(A_1 x_{max} + b)$$

and K_P serves as tuning parameter for closed-loop bandwidth. Weighting (also blending or membership) functions μ_u and μ_x are shown in Figure 1.

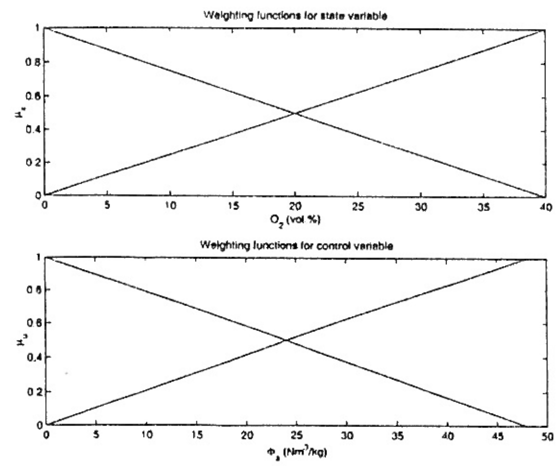

Fig. 1. Weighting functions for input and output signals

Closed-loop performance achieved by described design procedure was tested by computer simulation. The system closed-loop response with described controller is depicted in Figure 2 and associated control signal in Figure 3. It can be

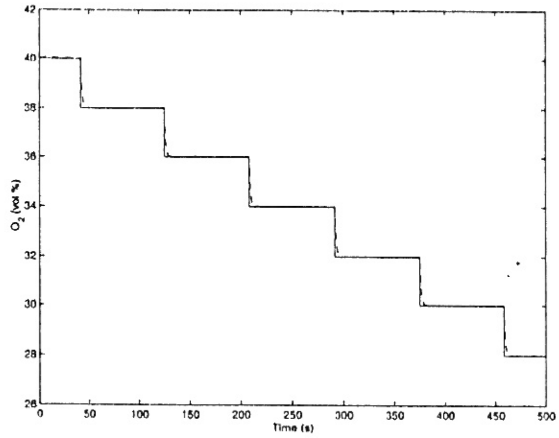

Fig. 2. Closed-loop response (dashed line) to a stair reference signal (full-line)

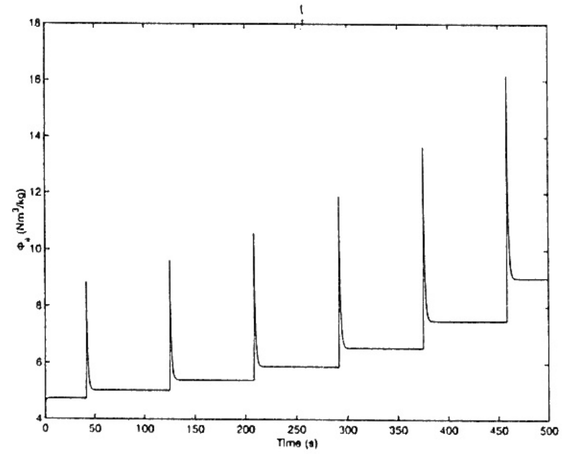

Fig. 3. Control signal to process

seen from Figure 2 that the closed-loop system performs equally well throughout the operating region. Control signal in Figure 3 reveals that process really has notable non-linear characteristics. It is necessary to remark that linear PI controller is not able to bring the closed-loop response even to vicinity of reference signal.

The situation when scheduling variable consist of variable x only, was also tested since this approach would simplify the controller structure. A relatively simple derivation of velocity-based linearised equations reveals that responses of higher order than those on Figure 2 are expected and that performance depends on operating point. The closed-loop response on Figure 4 confirms higher order responses while differences in response are not noticeable.

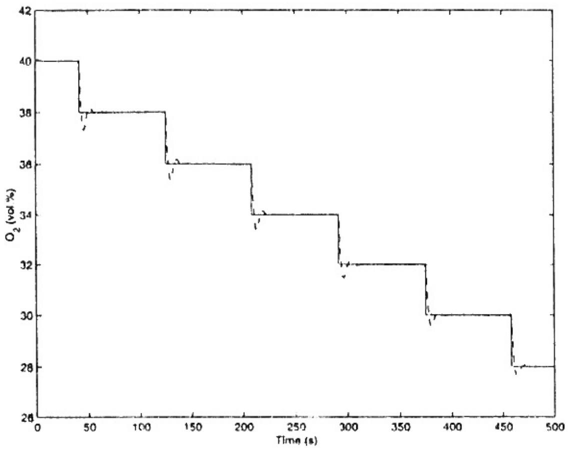

Fig. 4. Closed-loop response (dashed line) to a stair reference signal (full line) with controller using simplified scheduling variable vector

5. CONCLUSION

In this paper a possible approach for designing multiple blended controller for bilinear systems is presented. Though there exist several approaches for bilinear systems control and it does not represent unsolved control problem it is of notable interest that the control design of such systems can be facilitated with appropriate approach.

Velocity-based linearisation technique and blended controller approach was used in our case. Blended controllers are closely related to Takagi-Sugeno type of fuzzy controllers. This can especially be stated for controllers for bilinear systems. Velocity-based linearisation technique enables control design for non-linear systems where linear systems techniques and knowledge can be pursued. A care has to be exercised with selection of scheduling variable.

The approach was illustrated by one example in which control design for combustion process was pursued. It can be seen that described approach facilitated design procedure and resulted in controller complexity equal to the process complexity.

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