POLE PLACEMENT CONTROLLER DESIGN BASED ON FUZZY PROCESS MODEL

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Abstract: - An explicit self-tuning controller based on the Takagi-Sugeno fuzzy model of the process is proposed. The fuzzy model is represented as a linear regression model whose parameters are functions of some of the process variables. Such a model can be considered as a linear time-varying model whose parameter values are known at every moment. The pole placement design procedure modified for time-varying systems is applied to obtain the polynomial controller parameters that provide the desired closed-loop poles. The proposed algorithm is very simple, and thus suitable for on-line controller design in adaptive control systems.

Keywords: fuzzy control, fuzzy modeling, adaptive control.

1. INTRODUCTION

The representation of qualitative knowledge by fuzzy sets proposed by Zadeh (1965, 1973, 1975), and the application of the fuzzy set theory in control proposed by Mamdani (1974) have proved to be very successful in dealing with nonlinearities, parameter uncertainties and cases where it is difficult to obtain a useful mathematical model. Although the control strategies using fuzzy logic had been tested in practice, the lack of analytical tools for fuzzy system design was one of its main disadvantages for a long time.

Takagi and Sugeno (1985) proposed a process modeling strategy based on fuzzy logic in which the complex nonlinear process model is decomposed into several linear models by decomposing the variable space into subspaces and then approximating the process in each subspace by a simple linear regression model. This approach can be viewed as a means of using the linear system theory in the analysis and the control of nonlinear systems.

Tanaka and Sugeno (1992) introduced the method of analysis based on the Takagi-Sugeno fuzzy model in which sets of fuzzy rules are used to imply suitable local linear state space models from which local controllers can be determined. The stability of the overall system is determined by the Lyapunov stability analysis. The stability conditions require that for all local linear models a common positive-definite matrix should be found to satisfy the Lyapunov equation, and this is a very difficult problem.

Cao *et al.* (1995) suggested a way to avoid this problem. Their method is based on the linear uncertain system theory. In this method the stability analysis of a fuzzy control system is converted to the stability analysis of linear time-varying subsystems. Cao, *et al.* (1997) gave the necessary and sufficient conditions for the stabilization of the MIMO fuzzy control system and proposed the procedure for obtaining a stabilizing feedback control law based on the decomposition principle by which the design of a fuzzy discrete-time control system is decomposed into the design of "extreme" subsystems.

The methods proposed by Tanaka and Sugeno (1992) and Cao, *et al.* (1995, 1997) can be used in the analysis and design of a wide class of complex control systems. These methods can be used for off-line controller design, but they are not suitable for use as on-line controller design procedures in adaptive control because of their complexity.

Cupec *et al.* (1999a, b) proposed a simple controller design method for SISO systems, which is suitable for use in adaptive control. It is a version of the pole placement method (Åstrom and Wittenmark, 1995) modified for time-varying systems. The method is based on the Takagi-Sugeno input-output fuzzy model (Takagi and Sugeno, 1985), but it uses an approach that differs from the methods proposed in (Tanaka and Sugeno, 1992; Cao, *et al.*, 1995; Cao, *et al.*, 1997). Instead of designing local control laws for local linear models and then checking the stability of the overall system, in this method the overall fuzzy input-output model is considered as a linear regression model whose parameters are functions of some of the process variables. Such a model can be viewed as a linear time-varying model whose parameter values are known at every moment. In every step the parameters of the polynomial controller are obtained using the pole placement design procedure with cancellation of all process zeros, modified for time-varying processes with known parameters. In this way the closed-loop system behavior matches the desired dynamics. Cupec *et al.* (1999a, b) have proved the stability of the system assuming that the following conditions are fulfilled:

1) There is no time delay in the process;
2) All zeros of the discrete time process are time-invariant;
3) All zeros of the discrete time process are inside the unit circle.

In this paper it is shown that all the three conditions can be avoided using the modified pole placement procedure without zero cancellation, assuming that the premise variable is not correlated with either the control signal or the process output.

2. FUZZY PROCESS MODEL

Let us assume that the fuzzy model, proposed by Takagi and Sugeno (1985), and later expanded by Sugeno and Kang (1988) and Sugeno and Tanaka (1991) is used for description of the dynamic behavior of the process. The fuzzy model is composed of local linear models using the following inference rules:

$$R^i\text{: } \textbf{IF } \left[x_1(k) \text{ is } F_1^i\right] \textbf{ AND} \ldots \textbf{AND } \left[x_{nx}(k) \text{ is } F_{nx}^i\right]$$

$$\textbf{THEN } y^i(k+d+1) = -\sum_{j=1}^{na} a_j^i y_0(k-j+d+1) + \sum_{j=1}^{nb} b_j^i u(k-j+1) + c^i, \quad i = 1, \ldots, nr, \qquad (1)$$

where R^i denotes ith inference rule; x_j is the jth variable of the premises; F_j^i denotes the fuzzy set defined on the universe of discourse of the variable x_j, used in the ith inference rule; y^i output of the ith local model; y output of the model; u input of the model; a_j^i, b_j^i, c^i parameters of the ith local model (consequence parameters) and d the process dead time. The rules of the model consist of the premise which can have more than one variable and the consequence which is a local linear SISO model. The premise variables x_j can be the values of the output y, the input u at past time instants, or some other signals. Given the values of premise variables $x_j(k)$, the final output of the fuzzy process model is inferred by taking the weighted average of the local model outputs y^i:

$$y(k+d+1) = \sum_{i=1}^{nr} v^i(k)\left\{-\sum_{j=1}^{na} a_j^i y(k-j+d+1) + \sum_{j=1}^{nb} b_j^i u(k-j+1) + c^i\right\}, \qquad (2)$$

where

$$v^i(k) = \frac{\prod_{j=1}^{nx} \mu_j^i\left[x_j(k)\right]}{\sum_{i=1}^{nr}\prod_{j=1}^{nx} \mu_j^i\left[x_j(k)\right]}, \qquad (3)$$

μ_j^i - a membership function of the fuzzy set F_j^i,

nr - number of fuzzy rules.

Using the shift operator q^{-1}, the model (2) can be expressed in the transfer-function form:

$$\mathbf{A}(q^{-1},k)y(k+d+1) = \mathbf{B}(q^{-1},k)u(k) + c(k), \qquad (4)$$

where

$$\begin{aligned} \mathbf{A}(q^{-1},k) &= 1 + \sum_{j=1}^{na} a_j(k)q^{-j}, \\ \mathbf{B}(q^{-1},k) &= \sum_{j=1}^{nb} b_j(k)q^{-j+1}, \\ a_j(k) &\triangleq \sum_{i=1}^{nr} a_j^i v^i(k), \\ b_j(k) &\triangleq \sum_{i=1}^{nr} b_j^i v^i(k), \\ c(k) &\triangleq \sum_{i=1}^{nr} c^i v^i(k). \end{aligned} \qquad (5)$$

Equation (4) is the representation of the fuzzy model (1) in the form of a linear time-varying model. The fuzzy process model (4) can be presented as a linear regression:

$$y(k+1) = \left[\mathbf{m}^*(k)\right]^T \mathbf{p},$$

where:

$$\mathbf{m}^*(k) = \left[v^1(k)\mathbf{m}^T(k) \quad \cdots \quad v^{nr}(k)\mathbf{m}^T(k)\right]^T$$

$$\mathbf{m}(k) = [y(k)\ y(k-1)\ \cdots\ u(k)\ u(k-1)\ \cdots\ 1]^T$$

$$\mathbf{p} = \begin{bmatrix} a_1^1 & a_2^1 & \cdots & b_1^1 & b_2^1 & \cdots & c^1 \\ a_1^2 & a_2^2 & \cdots & b_1^2 & b_2^2 & \cdots & c^2 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ a_1^{nr} & a_2^{nr} & \cdots & b_1^{nr} & b_2^{nr} & \cdots & c^{nr} \end{bmatrix}.$$

The process model parameters a_j^i, b_j^i and c^i can be obtained using the recursive least-squares method (Takagi and Sugeno, 1985).

3. CONTROLLER DESIGN

In section 2 the fuzzy process model is presented as a linear time-varying model whose parameters are functions of process variables which can be measured or estimated. The basic idea of the pole placement design can be applied to determine a control law for the control of the process described by (4). The control law can be described by

$$\begin{aligned} \mathbf{R}(q^{-1},k)u(k) = -\mathbf{S}(q^{-1},k)y(k) + \\ +\mathbf{T}(q^{-1},k)y_r(k) - p(k). \end{aligned} \quad (6)$$

To accommodate process parameter variations, the controller parameters, i.e. the coefficients of the polynomials **R**, **S** and **T** should also be time-varying. The term p is used for the compensation of the offset c variations. The block diagram of the closed-loop system is shown in Fig. 1. It can be proven that equations (4) and (6) give the closed-loop system description:

$$\begin{aligned} &\{[\mathbf{R}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)]q^d + \\ &+\mathbf{B}(q^{-1},k)\mathbf{S}(q^{-1},k)q^{-1}\}y(k+1) = \\ &= \mathbf{B}(q^{-1},k)\mathbf{T}(q^{-1},k)y_r(k) - \mathbf{B}(q^{-1},k)p(k) + \\ &+\mathbf{R}(q^{-1},k)c(k) + \xi(k), \end{aligned} \quad (7)$$

where

$$\xi(k) = \mathbf{W}_1(k)u(k) + \mathbf{W}_2(k)y(k) - \mathbf{W}_3(k)y_r(k),$$

$$\mathbf{W}_1(k) = \sum_{i=0}^{nr}\sum_{j=1}^{nb}\left[r_i(k)b_j(k-i) - b_j(k)r_i(k-j+1)\right]q^{-i-j+1},$$

$$\mathbf{W}_2(k) = \sum_{i=2}^{nb}\sum_{j=0}^{ns} b_i(k)\left[s_j(k) - s_j(k-i+1)\right]q^{-i-j+1},$$

$$\mathbf{W}_3(k) = \sum_{i=2}^{nb}\sum_{j=0}^{nt} b_i(k)\left[t_j(k) - t_j(k-i+1)\right]q^{-i-j+1}.$$

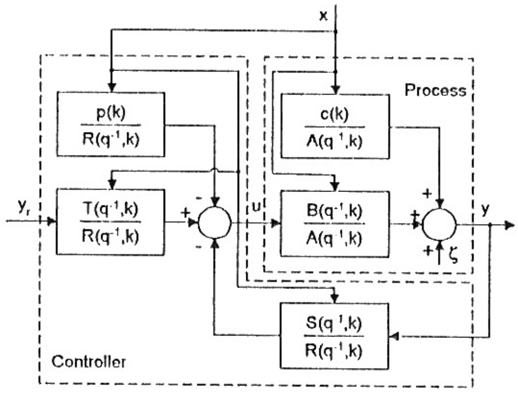

Fig. 1. A time-varying control system.

In equation (7) r, s and t denote the coefficients of the polynomials **R**, **S** and **T**, respectively, and "∘" denotes the operation given by the following definition:

Definition 1: Let **P** denote the set of all polynomials in the shift operator q^{-1} with time-varying coefficients, and let:

$$\mathbf{G}(q^{-1},k) = \sum_{j=0}^{ng} g_j(k)q^{-j} \qquad \text{and}$$

$$\mathbf{H}(q^{-1},k) = \sum_{j=0}^{nh} h_j(k)q^{-j}$$

be the elements of the set **P**. Operation $\circ : \mathbf{P}\times\mathbf{P}\rightarrow\mathbf{P}$ is defined by:

$$\mathbf{G}(q^{-1},k)\circ\mathbf{H}(q^{-1},k) = \sum_{i=0}^{ng}\sum_{j=0}^{nh} g_i(k)h_j(k-i)q^{-(i+j)}$$

Let the desired closed-loop system be described by

$$\mathbf{A}_M(q^{-1})y(k+d+1) = \mathbf{B}_M(q^{-1},k)y_r(k), \quad (8)$$

where

$$\mathbf{B}_M(q^{-1},k) = \mathbf{B}'(q^{-1},k)\mathbf{B}'_M(q^{-1}), \quad (9)$$

$$\mathbf{B}(q^{-1},k) = b_1(k)\mathbf{B}'(q^{-1},k).$$

The polynomial $\mathbf{B}'$ is a monic polynomial that contains all process zeros. Since the idea of the algorithm is not to cancel the process zeros, the desired closed-loop dynamics should include the process zeros, as stated in equation (9). As suggested by (7), to obtain the system with the dynamics (8), the following equations must hold:

$$
\begin{aligned}
&\mathbf{R}(q^{-1},k)\circ\mathbf{A}(q^{-1},k)+\\
&+\mathbf{B}(q^{-1},k)\mathbf{S}(q^{-1},k)q^{-(d+1)}=\\
&=\mathbf{A}_M(q^{-1})\mathbf{A}_O(q^{-1}),
\end{aligned}
\qquad (10)
$$

$$
\mathbf{B}(q^{-1})\mathbf{T}(q^{-1})=\mathbf{A}_O(q^{-1})\mathbf{B}_M(q^{-1}), \qquad (11)
$$

$$
-\mathbf{B}(q^{-1},k)p(k)+\mathbf{R}(q^{-1},k)c(k)=0, \qquad (12)
$$

where $\mathbf{A}_o$ is the so-called observer polynomial (Åstrom and Wittenmark, 1995). Equation (10) corresponds to the Diophantine equation of the time-invariant system. Equation (12) states that the purpose of the term p in the control law (6) is to compensate the effect of the change of the offset c. Full compensation can easily be obtained in the case when

$$
\mathbf{B}(q^{-1},k)=b_1(k).
$$

Then

$$
p(k)=\mathbf{R}(q^{-1},k)p^*(k), \qquad (13)
$$

$$
p^*(k)=\frac{c(k)}{b_1(k)}.
$$

In general, the exact fulfillment of (12) significantly complicates the controller structure. Partial compensation can be obtained by

$$
p^*(k)=\frac{c(k)}{\sum_{i=1}^{nb}b_i(k)}. \qquad (14)
$$

Then the closed-loop system is described by

$$
\begin{aligned}
&\mathbf{A}_M(q^{-1})\mathbf{A}_O(q^{-1})y(k+d+1)=\\
&=\mathbf{B}_M(q^{-1},k)\mathbf{A}_O(q^{-1})y_r(k)+\xi^*(k),
\end{aligned}
\qquad (15)
$$

$$
\xi^*(k)=\xi(k)-\tau(k). \qquad (16)
$$

It can be shown that the term τ in (16) is given by

$$
\begin{aligned}
\tau(k)=\sum_{i=1}^{nb}\sum_{j=0}^{m}\Big[&b_i(k)r_j(k-i+1)p^*(k-i-j+1)-\\
&-r_j(k)b_i(k-j)p^*(k-j)\Big]
\end{aligned}
$$

The coefficients of the polynomials $\mathbf{A}$ and $\mathbf{B}$ and coefficient c are functions of the premise variables x. The coefficients of the polynomials $\mathbf{R}$, $\mathbf{S}$ and $\mathbf{T}$ and the term p are calculated using the coefficients of the polynomials $\mathbf{A}$ and $\mathbf{B}$ and the offset c; hence they are also the functions of the premise variables x. Assuming that the premise variables x are not correlated with the process output y, the process input u and the referent value y_r, the polynomials $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_3$ in equation (7) and the term τ in equation (16) are not correlated with y, u and y_r either. Thus, variable ξ^* is likewise not correlated with y, u and y_r, and consequently, it can be considered as a disturbance signal. The calculation of the described pole placement controller can be implemented in five steps:

Step 1: Determine the coefficients $a_i(k)$, $b_i(k)$ and $c(k)$ using membership functions obtained by the fuzzyfication procedure and formulas (3) and (5).
Step 2: Choose the polynomials $\mathbf{A}_M$, $\mathbf{B}_M$ and $\mathbf{A}_o$.
Step 3: Solve equation (10) to obtain the coefficients of the $\mathbf{R}$ and $\mathbf{S}$ polynomials.
Step 4: Calculate the coefficients of the polynomial $\mathbf{T}$ using equation (11).
Step 5: Calculate the term p using equations (13) and (14).

In the above design procedure it is assumed that the values of the membership functions are known, which means that the membership functions do not depend on the current control signal value $u(k)$. Otherwise, the procedure is much more complicated. Instead of the numeric values of coefficients $a_i(k)$, $b_i(k)$ and $c(k)$, the expressions $a_i[u(k),k]$, $b_i[u(k),k]$ and $c[u(k),k]$ have to be used in all steps of the design procedure.

4. CASE STUDY: A LABORATORY LIQUID LEVEL RIG

The proposed controller is applied in the adaptive control of a laboratory liquid level rig shown in Fig. 2.

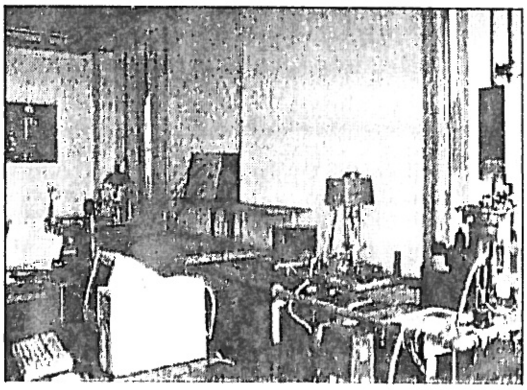

Fig. 2. Laboratory liquid level rig.

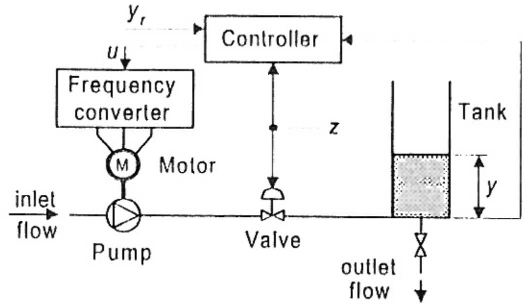

Fig. 3. Scheme of control system.

A pump with a squirrel cage induction motor controlled by a frequency converter provides the water flow. The control signal u is the frequency converter reference. The scheme of the control system is shown in Fig. 3.

The control system has been implemented using the MATLAB/SIMULINK program package and its Real-Time Workshop.

The fuzzy process model consists of several linear models that describe the process dynamics in different parts of the operating range.

The structure of the process model is chosen based on preliminary knowledge of the process. According to this knowledge, the variable that influences the process dynamics the most is the control valve opening. The process dynamics is approximated with a second order transfer function.

The operating range of the valve is divided in three areas that correspond to three fuzzy sets shown in Fig. 4. The membership functions have a triangular shape.

Every fuzzy set corresponds to a linear model that describes the process dynamics for different values of the valve opening. Thus, the fuzzy model is given by the following rules:

$$\begin{aligned} &R^1\text{: }\mathbf{IF}\ \left[z(k)\text{ is }F^1\right]\ \mathbf{THEN} \\ &\qquad y^1(k+1)=a_1^1y(k)+a_2^1y(k-1)+b_1^1u(k)+c^1 \\ &R^2\text{: }\mathbf{IF}\ \left[z(k)\text{ is }F^2\right]\ \mathbf{THEN} \qquad (17) \\ &\qquad y^2(k+1)=a_1^2y(k)+a_2^2y(k-1)+b_1^2u(k)+c^2 \\ &R^3\text{: }\mathbf{IF}\ \left[z(k)\text{ is }F^3\right]\ \mathbf{THEN} \\ &\qquad y^3(k+1)=a_1^3y(k)+a_2^3y(k-1)+b_1^3u(k)+c^3. \end{aligned}$$

The performance of the self-tuning control based on fuzzy process model is compared with the performance of the self-tuning control based on

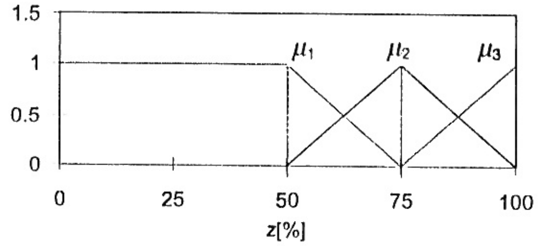

Fig. 4. The membership functions of the premise fuzzy sets.

linear process model given by the following equation:

$$y(k+1)=a_1y(k)+a_2y(k-1)+b_1u(k)+cz(k)\,.\quad(18)$$

The effect of the measurable disturbance signal is modeled with the last term of equation (18). The performance of the self-tuning control with linear process model is illustrated by Fig. 5. As can be seen, the effect of the disturbance on the closed-loop response of the system is significant. This is a consequence of the fact that changes of the disturbance signal cause changes in process parameters. Because the changes of the process parameters are large and frequent, the parameter estimation algorithm cannot follow them, resulting in inaccurate values of the model parameters.

The performance of the self-tuning controller with fuzzy process model is illustrated by Fig. 6. The system behavior is obviously much better than that obtained with the self-tuning controller based on linear model. The compensation of the disturbance is much better and the responses to the step changes of the reference signal y_r are without overshoots for any value of the valve opening in the considered range.

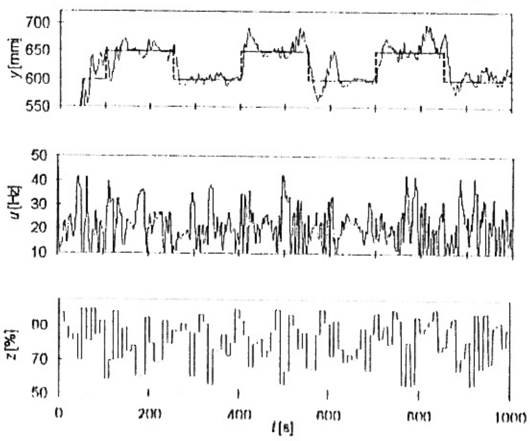

Fig. 5. The performance of the self-tuning controller with linear process model.

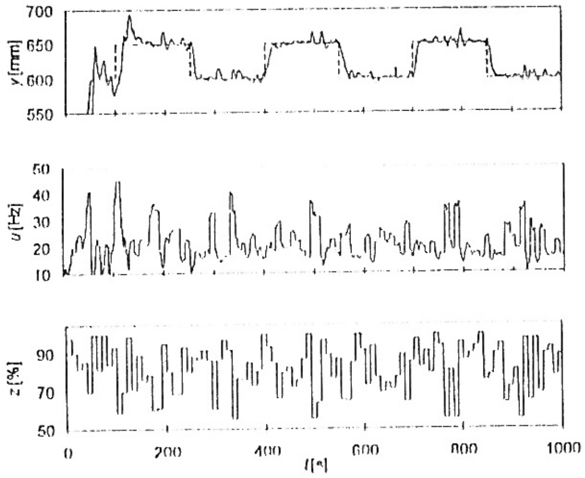

Fig. 6. The performance of the self-tuning controller with fuzzy process model.

Additionally, it can be seen that the control signal u is less active when the self-tuning controller with fuzzy model is used (see Fig. 6) than when the self-tuning controller with linear model is used (see Fig. 5). This feature is of great importance in real-world applications.

5. CONCLUSION

A self-tuning controller based on the Takagi-Sugeno fuzzy model of the process is proposed. The fuzzy model is represented as a linear time-varying model and the pole placement design procedure modified for time-varying systems is applied to obtain the desired closed-loop poles. Because of its simplicity, the proposed method is suitable for on-line controller design in adaptive control systems.

The proposed method is experimentally tested on the laboratory liquid level rig. The structure of the process model is chosen based on preliminary knowledge of the process. Although several process variables influence the process parameters, only one - the valve opening - is chosen for the construction of fuzzy rules. Only three fuzzy sets are defined in the operating range of the valve, and their shapes are chosen to be triangular, i.e. the simplest possible. Even with this simple process model, very good results are obtained.

It is shown that the system behavior obtained with the proposed controller is much better than that obtained with the self-tuning controller based on linear process model. The main advantage of the fuzzy model over the linear model is that the model parameters do not have to be modified as the operating area changes.

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