TRANSPARENT FUZZY SYSTEMS AND MODELLING WITH TRANSPARENCY PROTECTION

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Abstract: Fuzzy systems have gained enormous popularity in numerous applications capable of handling uncertainty and complexity of the real world in contrast to conventional techniques. It is the dual nature of fuzzy systems that makes them attractive for modelling and control purposes - in addition to good approximation capabilities, the information saved into the fuzzy system can be processed in linguistic terms, much similar to human reasoning. Interpretability that derives from the use of linguistic rulebase is, however, interconnected with the transparency of the fuzzy system, and the latter not being the default property of fuzzy systems, is often ignored by the developers. This paper defines the transparency conditions for standard fuzzy systems and describes the techniques available for obtaining transparent fuzzy systems from experimental data along with examples and comments.

Keywords: Fuzzy systems, modelling, linguistic variables.

1. INTRODUCTION

The first fuzzy controllers were obtained by translating human actions into the language of fuzzy systems (Mamdani and Assiliani, 1975). Though this approach has produced many successful applications, there is no well-defined methodology for obtaining such controllers. Algorithms utilising input-output measurements for adjusting fuzzy system parameters (most notably ANFIS (Jang, 1993)) have aroused significant interest lately. Besides the approximation properties of such algorithms, usually little attention is paid to the transparency of the trained system. This, however, can be considered a serious drawback.

Transparency and interpretability, as discussed here, are not matching terms. Standard fuzzy systems are interpreted by using natural language and qualitative terms, while first-order Sugeno-Takagi systems can be interpreted in terms of the local linear models of the system (Babuška, 1997). Such interpretation is valid, however, only if the fuzzy system is transparent (section 3). Sugeno-Takagi systems which are widely used for training algorithm implementation, usually employ smooth (Gaussian, generalised bell) input membership functions (MFs). System transparency is protected by the MF parameter constraints that in the case of smooth functions cannot be established reliably. This allows us to define transparency constraints (section 4) for standard (Mamdani) fuzzy systems with triangular MFs.

2. SYSTEM DEFINITION

We observe multi input/multi output (MISO) standard fuzzy systems with triangular membership functions, employing centroid defuzzification given by the rulebase, the r^{th} rule of which ($r = 1 \ldots R$) determines the linguistic relationship between the inputs U_i ($i = 1 \ldots N$) and the output V of the system via their linguistic labels A_{ir}, B_r.

IF U_1 is A_{1r} AND U_2 is A_{2r} … AND U_i is A_{ir} … AND U_N is A_{Nr} THEN V is B_r

Fuzzy output of the system is computed by the inference algorithm, consisting of fuzzification, conjunction, implication, and aggregation

$$F(y)=\bigcup_{r=1}^{R}\left(\left(\bigcap_{i=1}^{N}\mu_{ir}(x_i)\right)\cap\gamma_r\right) \qquad (1)$$

where μ_{ir} and γ_r denote the MFs of the i^{th} input variable and the output associated with the r^{th} rule, respectively; x_i denotes the numerical value of the i^{th} input variable, and $\cap$, $\cup$ denote the operators called t-norm and t-conorm, respectively.

Fuzzy output of the system (1) is thereafter defuzzified with centre-of-gravity defuzzification.

$$y=\frac{\int_{y_{min}}^{y_{max}} yF(y)dy}{\int_{y_{min}}^{y_{max}} F(y)dy} \qquad (2)$$

3. SYSTEM TRANSPARENCY

Definition: r^{th} rule of the standard MISO fuzzy system (2) is transparent if it's activation degree

$$\tau_r=\bigcap_{i=1}^{N}\mu_{ir}(x_i)=1, \qquad (3)$$

results in the system output

$$y=b_r, \qquad (4)$$

where b_r is the centre of the output MF γ_r associated with the activated rule.

Obviously, $\tau_r=1$ only if $\forall x_i=b_{ir}$, where b_{ir} is the centre of the MF of the i^{th} input associated with the r^{th} rule.

System (2) is transparent only if all its rules are transparent. For a SISO standard system such

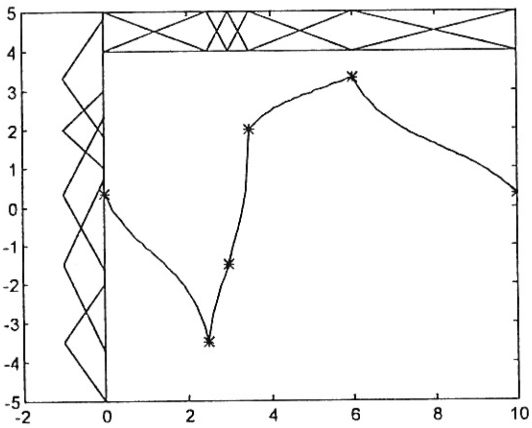

Fig. 1. Transparent fuzzy system.

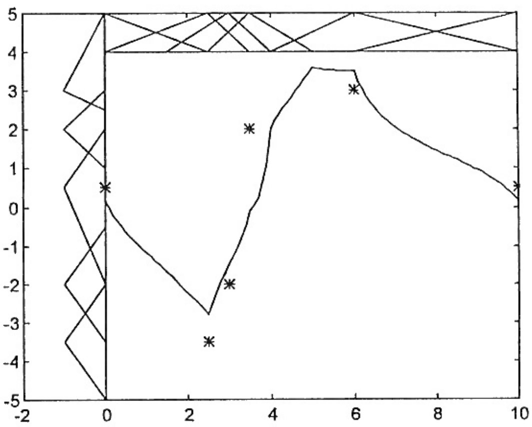

Fig. 2. Non-transparent fuzzy system.

situation is depicted in Fig. 1, where the asterisks denote transparency check points (3-4). Transparency is not the default property of a fuzzy system and non-transparent fuzzy system is a common result without constraints to MFs (Fig. 2).

4. HOW TO PRESERVE TRANSPARENCY

System transparency can be protected by the constraints applied to MFs. The constraints for input and output MFs are observed separately.

4.1 Inputs

The i^{th} input of the system (2) is partitioned into n_i triangular fuzzy sets (Fig. 3)

$$\mu_i^j(x_i)=\max\left(\min\left(\frac{x_i-a_i^j}{b_i^j-a_i^j},\frac{c_i^j-x_i}{c_i^j-b_i^j}\right),0\right), \qquad (5)$$
$$j=1,\dots,n_i$$

System transparency is protected if

$$c_i^{j-1}\le b_i^j\le a_i^{j+1}, \qquad (6)$$
$$i=1,\dots,N,\ j=2,\dots,n_i-1.$$

For the sake of convenience, strict condition

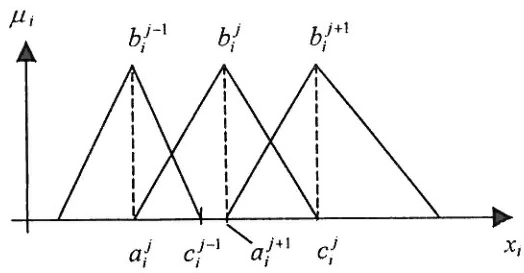

Fig. 3. Partition of the input(s).

$$c_i^{j-1} = b_i^j = a_i^{j+1}, \tag{7}$$

can be used.

4.2 Outputs

System (2) output transparency is protected if the centre of gravity of each output function γ_r equals its centre b_r:

$$\frac{\int_{y_{min}}^{y_{max}} y\gamma_r(y)dy}{\int_{y_{min}}^{y_{max}} \gamma_r(y)dy} = b_r \tag{8}$$

(8) is satisfied by

a) symmetric triangular membership functions that are determined by two parameters - centre b^j and spread $d^j > 0$ ($j = 1,\dots,m$)

$$\gamma^j(y) = \max\left(\min\left(\frac{2y - 2b^j + d^j}{d^j}, \frac{2b^j + d^j - 2y}{d^j}\right), 0\right) \tag{9}$$

b) singleton output functions (i.e. crisp numbers)

- A standard fuzzy system with singleton output functions can be considered equivalent to a zero-order Sugeno system. Transparency analysis of Sugeno systems in general, however, remains outside the scope of the present paper.
- Using mean-of-maxima defuzzification instead of centre-of-gravity method preserves output transparency, regardless of the type of output membership functions because $\max(\gamma_r(y)) = b_r$.

5. ROLE OF INFERENCE OPERATORS

The inference parameters commonly used (minimum, algebraic product, maximum, algebraic sum) do not distort system transparency, rather determine how the system output is interpolated in regions where $\tau_r < 1$.

The most nonlinear system is obtained with min-min-max inference (Fig. 4) (first min stands for minimum conjunction, second for minimum implication and max for maximum aggregation, respectively).

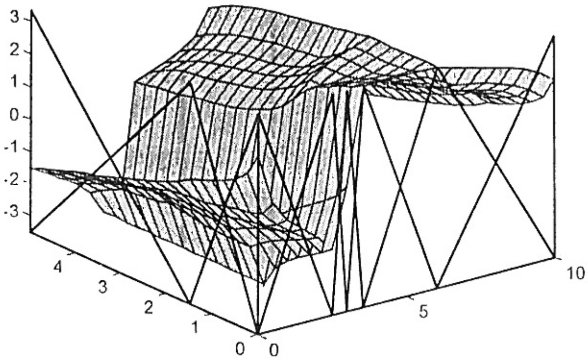

Fig. 4. 2-input/single output min-min-max system.

With the singleton model

a) output membership functions are crisp, thus $\tau_r \cdot \gamma_r \equiv \min(\tau_r, \gamma_r)$

b) $\sum_{r=1}^{R} \tau_r \cdot \gamma_r \equiv \max(\tau_1 \cdot \gamma_1, \dots, \tau_R \cdot \gamma_R)$ if output membership functions γ_r do not match, i.e. in most cases.

Thus, the nature of interpolation in the case of singleton model is basically determined by the conjunction operator. When the latter is product we obtain the system that resembles a piecewise linear system (Fig. 5). With minimum conjunction operator non-linearity is increased.

6. LEARNING ALGORITHMS THAT PROTECT TRANSPARENCY OF FUZZY SYSTEMS

Despite insufficient attention paid to the transparency of fuzzy systems, methods for training transparent fuzzy systems are available or can be obtained by a simple modification of the existing ones. This is due to two factors.

In engineering practice, the amount of overlap between fuzzy MFs is noted as "an important factor affecting accuracy". A minimum of 25% and a

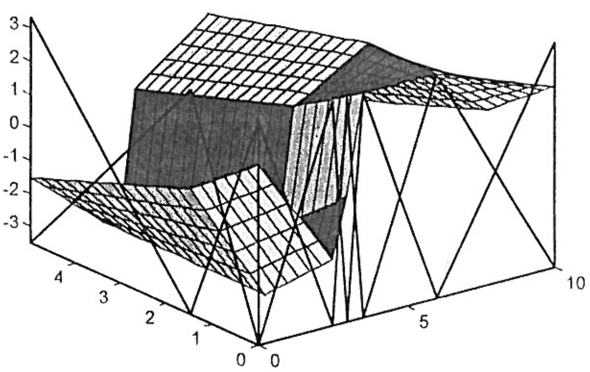

Fig. 5. 2-input/single output prod-prod-sum system.

maximum of 75% have been established experimentally (Shaw, 1998). Frequently, 50% overlap is a reasonable compromise. Via the latter, condition (7) is satisfied.

Singleton output functions that make the optimisation of the system easier, are very popular. So, it is the engineering experience in combination with the limits of mathematics that leads to the algorithms described below.

6.1 Least squares estimation

Least squares (LS) method is a procedure for determining the output singletons from experimental data for a given input partition (Passino and Yurkovich, 1998). Prod implication and sum aggregation for the model to be identified are the required operators, that, along with weighted average defuzzification (to what centre-of-gravity method transforms with output singletons), results in the system:

$$y = \sum_{r=1}^{R} b_r \tau_r \Big/ \sum_{r=1}^{R} \tau_r \qquad (10)$$

or

$$y = \Phi\theta , \qquad (11)$$

where

$$\theta = [b_1, b_2, ..., b_R]^T \; \Phi = [\phi_1, \phi_2, ..., \phi_R],$$

$$\phi_r = \tau_r \Big/ \sum_{r=1}^{R} \tau_r ,$$

from where the output singletons are computed as

$$\theta = [\Phi^T \Phi]^{-1} \Phi^T y . \qquad (12)$$

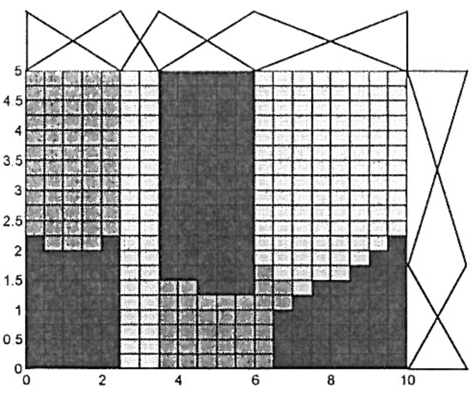

Fig. 6. Input membership functions derived form GK clusters.

In combination with Gustafson-Kessel (GK) clustering for the definition of the input partition, we obtain the tool (Babuška, 1997) for identifying the complete system. The centres of input membership functions are selected so that they coincide with the lines where the surfaces of clusters approximately intersect (Fig. 6).

Input constraint (7) is applied here by default, output MFs satisfy (8), and thus the resulting system is transparent.

6.2 Gradient descent

Gradient descent (GD) adaptation methods, based on the minimisation of the error function,

$$\varepsilon = \frac{1}{2}[y - \tilde{y}]^2 , \qquad (13)$$

where y denotes the output of the model and $\tilde{y}$ is the reference output, have been applied to the fuzzy systems by several authors.

To minimise the error through the modifications of the MF parameters, differential calculus is used, provided that the error equation (13) operates on differentiable equations.

$$\Delta c_{ir} = -\alpha \frac{\partial \varepsilon}{\partial c_{ir}} , \qquad (14)$$

where c_{ir} is the updated parameter and α is the learning rate.

With triangular MFs special attention must be paid to the points where the derivative does not exist. Reader may refer to (Passino and Yurkovich, 1998) for a detailed description of the algorithm derivation procedure and specifically to (Jager, 1995) or (Riid and Rüstern, 2000) for the algorithm for training the system with triangular MFs. Transparent system is obtained then by the Jager approach (Jager, 1995).

6.3 Genetic algorithms

Both of the described approaches limit the selection of inference operators and types of MFs. In case we do not want to restrict ourselves to singleton systems and product-implication/sum-aggregation inference, genetic algorithms may be a solution.

Genetic algorithms (GAs) are stochastic search techniques that operate without knowledge of the task domain, utilising only the fitness of the evaluated individuals (Whitley, 1993), (Ortega and Giron-Sierra, 1998). It is because of the latter why the type of inference and/or membership functions can be freely selected.

A standard GA is characterised by a number of parameters: population size N, maximum number of generations M, the probability of crossover (P_c) and probability of mutation (P_m).

In the present approach, GA is used for tuning the membership functions of the systems. The membership function parameters of a number of randomly generated fuzzy systems are decoded into binary sequences (chromosomes), each chromosome representing an individual fuzzy system. The initial population consisting of N chromosomes evolves to the next generation through the genetic operations of crossover and mutation. After each step, chromosomes are decoded and the fitness of the resulting systems is evaluated. Chromosomes with better fitness have higher probability to survive. The constraints needed to protect transparency are introduced in the decoding phase. The fitness function is computed as follows:

$$f = 100 - (\varepsilon \cdot 100) / \mathrm{abs}(y_{max} - y_{min}). \quad (15)$$

7. EXAMPLES

Fuzzy systems introduced in section 5 are modeled to demonstrate the capabilities of the methods described in section 6. It is assumed that no precise rule information is available, therefore initial rulebase is constructed so that it includes all possible combinations of input membership functions, and unique output membership function is assigned to each rule consequent. The modelling root-mean-square errors (RMSEs) are shown in Table 1.

With the test data generated by the system depicted in Fig. 5, the best-known fuzzy approximation tool ANFIS reduces the RMSE through 200 training epochs to 0.0335 but has more problems with the modelling of highly non-linear system in Fig. 4.

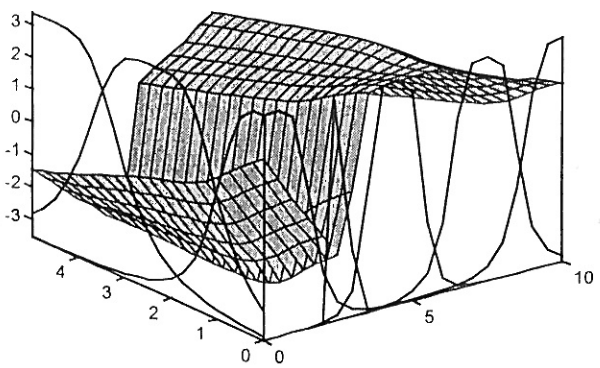

Fig. 7. ANFIS approximation of prod-prod-sum system

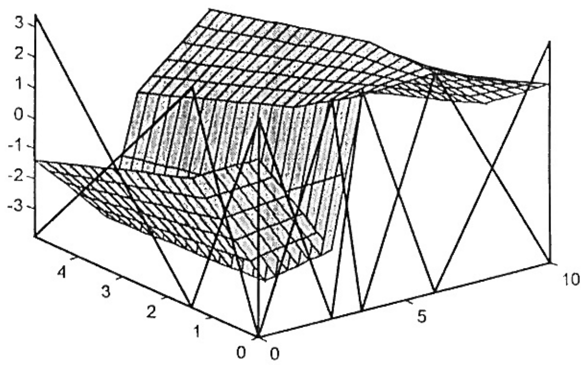

Fig. 8. Least squares approximation of prod-prod-sum system.

Table 1. Modelling root-mean-square-errors.

Modelling algorithm	RMSE (prod-prod-sum system)	RMSE (min-min-max system)
GK/LS	0.2398	0.3925
GD	0.0850	0.3711
GA	0.2058	0.3182
ANFIS	0.0335	0.1438

ANFIS is a combination of GD and LS, based on the Sugeno structure and the transparency of the trained system is rarely obtained

GK clustering with LS produces transparent models, but the approximation error is 3-5 times greater than in the previous case, depending on the type of the modelled system. Derivation of input-output partition cannot be fully automated efficiently and it works best if human judgement is involved. The computed output singletons (12) are optimal for the given input partition. Smaller number of tuning parameters compared to ANFIS, however, implies that highly non-linear systems cannot be modelled very well.

Approximation with GD method is much similar to the LS. The approximation error is somewhat smaller, though, and human judgement is not required. The results are obtained with 100 training epochs.

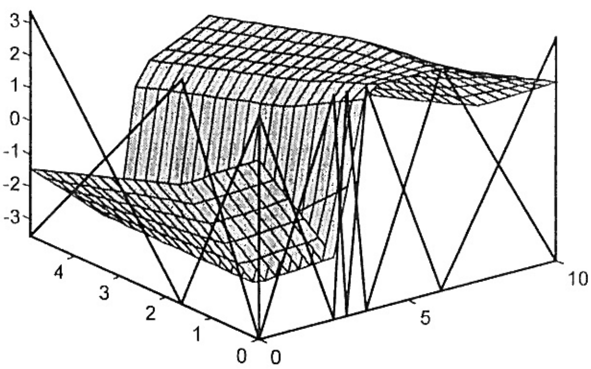

Fig. 9. GD approximation of prod-prod-sum system.

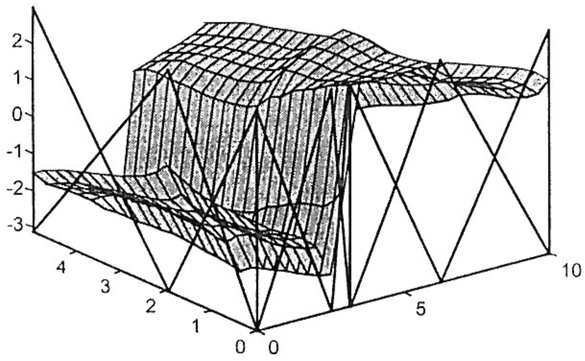

Fig. 10. GA approximation of min-min-max system.

Compared to other techniques, genetic algorithm is the most costly computationally. It is, however, the only method of those described above that is capable of reasonably good transparent approximation of a highly non-linear system.

In summary, it is worth pointing out that GD at modelling of prod-prod-sum system and GA at modelling of min-min-max system are capable of restoring the input partition very close to the original systems.

8. CONCLUSIONS

In this paper the transparency condition (3-4) for standard fuzzy systems was suggested and the constraints required for protecting the transparency of standard fuzzy systems with triangular membership functions were derived. An overview of available techniques with transparency protection for obtaining fuzzy systems from experimental data, was given along with the experiments confirming these properties.

Learning algorithms that not only address the issue of approximation quality but protect the transparency of the approximated model as well, could be used for a variety of applications. To be taken seriously, though, these algorithms should be explored further because of the gap between transparency and adaptability.

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