# ADAPTIVE CONTROL OF STOCHASTIC NONLINEAR SYSTEMS USING NEUROFUZZY NETWORKS

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Abstract: An adaptive control scheme is proposed for stochastic nonlinear systems with linear noise-generating mechanism. It is assumed that the orders and the delay of the system are known, whilst the nonlinearity is unknown. The controller is derived based on minimum variance control of locally linear models, and is implemented using neurofuzzy networks. The proposed controller is trained on line using recursive least squares method. The stability of the adaptive control scheme is analyzed. It is shown that under certain conditions, the proposed control scheme has attractive convergence properties. A simulation example is presented to illustrate its performance. 

Keywords: Adaptive control, Fuzzy logic, Neural networks, Nonlinear systems, Stochastic systems

## 1. INTRODUCTION

Although many successful applications of self-tuning controllers (Åström and Wittenmark, 1973; Clarke and Gawthrop, 1975) have been reported in the literature, their performance can deteriorate rapidly when they are applied to control nonlinear systems. To overcome this problem, several self-tuning schemes for nonlinear systems are proposed (Agarwal and Seborg, 1987; Anbumani, *et al.*, 1981; Svoronos, *et al.*, 1981). However, most of these schemes require the knowledge of the nonlinearity, and the controls are often designed on a case by case basis. When the nonlinearity of the system is unknown, adaptive controller derived based on neurofuzzy networks is proposed (Yeung, *et al.*, 1999). Neurofuzzy networks are used here, as they possess many attractive properties (Brown and Harris, 1994). For example, they can model nonlinear functions with arbitrary accuracy (Kosko, 1992) and can be trained on line using linear parameter estimation algorithms. The aim of this paper is to derive the stability of this controller, which is not given in (Yeung, *et al.*, 1999).

Significant progress in the global convergence results for linear adaptive control algorithms has been made in the eighties (Goodwin and Sin, 1984; Sin and Goodwin, 1982). It is shown that these results can be extended to the adaptive nonlinear controllers presented here. Similar to the linear adaptive controllers, it is assumed that the noise-generating mechanism of the systems is linear. Further, the controller is derived using minimum variance control law and is trained using recursive least squares (RLS) algorithm (Ljung and Söderström, 1983).

The organization of the paper is as follows. In Section 2, the problem is formally presented. The adaptive nonlinear controller is derived in Section 3. The training of the neurofuzzy controller is discussed in Section 4. The global convergence result of the proposed control algorithm is established in Section 5 and is illustrated by a simulation example, which is presented in Section 6.

## 2. PROBLEM STATEMENT

Consider the following nonlinear system

$$y(t) = f[y(t-k), \cdots, u(t-k), \cdots] + \eta(t) \qquad (1)$$

where $\eta(t)$ is a stationary stochastic process given by

$$A(z^{-1})\eta(t) = C(z^{-1})e(t) \qquad (2)$$

In (1), $\{y(t)\}$ and $\{u(t)\}$ are respectively the output and the input sequences; and $f(.)$ is a smooth nonlinear function. In (2), $e(t)$ is a white noise process and $A(z^{-1})$ and $C(z^{-1})$ are polynomials in the backward shift operator $z^{-1}$, i.e.,

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n_a} \quad (3)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_n z^{-n_c} \quad (4)$$

From (2), the nonlinear system given by (1) can be rewritten as the Nonlinear Auto-Regressive Moving-Average model with eXogenous (NARMAX) model,

$$A(z^{-1})y(t) = h(t-k) + C(z^{-1})e(t) \quad (5)$$

where h(t-k) is an unknown nonlinear function given by

$$h(t-k) = A(z^{-1})f[y(t-k), \cdots, u(t-k), \cdots] \quad (6)$$

A system can always be described by (5), if it is finite-dimensional, and if the signal-generating mechanism is linear. The noise sequence $\{e(t)\}$ is a stochastic process defined on a probability space $(\Omega, F, P)$ and adapted to the sequence of increasing sub-sigma algebras $(F_t, t \in N)$, where $F_t$ is generated by the observations up to and including time t. The initial condition of $F_t$ is denoted by $F_0$. The sequence $\{e(t)\}$ is assumed to satisfy the following conditions,

**A1** $E\{e(t) \mid F_{t-1}\} = 0$ a.s. (7)

**A2** $E\{e^2(t) \mid F_{t-1}\} = \sigma^2$ a.s. (8)

**A3** $\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} e^2(t) < \infty$ a.s. (9)

These assumptions play an important role in the proof of convergence of the adaptive algorithm. It is assumed that the coefficients of $A(z^{-1})$, $C(z^{-1})$, and $\sigma^2$ are unknown and that only the input sequence $\{u(t)\}$ and the output sequence $\{y(t)\}$ are available. It is further assumed that (5) satisfies,

**A4** the nonlinear function h(t) is first order differentiable with respect to its arguments.

**A5** the delay of the system k is known.

**A6** the system orders $n_y$, $n_u$ and $n_e$ are known.

**A7** $C(z^{-1})$ has all its zeros inside the closed unit circle.

Assumption **A4** implies that the nonlinear system can be approximated by the linear terms of the Taylor series expansion of h(t). Adaptive controller can then be derived for the locally linear model using existing techniques (Åström and Wittenmark, 1973; Clarke and Gawthrop, 1975). Assumptions **A5** to **A7** are technical assumptions required in deriving the stability result given in Section 5.

In this paper, the control law is designed such that the output of the system tracks the reference input w(t) with a minimum mean square error. Specifically, the control objective is to achieve with probability 1,

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} y^2(t) < \infty \quad (10)$$

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} u^2(t) < \infty \quad (11)$$

and to minimize the limit,

$$\lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E\{[y(t) - w(t)]^2 \mid F_{t-k}\} \quad (12)$$

where $\{w(t)\}$ is the reference sequence. To ensure that y(t) is bounded, it is important that the reference input w(t) is also bounded, as it is clear that a system tracking an unstable w(t) is unstable. Consequently, a further assumption on the set-point is made.

**A8** w(t+k) is computable at time t and that

$$|w(t)| < m_1 < \infty \quad \text{for all } t \quad (13)$$

## 3. DERIVATION OF THE NONLINEAR CONTROLLER

In this section, preliminary discussion on the self-tuning controller for linear systems is presented. Its application to linearized nonlinear systems is discussed, followed by the derivation of the nonlinear controller.

### *3.1 Minimum variance control*

As $C(z^{-1})$ is assumed to be asymptotically stable, the optimal k-step-ahead predictor for the system (5) can be expressed in the following form

$$C(z^{-1})[y(t+k) - v(t+k)] = G(z^{-1})y(t) + F(z^{-1})h(t) \quad (14)$$

where $G(z^{-1})$ and $F(z^{-1})$ are polynomials satisfying

$$C(z^{-1}) = F(z^{-1})A(z^{-1}) + z^{-k}G(z^{-1}) \quad (15)$$

$$F(z^{-1}) = 1 + f_1 z^{-1} + \cdots + f_{k-1} z^{-k+1} \quad (16)$$

$$G(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{n-1} z^{-n+1} \quad (17)$$

The optimal k-step-ahead prediction of y(t+k) given $F_t$ is

$$y^0(t+k \mid t) = y(t+k) - v(t+k) = E\{y(t+k) \mid F_t\} \quad (18)$$

where $v(t+k)$ is a moving average given by

$$v(t+k) = \sum_{i=0}^{k-1} f_i e(t+k-i) \quad (19)$$

It implies that

$$E\{v(t+k) \mid F_t\} = 0 \quad \text{a.s.} \quad (20)$$

$$\gamma^2 = E\{v^2(t+k) \mid F_t\} = \sigma^2 \sum_{i=0}^{k-1} f_i^2 \quad \text{a.s.} \quad (21)$$

where $\gamma$ is the minimum mean square control error achievable by any causal feedback.

The minimum variance control law is obtained by minimizing the following cost function,

$$J(t+k) = E\{[y(t+k) - w(t+k)]^2 \mid F_t\} \quad (22)$$

Subtracting $C(z^{-1})w(t+k)$ from both sides of (14) gives

$$C(z^{-1})[\varepsilon(t+k) - v(t+k)] = G(z^{-1})y(t) + F(z^{-1})h(t) - C(z^{-1})w(t+k) \quad (23)$$

where $\varepsilon(t+k)=y(t+k)-w(t+k)$ is the tracking error of the system output. The cost function $J(t+k)$ is minimized by setting $y^0(t+k|t)$ to the desired value $w(t+k)$ or setting the left-hand side of (23) to zero, giving the minimum variance control law,

$$F(z^{-1})h(t) = C(z^{-1})w(t+k) - G(z^{-1})y(t) \quad (24)$$

### *3.2 Derivation of the control law based on linearization*

For the nonlinear system given by (5), the minimum variance control given by (24) is valid only at its locally linearized models at specific operating points. From assumption **A4**, the $i^{th}$ linearized model exists, and is denoted by the superscript i in the following discussion. For the $i^{th}$ model, the nonlinear function $h(t)$ in (24) becomes,

$$h^{(i)}(t) = B^{(i)}(z^{-1})y(t) + D^{(i)}(z^{-1})u(t) + \alpha^{(i)} \quad (25)$$

where $B^{(i)}(z^{-1})$ and $D^{(i)}(z^{-1})$ are polynomials in the backward shift operator $z^{-1}$ for the $i^{th}$ operating point, i.e.,

$$B^{(i)}(z^{-1}) = b_0^{(i)} + b_1^{(i)}z^{-1} + \cdots + b_{n_y}^{(i)}z^{-n_y} \quad (26)$$

$$D^{(i)}(z^{-1}) = d_0^{(i)} + d_1^{(i)}z^{-1} + \cdots + d_{n_u}^{(i)}z^{-n_u} \quad (27)$$

and $\alpha^{(i)}$ is a constant. Substituting (25) into (24), the control law is given by

$$\overline{F}^{(i)}(z^{-1})u(t) = C(z^{-1})w(t+k) - \overline{G}^{(i)}(z^{-1})y(t) - \overline{\alpha}^{(i)} \quad (28)$$

where

$$\overline{F}^{(i)}(z^{-1}) = F(z^{-1})D^{(i)}(z^{-1}) \quad (29)$$

$$\overline{G}^{(i)}(z^{-1}) = G(z^{-1}) + F(z^{-1})B^{(i)}(z^{-1}) \quad (30)$$

$$\overline{\alpha}^{(i)} = F(z^{-1})\alpha^{(i)} \quad (31)$$

Normalizing (28) by $d_0^{(i)}$, and rearranging gives

$$u(t) = \sum_{j=0}^{n_e+k-1} \overline{c}_j^{(i)} w(t+k-j) - \sum_{j=0}^{n_y+k-1} \overline{g}_j^{(i)} y(t-j) - \sum_{j=1}^{n_u+k-1} \overline{f}_j^{(i)} u(t-j) - \overline{\alpha}^{(i)} \quad (32)$$

Since the control law given by (32) is a function of the operating point, it is necessary to develop a nonlinear controller that can operate over the full range of the operating points of the system.

### *3.3 Implementation of the nonlinear controller using neurofuzzy networks*

Since neurofuzzy networks have the ability to approximate smooth nonlinear functions with arbitrary accuracy (Kosko, 1992), they are used here to implement the adaptive nonlinear controller as proposed in (Yeung, *et al.*, 1999). Define a vector consisting of the input of the B-spline neurofuzzy network (Brown and Harris, 1994) given by (32),

$$x(t) = [y(t), \cdots, y(t-n_y-k+1), u(t-1), \cdots, u(t-n_u-k+1), w(t+k), \cdots, w(t-n_e+1)]^T \quad (33)$$

Note that the constant term $\overline{\alpha}$ is absorbed into the fuzzy mapping, and is not included explicitly in (33). The dimension of $x(t)$, denoted by n, is

$$n = n_y + n_u + n_e + 3k - 1 \quad (34)$$

Let $\theta$ be a vector consisting of the weights of the neurofuzzy network, where

$$\theta = [\theta_1, \theta_2, \cdots, \theta_p]^T \quad (35)$$

Then the nonlinear control obtained from the neurofuzzy network is

$$u(t) = a^T(t)\theta \quad (36)$$

where $a(t)$ is the transformed input vector given by

$$a(t) = [a_1(x(t)), a_2(x(t)), \cdots, a_p(x(t))]^T \quad (37)$$

and $\{a_i(x(t)),\ i = 1, \cdots, p\}$ are the tensor products of the B-spline membership functions, $\mu_{A_k^i}(x_k(t))$,

$$a_i(x(t)) = \prod_{k=1}^{n} \mu_{A_k^i}(x_k(t)) \quad (38)$$

The schematic diagram of the neurofuzzy controller is shown in Fig. 1.

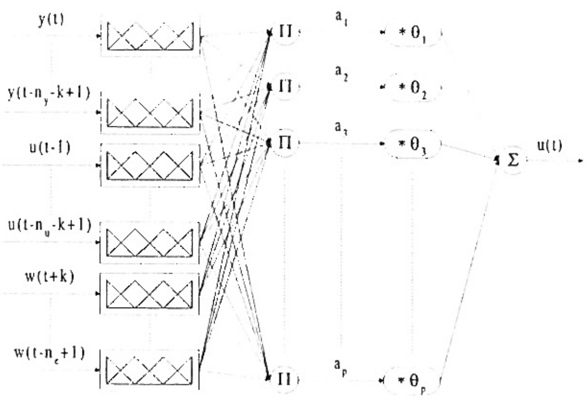


Fig. 1 The neurofuzzy controller

## 4. TRAINING OF THE NEUROFUZZY CONTROLLER

In this section, the target for the training of the nonlinear controller is derived, followed by the on-line training algorithm.

### *4.1 Derivation of the training target*

Since the output of the neurofuzzy network given by (36) is linear in the weights of the network, the optimal weights can be estimated using any linear parameter estimation algorithms using the training target as derived below. Rewrite the cost function given by (22) using (36), such that it is an explicit function of $u(t)$,

$$J(t+k)=E\{[\lambda(u(t)-a^T(t)\theta)+(y(t+k)-w(t+k))]^2 \mid F_t\} \quad (39)$$

where $\lambda$ is an arbitrary real constant. Rearranging (39) gives

$$J(t)=\lambda^2 E\{[\bar{u}(t)-a^T(t-k)\theta]^2 \mid F_{t-k}\} \quad (40)$$

where

$$\bar{u}(t)=u(t-k)+\frac{1}{\lambda}(y(t)-w(t)) \quad (41)$$

The cost function given by (40) is minimized at time t, if the control generated from the network at time t-k approaches $\bar{u}(t)$. Therefore, a natural choice of the training target for the nonlinear controller based on the neurofuzzy network is $\bar{u}(t)$. To ensure that the weights in the network are adapted in a direction that minimizes the cost function, $\lambda$ should have a sign opposite to that of $d_0^{(i)}$ in (27). The magnitude of $\lambda$ is set to 1 in this paper. However, it should be noted that the choice of $\lambda$ affects the convergence rate in the estimation of the weights.

### *4.2 On-line training algorithm*

Let $\hat{\theta}(t)$ be the estimate of $\theta$ at time t,

$$\hat{\theta}(t)=[\hat{\theta}_1(t),\hat{\theta}_2(t),\cdots,\hat{\theta}_p(t)]^T \quad (42)$$

The weights are computed recursively using the recursive least squares (RLS) method as follows (Ljung and Söderström, 1983),

$$\hat{\theta}(t)=\hat{\theta}(t-1)+\frac{P(t-1)a(t-k)[\bar{u}(t)-a^T(t-k)\hat{\theta}(t-1)]}{1+a^T(t-k)P(t-1)a(t-k)} \quad (43)$$

and the covariance matrix P(t) by

$$P(t)=P(t-1)-\frac{P(t-1)a(t-k)a^T(t-k)P(t-1)}{1+a^T(t-k)P(t-1)a(t-k)} \quad (44)$$

In summary, the implementation of the proposed adaptive controller involves training the neurofuzzy network on line using (41), (43) and (44). The control is then computed by (36) using the updated parameters of the neurofuzzy network.

## 5. STABILITY ANALYSIS

It is shown in this section that the adaptive nonlinear control based on the neurofuzzy networks presented in section 4 has attractive convergence properties. Before proceeding further, the following assumptions are made.

**A9** The predefined ranges of the input fuzzy sets of the neurofuzzy controller cover all possible states.

**A10** The accuracy of approximation of the neurofuzzy network is sufficiently high.

**A11** All the zeros of $D^{(i)}(z^{-1})$ in each locally linear model are strictly inside the closed unit disk.

**A12** The sign of $d_0^{(i)}$ is known and is fixed at all operating points.

**A13** The transfer function $\left[\frac{|d_0^{(i)}|}{C(z)}-\frac{1}{2}\right]$ is very strictly passive for all i $(i=1,\cdots,p)$.

**A14** The condition number of P is bounded, i.e.,

$$\limsup_{N\to\infty}\frac{\lambda_{max}P(N)}{\lambda_{min}P(N)}<\infty \quad (45)$$

Assumption **A9** ensures that the transformed input a(t) of the neurofuzzy network is well defined for all t. Assumption **A10** is not restrictive, and can readily be achieved by a suitable choice of the number and the orders of input fuzzy sets (Kosko, 1992). The minimum phase property given by assumption **A11** is necessary to ensure the control is bounded. Hence there exists an optimal weight $\theta_0$ such that the modelling errors are negligible. Assumptions **A12** to **A14** are required to establish the convergence properties of the RLS algorithm (43) and (44) using the Martingale convergence theorem as presented below.

**Lemma 1** Subject to assumptions **A1** to **A14**, the algorithm given by (41), (43) and (44) ensures that, with probability 1,

(i) Parameter difference convergence

$$\lim_{N\to\infty}\sum_{t=1}^{N}\|\hat{\theta}(t)-\hat{\theta}(t-k)\|^2<\infty \quad (46)$$

for finite k.

(ii) Normalized prediction error convergence

$$\lim_{N\to\infty}\sum_{t=1}^{N}\frac{[\zeta(t)-v(t)]^2}{r(t)}<\infty \quad (47)$$

where

$$\begin{aligned}\zeta(t)&=\bar{u}(t)-a^T(t-k)\hat{\theta}(t-1)\\&=a^T(t-k)\hat{\theta}(t-k)+(y(t)-w(t))/\lambda\\&\quad-a^T(t-k)\hat{\theta}(t-1)\\&=\varepsilon(t)/\lambda+a^T(t-k)[\hat{\theta}(t-k)-\hat{\theta}(t-1)]\end{aligned} \quad (48)$$

and

$$r(t)=r(t-1)+a^T(t-k)a(t-k),\quad t\geq k+1 \quad (49)$$

where

$$r(0)=\mathrm{trace}\,P^{-1}(0) \quad (50)$$

**Proof** The proof of lemma 1 follows closely to that given in (Goodwin and Sin, 1984). Note that the following relationships hold.

$$\zeta'(t)=\frac{\zeta(t)}{1+a^T(t-k)P(t-1)a(t-k)} \quad (51)$$

where

$$\zeta'(t)=\bar{u}(t)-a^T(t-k)\hat{\theta}(t) \quad (52)$$

and

$$C(z^{-1})z(t)=|d_0^{(i)}|\,b(t) \quad (53)$$

where

$$z(t) = \zeta'(t) - v(t) \quad (54)$$

$$h(t) = -a^T(t-k)\tilde{\theta}(t) \quad (55)$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta_0 \quad (56)$$

In deriving the properties of the RLS algorithm given in Lemma 1, $\{a(t)\}$ is assumed to be known, which is readily satisfied by a(t) given by (37) in the neurofuzzy network. The global convergence result of the adaptive nonlinear controller based on the neurofuzzy networks is given in Theorem 1.

**Theorem 1** Subject to assumptions **A1** to **A3** on the noise, assumptions **A4** to **A8** on the system and the signal, and assumptions **A9** to **A14**, the adaptive control algorithm given by (36), (41), (43) and (44) is globally convergent in the following sense,

(i) $$\lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E\{[y(t)-w(t)]^2 \mid F_{t-k}\} = \gamma^2 \quad \text{a.s.} \quad (57)$$

(ii) $$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} y^2(t) < \infty \quad \text{a.s.} \quad (58)$$

(iii) $$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} u^2(t) < \infty \quad \text{a.s.} \quad (59)$$

**Proof** (i) Since the outputs of the fuzzy membership functions must be between 0 and 1, the transformed inputs are also between 0 and 1, i.e.,

$$0 \le a_i(x(t-k)) \le 1 \qquad \text{for } i = 1,\cdots,p \quad (60)$$

Further, the partition of unity assumption of neurofuzzy networks (Brown and Harris, 1994) implies that

$$\sum_{i=1}^{p} a_i(x(t-k)) = 1 \quad (61)$$

Hence

$$a^T(t-k)a(t-k) \le 1 \quad (62)$$

Consequently, the following relation holds if the input $x(t-k)$ remains within the predefined operating regions,

$$\frac{r(N)}{N} = \frac{1}{N}[\operatorname{trace} P^{-1}(0) + \sum_{t=k+1}^{N} a^T(t-k)a(t-k)] \quad (63)$$
$$\le \frac{1}{N}[\operatorname{trace} P^{-1}(0) + N] = K_1$$

where $K_1$ is a finite positive constant. Substituting (48) into (47) yields

$$\lim_{N\to\infty} \sum_{t=1}^{N} \frac{[\varepsilon(t)/\lambda + a^T(t-k)[\hat{\theta}(t-k) - \hat{\theta}(t-1)] - v(t)]^2}{r(t)} < \infty \quad (64)$$

From (46) and setting λ=1, (64) reduces to

$$\lim_{N\to\infty} \sum_{t=1}^{N} \frac{[\varepsilon(t) - v(t)]^2}{r(t)} < \infty \quad (65)$$

If r(t) is bounded, (65) implies that

$$\lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} [\varepsilon(t) - v(t)]^2 = 0 \quad \text{a.s.} \quad (66)$$

If r(t) is unbounded, then

$$\sum_{t=1}^{N} \frac{[\varepsilon(t) - v(t)]^2}{r(t)} \quad (67)$$

is non-decreasing, Kronecker's lemma (Neveu, 1975) can be applied to obtain

$$\lim_{N\to\infty} \frac{N}{r(N)} \frac{1}{N}\sum_{t=1}^{N} [\varepsilon(t) - v(t)]^2 = 0 \quad \text{a.s.} \quad (68)$$

Substituting (63) into (68) gives

$$\lim_{N\to\infty} \frac{1}{K_1} \frac{1}{N}\sum_{t=1}^{N} [\varepsilon(t) - v(t)]^2 = 0 \quad \text{a.s.} \quad (69)$$

Since $0 < K_1 < \infty$, (66) follows immediately.

Note that

$$E\{\varepsilon^2(t) \mid F_{t-1}\} = E\{[y(t) - w(t) - v(t) + v(t)]^2 \mid F_{t-1}\}$$
$$= E\{[y(t) - w(t) - v(t)]^2 + 2[y(t) - w(t) - v(t)]v(t) + v^2(t) \mid F_{t-1}\} \quad (70)$$

Since $y(t)-v(t)$ and $w(t)$ are $F_{t-1}$ measurable, it follows from (20) that

$$E\{[y(t) - w(t)]^2 \mid F_{t-1}\} = [\varepsilon(t) - v(t)]^2 + E\{v^2(t) \mid F_{t-1}\} \quad (71)$$

Equation (57) follows from (21), (66) and (71).

(ii) Equation (58) follows from (13) and (57).

(iii) From (46) and assumption **A11**, the weight vector is bounded and its estimate converges. From (36) and (62), the input u(t) is bounded for all t, and hence (59) follows.

## 6. EXAMPLE

Consider the following nonlinear system,

$$(1 + 0.4z^{-1})y(t) = \frac{-0.9y(t-1) + u(t-1)}{1 + y^2(t-1)} + (1 - 0.7z^{-1})e(t) \quad (72)$$

where e(t) is a white noise with a variance of 0.01 satisfying assumptions **A1** to **A3**. The open-loop response of the system subject to a unit step input is quite oscillatory, as shown in Fig. 2.

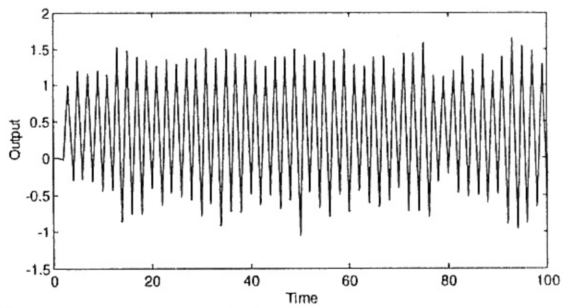


Fig. 2 Open-loop unit step response

It is assumed that only the orders and the delay of the system are known, i.e., $n_y$=0, $n_u$=0, $n_e$=1 and k=1, whilst the nonlinearity of the system is unknown. The adaptive nonlinear controller discussed in Sections 3 and 4 is implemented with y(t), w(t) and w(t+1) as its inputs. Four second-order basis functions are selected for each input, giving a total of

64 weights. The range of the fuzzy sets of {y(t)} is between -1 and 1, while that of {w(t)} is between -0.5 and 0.5. From (72), the sign of $d_0^{(1)}$ must be positive. From (41), the training target of the neurofuzzy network is

$$u(t-1)-y(t)+w(t) \quad (73)$$

The initial values of the weights were set to 0.1, and the initial covariance matrix to 100**I**, where **I** is an identity matrix. The set-point of the system is

$$w(t+1)=0.4\sin(t/10)\cos(t/3) \quad (74)$$

The system is simulated for 10000 samples. As shown in Fig. 3, the system output converges towards the set-point quickly. Satisfactory control and set-point tracking are achieved after the weights converge, as shown in Fig. 4. From (15) and (21), $F(z^{-1})=1$ and hence $\gamma^2=0.01$. The variance of the output error, shown in Fig. 5, is given by

$$\frac{1}{t}\sum_{i=1}^{t}[y(i)-w(i)]^2 \quad (75)$$

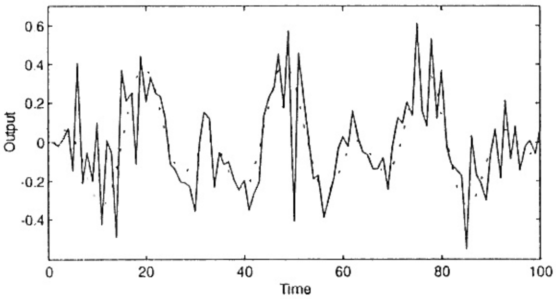


Fig. 3 Performance of the proposed controller at t=[1,100], solid line – y(t), dotted line – w(t)

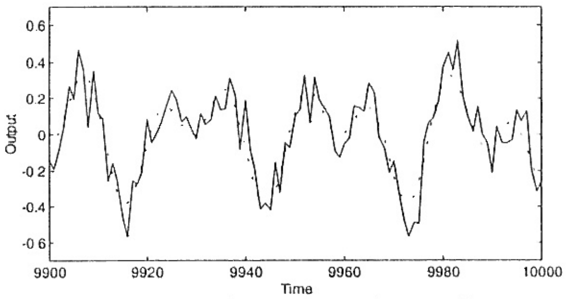


Fig. 4 Performance of the proposed controller at t=[9900,10000], solid line – y(t), dotted line – w(t)

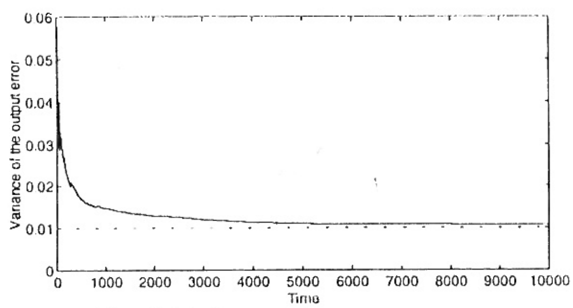


Fig. 5 Variance of the output error

It is clear that it converges to the variance of the noise sequence. For example, the estimated variance is 0.0107 at t=10000, which is very close to that of e(t) at a value of 0.01, illustrating the convergence properties of the adaptive nonlinear controller given in Theorem 1.

## 7. CONCLUSION

An adaptive nonlinear controller is presented for stochastic nonlinear systems contaminated by white noise. The controller is designed based on minimum variance control of locally linearized models, and is implemented using neurofuzzy networks. Since the output of the controller is linear in its weights, it can be trained on line using recursive least squares method. It is shown in this paper that under certain assumptions, the variance of the output error converges to the minimum mean square control error achievable by any causal feedback. This property is illustrated by the simulation example.

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