

INFORMATION AGGREGATION IN FLC USING EVOLUTIONARY OPERATORS

Márta Takács*, Lőrincz Katalin**, Anikó Szakál**

Budapest Polytechnic,

*John von Neumann Faculty of Informatics,

**Bánki Donát Faculty of Mechanical

H-1081 Budapest Népszínház u. 8., Hungary

Abstract: In this paper information aggregation of generalised max and min operators is outlined, where some new operators from this operator-group, (the maximum distance minimum and the maximum distance maximum operators for example), are uninorms and compensative ones as well. A Mamdani type FLC inference mechanism is represented, and the fuzzy rule system uses the new operator group. Several pairs of the evolutionary T t-norms (for rule outputs obtaining) and evolutionary S t-conorms (for FLC output obtaining) have been tried out. Some investigation results are given after the simulation. *Copyright©2000 IFAC*

Keywords: Fuzzy control, Fuzzy models, Fuzzy systems, Operators, Simulation.

1. INTRODUCTION

Since 1965 several classes of operations of intersection, union and complement operators, satisfying appropriate axioms, have been introduced. By accepting some basic conditions, a broad class of set of operations for union and intersection is formed by t-operators (J. Fodor, M. Roubens, 1994).

For a kind of generalisation of t-norms and t-conorms the concept of uninorm was introduced by Yager and Rybalov (1998), Fodor, J., Yager, R., and Rybalov, (1997), De Baets, (1998.). Uninorms are such kind of generations of t-norms and t-conorms where the neutral element can be any number from the unit interval. The neutral element e is clearly unique. The case $e = 1$ leads to t-conorm and the case $e = 0$ leads to t-norm .

For any t-norms and t-conorms the inequality $T \leq T_M \leq S_M \leq S_W$ holds, which means that there are no t-operators lying between the minimum and

maximum operators. This could be a disadvantage of the application of t-operators as aggregation operators in several intelligent systems. To handle the problem Zimmermann and Zysno (1980) has introduced the so-called γ -operator as the first compensatory operator. Since than compensative operators have been studied by several authors.

An operator M is said to be a *compensative* if and only if

$$\min(x, y) \leq M(x, y) \leq \max(x, y), \quad \forall (x, y) \in [0, 1]^2$$

In many applications where for example compensation behaviour is required conventional operators do not work well.

In this paper information aggregation of generalised max and min operators is outlined, where some new operators from the introduced operator-group, (the maximum distance minimum and the maximum distance maximum operators for example), are uninorms and compensative ones as

well (I. Rudas, 1998.c, 1999.a,1999.c).

The new operators are evolutionary types in the sense that if e is increasing starting from zero till $e=1$ the min operator is developing into the max operator, while on the other side the max is transformed into the min operator (where e is an arbitrary element of the unit interval $[0,1]$), (I. Rudas 1999.b.). Beside this, operators make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker, and they are *evolutionary operators* in this sense as well. In the paper an information aggregation in FLC using evolutionary operators is given, with special emphasis on this behaviour.

The structure of the paper is as follows: at first the distance-based evolutionary operators are introduced. It is shown that the novel operations can be constructed from the conventional min and max operations and their properties are discussed and showed by graphs.

After the review of background properties the Mamdani type FLC inference mechanism is represented in a fuzzy rule system using new operator group. The simulation results are summarised, and let shown, that the evolutionary types of operators, when e is increasing starting from zero till $e=1$, and the min operator is developing into the max operator, while on the other side the max is transformed into the min operator, the desired state of the system is obtained in every step increasingly. It illustrated, that this operators make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker.

2. EVOLUTIONARY OPERATORS REVIEW

The maximum distance minimum operator with respect to $e \in [0,1]$ is defined as

$$T_e^{\max} = \begin{cases} \max(x, y), & \text{if } y > 2e - x \\ \min(x, y), & \text{if } y < 2e - x \\ \min(x, y), & \text{if } y = x \text{ or } y = 2e - x \end{cases}$$

The maximum distance maximum operator with respect to $e \in [0,1]$ is defined as

$$S_e^{\max} = \begin{cases} \max(x, y), & \text{if } y > 2e - x \\ \min(x, y), & \text{if } y < 2e - x \\ \max(x, y), & \text{if } y = x \text{ or } y = 2e - x \end{cases}$$

The minimum distance minimum operator with respect to $e \in [0,1]$ is defined as

$$T_e^{\min} = \begin{cases} \min(x, y), & \text{if } y > 2e - x \\ \max(x, y), & \text{if } y < 2e - x \\ \min(x, y), & \text{if } y = x \text{ or } y = 2e - x \end{cases}$$

The minimum distance maximum operator with respect to $e \in [0,1]$ is defined as

$$S_e^{\min} = \begin{cases} \min(x, y), & \text{if } y > 2e - x \\ \max(x, y), & \text{if } y < 2e - x \\ \max(x, y), & \text{if } y = x \text{ or } y = 2e - x \end{cases}$$

The structures of the evolutionary operators are illustrated in 2D (I. Rudas, 1999.d) and in 3D. The structures of the maximum distance maximum operator with $e=0.2$ is given in Figure 1., and the 3D representation in Figure 2.

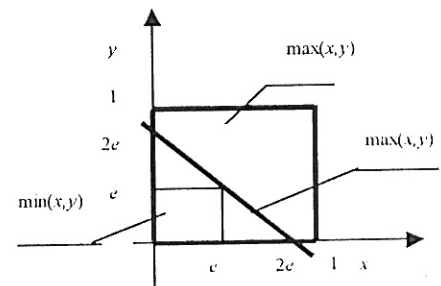


Fig. 1. Maximum distance maximum

The representation of the maximum distance minimum operator is very similar to the representation of maximum distance maximum. The first is different from the second just on the boundary of the minimum domain and maximum domain, on the line $y = -x + 2e$ (the first has minimum values, the second maximum values). This property can be employed by the simulation.

The structures of the minimum distance minimum operator with $e=0.2$ is given in Figure 3., and the 3D representation in Figure 4.

The representation of the minimum distance minimum operator and the representation of minimum distance maximum differ on the

boundary of the minimum domain and maximum domain.

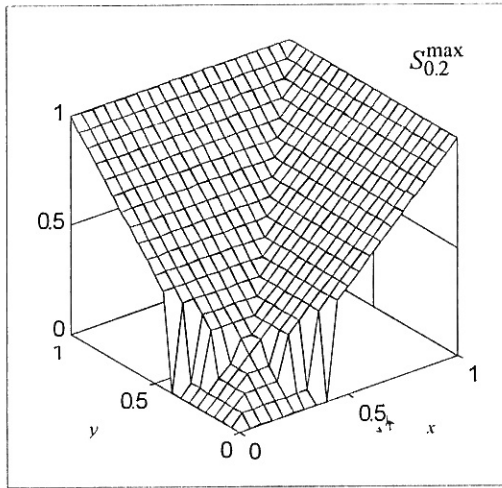


Fig. 2. The 3D representation of $S_{0.2}^{\max}$ operator.

With a continual changing of e it can be shown in 3D, how develops the min to the max operator (observed the T_e^{\min} or S_e^{\min} , where e is increasing from 0 till 1), and how develops the max to the min operator (observed the T_e^{\max} or S_e^{\max} , where e is increasing from 0 till 1).

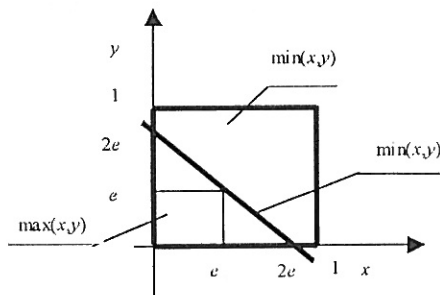


Fig. 3. Minimum distance minimum

3. INVESTIGATION RESULTS

After the review of background properties the Mamdani type FLC inference mechanism is represented in a fuzzy rule system using new operator group.

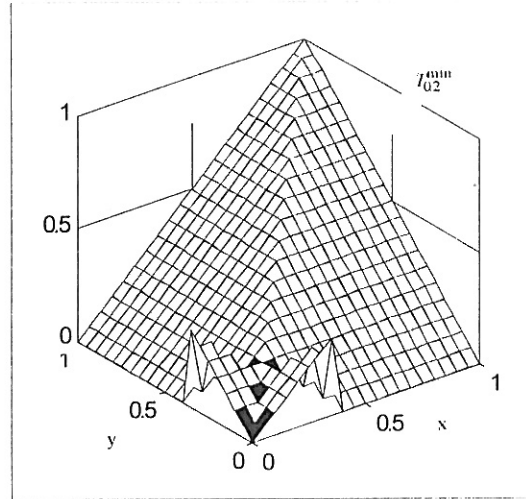


Fig. 4. The 3D representation of the $T_{0.2}^{\min}$ operator.

It is known, that for an *if A then B* type rule, with rule input A' , the rule output B' is obtained with expression

$$B' = T_1(B, T_2(A, A')), \quad (1)$$

where T_1 and T_2 are t-norms. For several substitutions of the T_1 and T_2 t-norms with distance-based evolutionary t-norms T_e^{\max} and T_e^{\min} (for increasing e form $[0,1]$), the obtained B' fuzzy set is shown. Using the unvaried triangular fuzzy sets for rule premise A , rule consequence B and rule input A' , we can observe several B' rule-outputs. The consequence is, that the evolutionary type t-norms make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker. We can recognise this property rather using the general case of evolutionary t-norms than using classical $T_1^{\max}(x, y) = \min(x, y)$ operator.

3.1. The FLC System

Using previous experience, a simple FLC system is constructed into a simulation system. The source of the system is a step function, with the starting value 0, and the final value 1. The system to be controlled is a first order differential equation, $q' = k_1(y + k_2q)$. This simple system proved

useful in earlier investigations of the T norms and S conorms (J.K. Tar, at all, (1998.)).

The FLC is based on three rules:

IF e is N THEN y is N OR y is Z

IF e is Z THEN y is Z

IF e is P THEN y is P OR y is Z

This system, without feedback of error-change \dot{e} has been tested in earlier papers as well, and it proved useful and simple too. (Takacs, 1997.).

It is a Mamdani type controller, so the FLC output is obtained as centre of gravity of the fuzzy set $S(\text{ruleoutput}_1, S(\text{ruleoutput}_2, \text{ruleoutput}_3))$, where S is an evolutionary t-conorm, and rule-outputs are obtained with previously represented methods (see (1)).

3.2. The simulation results

Several pairs of the evolutionary T t-norms (for rule outputs obtaining) and evolutionary S t-conorms (for FLC output obtaining) have been tried out, with special emphasis on the pairs $(T_e^{\max}, S_{1-e}^{\max})$ and $(T_e^{\min}, S_{1-e}^{\min})$. Figure 5. shows the simulation system output, using this evolutionary norms.

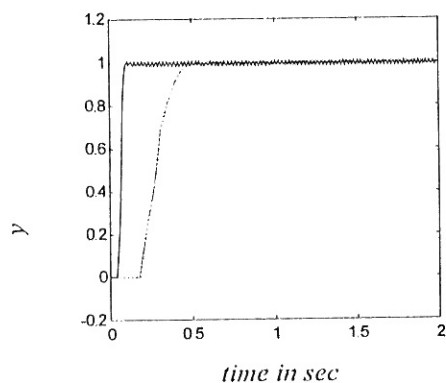


Fig. 5. Simulation results

4. CONCLUSION

As the conclusion a general aggregation of the experiences is given for evolutionary operators using in FLC. The choosing of pairs $(T_e^{\max}, S_{1-e}^{\max})$ and $(T_{1-e}^{\min}, S_e^{\min})$ by the

simulation, using the same e value, gives results with negligible difference. So it was sufficient trying out the pairs $(T_e^{\max}, S_{1-e}^{\max})$ for example.

The choosing of the pair $(T_e^{\max}, S_{1-e}^{\max})$, where e is near zero, return in short time the desired state of the system, but it is not stable. If e is near 1, the situation is known, because it develops to choosing of pair (min, max). The desired state is obtained easier, and the system stay stable. It can be observed, that continual choosing of e from zero till 1 results continual improvement in stability, and continual increasing time of obtaining desired state in system. The choosing of pair $(T_{0.5}^{\max}, S_{0.5}^{\max})$ gives acceptable result by both criteria. It proofs again, that the reconciling of the T nom and S conorm in FLC, where this pair is dual in fuzzy theory sense, gives the best results in a simple simulation system.

REFERENCES

- De Baets, B. (1998.): Uninorms: the known classes, *Fuzzy Logic and Intelligent Technologies for Nuclear Science and Industry*. (D. Ruan, H.A. Abderrahim, P.D'hondt and E. Kerre, eds). *Proc. Third Int. FLINS Workshop* (Antwerp. Belgium), World Scientific Publishing, Singapore, pp. 21-28.
- Fodor J., M. Roubens, (1994.): Fuzzy Preference Modelling and Multicriteria Decision Support, *Kluwer Academic Publishers, The Netherlands*.
- Fodor, J., Yager, R., Rybalov, A. (1997): Structure of uninorms, *International Journal Uncertainty, Fuzziness, and Knowledge Based Systems*. 5, pp. 411-427.
- I.J. Rudas, M.O. Kaynak: Entropy-Based Operations on Fuzzy Sets, (1998.a). *IEEE Transactions on Fuzzy Systems*, vol.6, no. 1, February 1998. pp. 33-40.
- I.J. Rudas, O.Kaynak: New Types of Generalized Operations, (1998.b). *Computational Intelligence: Soft Computing and Fuzzy-Neuro Integration with Applications. Springer NATO ASI Series. Series F: Computer and Systems Sciences, Vol. 192. 1998. (O. Kaynak, L. A. Zadeh, B. Türksen, I. J. Rudas editors)*, pp. 128-156.

- ✓ I.J. Rudas, M.O. Kaynak: Minimum and maximum fuzziness generalized operators, (1998.c), *Fuzzy Sets and Systems* 98 83-94.
- Imre Rudas: Generalization of Fuzzy t-operators, (1999.a). *Proceedings of the Bánki Donát Politechnic Jubilee International Conference*, Budapest, , pp. 51-55.
- ✓ Imre J. Rudas: Towards the generalization of t-operators: a distance based approach, (1999.b). *Proceedings of the International Conference on Intelligent Information Systems 1999*, (IIS'99). Varazdin, September 24-26,.
- ✓ Imre Rudas: Generalization of Fuzzy t-operators, (1999.c). *Proceedings of the Bánki Donát Politechnic Jubilee International Conference*, Budapest, 1999, pp. 51-55.
- ✓ Imre J. Rudas: Evolutionary Operators, (1999.d). *Proceedings of INES' 99, IEEE International Conference on Intelligent Engineering Systems*, Poprad, High Tatras, Stara Lesna, Slovakia, November 1-3, 1999, pp. 331-335 (Plenáris előadás).
- M. Takacs, (1997) Investigation on a special Group of Fuzzy Implication Operators and fuzzy Inference mechanisms Using a Simplified Rule Base System, *Progress in Connectionist-Based Information System, Proc. Of the 1997 International Conference on Neural Information Progressing and Intelligent Information Systems*, Springer-Verlag, Vol.2, pp. 793-796.
- ✓ J.K. Tar, O.M. Kaynak, J.F. Bitó, I.J. Rudas, T. Kégl, (1998.): The Use of Uniform Structures in Adaptive Dynamic Control of Robots. *Proc. of IEEE International Symposium on Industrial Electronics (ISIE'98)*, Pretoria, South Africa, 7-10 July, 1998, Vol. 1, pp. 137-142.
- Yager, R., Rybalov, A., (1998): Uninorm aggregation operators. *Fuzzy Sets and Systems* 80, pp. 105-136.
- Zimmermann, H.-J., Zysno, P. (1980), "Latent connectives in human decision making". *Fuzzy Sets and Systems* 4 .37-51.