ROBUST $\mathcal{H}_\infty$ CONTROL OF RAILROADS LINING SYSTEM

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ABSTRACT: The robust control based $\mathcal{H}_\infty$ norm minimization is considered to achieve robust stability and enhance the performance of the lining system, which is a servosystem applied for railroads laying machines. The system is characterized by the inability of determining its parameters accurately; specifically, load, friction, leakage and the disturbance force. With the characteristics of the proportional control valve (42 LPM, bandwidth of 6 Hz), a robust controller is designed and implemented upon the lining system. While the robust performance is satisfied; with a maximum overshoot of 14% and steady state error not greater than 0.5%; the system bandwidth is limited to maintain the system robustness. Otherwise, a larger valve capacity (85 LPM, 10 Hz) is required to increase the system bandwidth. While the $\mathcal{H}_\infty$ requires lengthy messy mathematical expressions connected with the problem formulation; the closed loop objectives met robustly over the whole range of uncertain parameters.

Keywords: Railways, Robust Control, Servo Hydraulics

1. SYSTEM DESCRIPTION

The aim is to enhance the straightness irregularities of the railroad track; straight, curved, and also the switchs in which the cross section of the rails varying with the track coordinate and even differs between the right and left rail. The track is measured at four points and two versines are compared to control the lining system. The measuring axles (A, B, C, D) are pneumatically preloaded to the selected reference rail, the chord stretched between (A & D) represents the direction of the alignment. Depending on the track alignment at the contact points, the potentiometers placed at (B & C) are operated by the chord; in this way, the versines are measured as analogue electronic signals, as shown in figure (1).

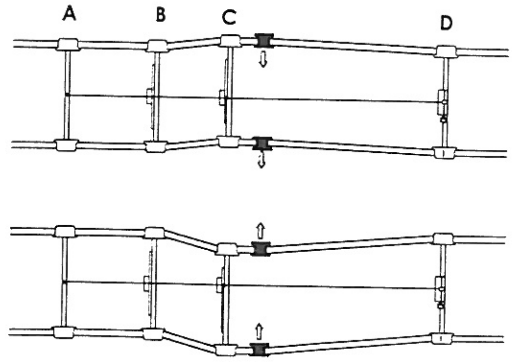

Fig. (1): Rail straightness measuring system

The laser device is fixed to the track in the front of the machine (at some fixed point) and is laterally adjusted according to the lining error as shown in figure (2). In the beginning of work, the laser device is pointed to the centerline of the laser receiver and fixed in this position for the rest of operation. By means of an automatic follow up control, the laser receiver is always positioned at the center of the laser beam and therefore determines the input of the reference values (Lining error).

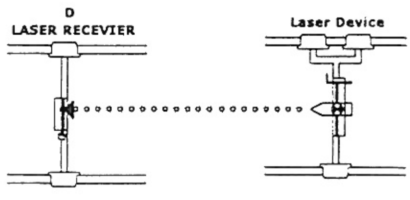

Fig. (2): Adjusting the laser device

The hydraulic actuating system is shown in figure (3), which consists of the proposed proportional control valve and two symmetrical cylinders.

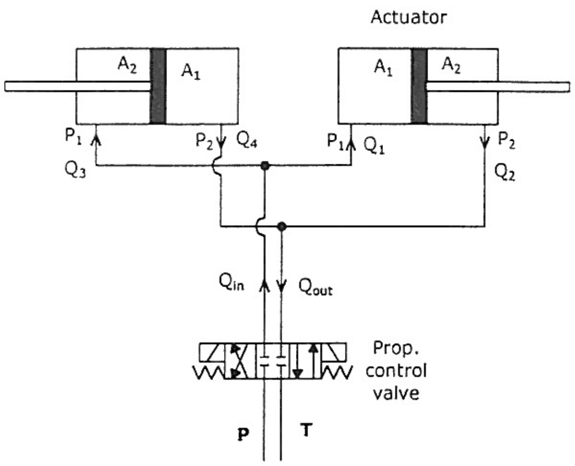

Fig. (3): Hydraulic system

2. DYNAMIC MODEL

Based on the hydraulic system shown in figure (3); the state space form in the packed matrix array of the valve, actuator and the load (all in series) are as:

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] =$$

$$\left[\begin{array}{cccc|cc} -\dfrac{\left(\frac{k_p}{k_3}\right)+2L}{(c_1+c_2)} & \dfrac{2L}{(c_1+c_2)} & -\dfrac{(A_1-A_2)}{(c_1-c_2)} & 0 & \dfrac{k_x k_1}{(c_1+c_2)} & 0 \\ \dfrac{2L}{(c_1+c_2)} & -\dfrac{\left(\frac{k_p}{k_4}\right)+2L}{(c_1+c_2)} & \dfrac{(A_1+A_2)}{(c_1+c_2)} & 0 & -\dfrac{k_x k_2}{(c_1+c_2)} & 0 \\ \dfrac{(A_1+A_2)}{M} & -\dfrac{(A_1+A_2)}{M} & -\dfrac{D_h}{M} & -\dfrac{k_s}{M} & 0 & -\dfrac{1}{M} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{1}$$

with a state vector of: $[p_1 \quad p_2 \quad \dot{y} \quad y]'$, and the no-load flow and pressure gain are: $k_x = k_f\sqrt{\left(\frac{p_s}{2}\right)}$ and $k_p = \dfrac{k_f x_{ss}}{2\sqrt{p_s}}$ respectively. The linearized constants k_1, k_2, k_3, k_4 are as:

$$k_1 = k_2 = \sqrt{1-\bar{F}} \quad , \quad k_3 = k_4 = \frac{1}{\sqrt{\left(1-\bar{F}/2\right)}}$$

$$\text{where, } \bar{F} = \frac{F}{p_s\,(A_1 + A_2)}$$

The dynamic equation of the proportional solenoid can be expressed in packed matrix array as:

$$G_s = \left[\begin{array}{c|c} -\dfrac{1}{T_{sp}} & \dfrac{K_n}{T_{sp}} \\ \hline 1 & 0 \end{array}\right] \tag{2}$$

The system block diagram is shown in figure (4).

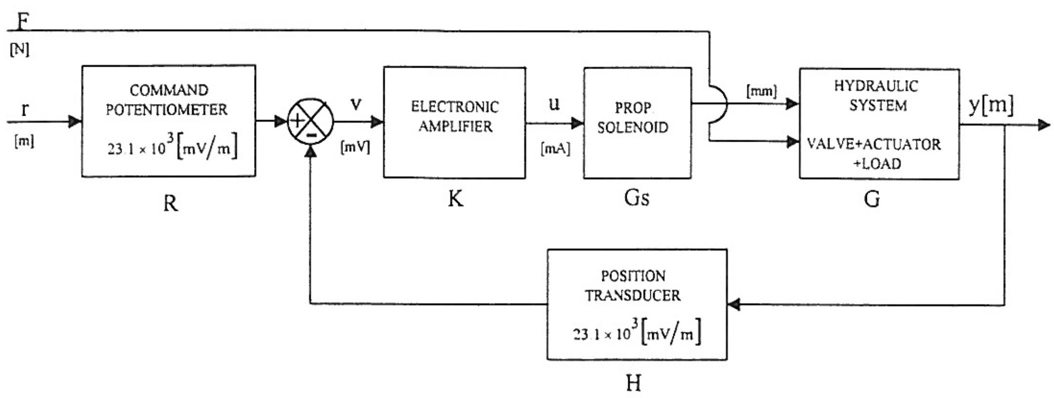

Fig. (4): Block diagram of the proportional lining system

3. SYSTEM ANALYSIS

The system response with the parameter nominal values is shown in figure (5). Although the system operated using a servovalve (MOOG series 62, flapper-nozzle type, valve bandwidth of 50 Hz) introduce a satisfactory performance (system bandwidth of 0.19 Hz, zero steady state) rather than the system operated with proportional control valve; the contamination effect and the low-grade maintenance may limit this performance. Also the system is sensitive to disturbance forces (introduce a steady state error of 4.25% due to applying 100 KN). While the proportional system is too slow (the uncompensated system bandwidth of 0.06 Hz); which directly affect the machine productivity, and shows a larger steady state error of 18%.

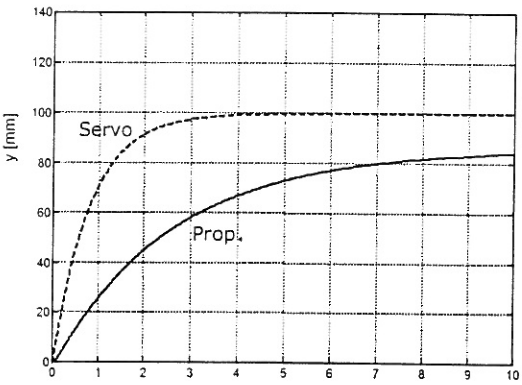

Fig. (5): Time response of the system

The effect of applying 100 [KN] as a disturbance input, which may arise when lining a track beside the platforms, on the time response of the system operated with the proportional control valve is shown in figure (6), which indicated a great sensitivity to the disturbance forces.

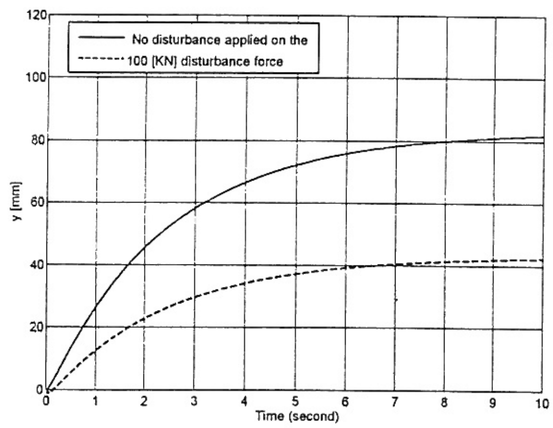

Fig. (6): Effect of 100 [KN] disturbance force

The effect of leakage flow is clearly visualized in figure (7); beside a considerable increase of the amount of damping, the leakage flow cause a deviation in the steady state position. Also, leakage flow plays an important role in the nonlinearity of the open loop dynamics of the hydraulic actuator. The effect of variation in the load mass is shown in figure (8), which indicates a considerable increase in the time constant and shows an increase in the steady state error with a heavy load.

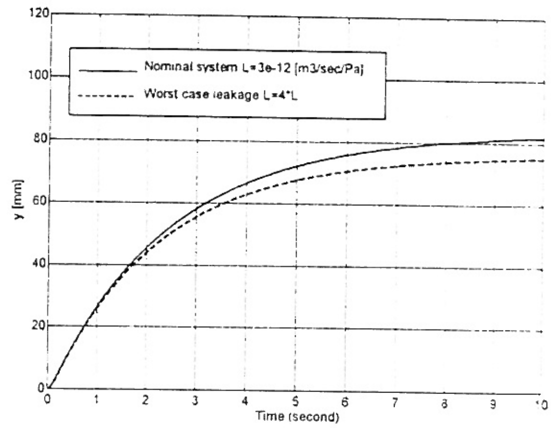

Fig. (7): Effect of leakage on the system response

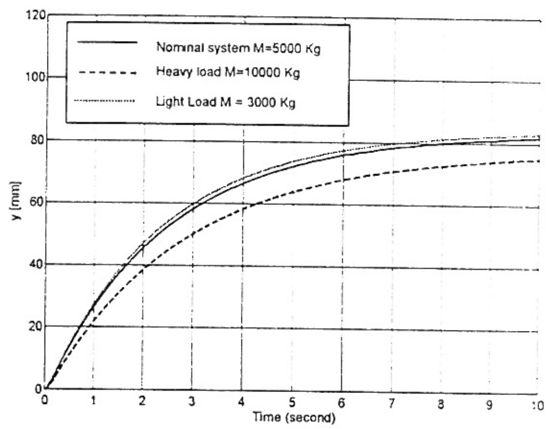

Fig. (8): Effect of load mass variation on the system

It can be concluded that the proportional system is more sensitive to disturbance and also to the parameter variation (leakage, friction, load mass).

4. PROBLEM DESCRIPTION

With the characteristics of the proportional control valve, it is required to seek a system with good performance under variations of system parameters, adequate rejection of external disturbances, good tracking characteristics, and also zero steady state error to step input. The closed loop objectives must be met robustly, i.e., over the whole range of uncertain parameters. To this end, the $\mathcal{H}_\infty$ robust control method for controller design is used. The objectives that aimed to achieve are summarized in the following:

1. Fast response with zero steady state error, and adequate rejection of external disturbances, while considering that the largest disturbance forces does not exceed 100 [KN], which may arises when lining tracks beside the platforms.

2. The controller action (control signal) must obey the saturation limit of the proportional valve (1.5 A).

3. The system must met the robust stability and robust performance conditions, with the parameters varies in the specified ranges as:

- The mass changes from 3000 to 6000 [kg]; according to the material of the sleepers (wood, concrete, steel) and the rail type. The heavy load mass can be set as 10000[kg]; when lining a switch.
- L: from (L) to (4*L), i.e., the leakage may increase four times as the nominal value.
- μ (Friction coefficient): from 0.2 to 0.7; according to the ballast kind on road viaduct.
- Unlike the servovalve; which mainly operates in the null region when it is controlling the hydraulic actuators and its operation points was the steady state equilibrium at which the steady state input (current) is zero. It is something difficult to estimate the steady state spool position, when the proportional valve considered. By considering both the no-load flow and pressure gains are uncertain parameters; and estimate the nominal values for both, one can deal with the non-linearities as a source of parametric uncertainties. The flow and pressure gains of the proportional control valve may be as follows: Flow gain (nominal): 3.954×10^{-4} [m^3/sec/mm] and its range is from 2.864×10^{-4} to 4.946×10^{-4} [m^3/sec/mm], Pressure gain (nominal): 3.52×10^{-11} [m^3/sec/N/m^2] and its range is from 3.488×10^{-12} to 7.812×10^{-11} [m^3/sec/N/m^2]

As required by the method, the combination of objectives and uncertainty are mapped into the frequency domain as a set of bounds that must be achieved by the nominal loop transfer functions.

5. CONTROLLER DESIGN

The $\mathcal{H}_\infty$ control design methods aims to press down the peak(s) of one or more selected transfer function(s) over all the frequency range specified. The block diagram in figure (9) is the same as figure (4), but with additional weights selected to shape the specified loops. It is suggested to add three weighting blocks; W_p: to restrict the behavior of the sensitivity transfer function and maximized to reach the best achievable performance, W_u: to limit the control action to the valve saturation limit. Next the demand for robustness is added by including the uncertainty weight W_T with the standard plant; to maintain the performance and stability with parameters variation over the specified range.

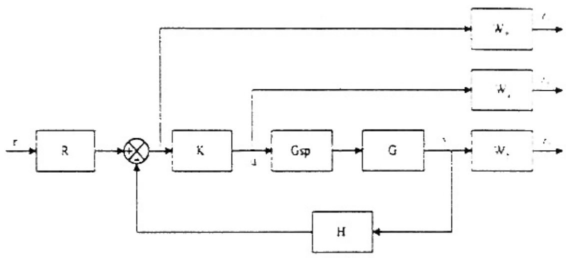

Fig. (9): Block diagram with the proposed weights

Identifying the outputs z and inputs w as:

$$z=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}=\begin{bmatrix} W_p e \\ W_u u \\ W_T y \end{bmatrix} \text{ and } w=[r]$$

the closed loop transfer matrix should take the form

$$\mathcal{T}=\begin{bmatrix} W_p RS \\ W_u KRS \\ W_T T \end{bmatrix} \tag{3}$$

The standard $\mathcal{H}_\infty$ optimal regulation problem is the problem of determining a compensator with transfer matrix K that stabilizes the closed loop system and minimizes the infinity norm of the closed loop transfer matrix $\mathcal{T}$ from the external input to the outputs specified. Thus, the $\mathcal{H}_\infty$ control problem should take the form as: find K that stabilizes the system such that:

$$\left\| \begin{matrix} W_p RS \\ W_u KRS \\ W_T T \end{matrix} \right\|_\infty \le 1 \tag{4}$$

which is a version of what is known as the mixed sensitivity problem. The name derives from the fact that the optimization involves both sensitivity and complementary sensitivity functions.

5.1. WEIGHTS SELECTION

PERFORMANCE WEIGH: For good tracking accuracy in each of the controlled outputs, the sensitivity function is required to be small. This suggests forcing integral action into the controller. While a pure integrator can not be included in W_p

anyway (since the standard $\mathcal{H}_\infty$ optimal control problem would not then be well posed); these weight must give a finite gain at low frequencies. A second order weight (selected to ensure a step changes) of the inverse of the performance weight can be formed as:

$$W_P^{-1} = 5\times10^{-3}\frac{\left(\frac{s}{0.045}+1\right)^2}{\left(\frac{s}{1.43}+1\right)^2} \qquad (5)$$

with such a weight the minimum bandwidth frequency required should be 0.64 [rad/sec] against 0.4 [rad/sec] for the uncompensated system, and the steady state error does not exceed 5×10^{-3} (0.5%), which is the low frequency gain.

INPUT WEIGHTING FUNCTION: By considering the control signal equation $u = KS\cdot r$. The magnitude of $|KS|$ in the low frequency range is essentially limited by the saturation limit of the actuator; Hence, the maximum gain $|KS|$ can be fairly large, while the high frequency gain is limited by the controller bandwidth. The first order high pass filter is used with a corner frequency to limit input magnitudes at high frequencies and thereby limit the closed loop bandwidth. Also, it is recommended to design a fast controller as fast as possible, so the controller bandwidth should be selected to be 1 [MHz], The inverse of the control signal weight has the form as in equation (6):

$$W_u^{-1} = 15\times10^{3}\frac{\left(\frac{s}{12.5\times10^{9}}+1\right)}{\left(\frac{s}{418}+1\right)} \qquad (6)$$

ROBUSTNESS WEIGHT: The multiplicative input uncertainty represented by:

$$G_p = G(I+W_T\Delta)\ ,\ |\Delta|\le1 \qquad (7)$$

should be selected to model the parametric uncertainties with an unstructured manner. It should be mentioned here that the choice of the multiplicative input uncertainty results from the robustness condition, $\|W_T T\|_\infty \le 1$; to avoid the objectives trade off. According to the parameter ranges listed in section (4); a rational transfer function weight $W_T(s)$ is constructed such that:

$|W_T(\omega)| \ge \ell(\omega) \quad \forall\ \omega$, where:

$$\ell(\omega) = \max\left|\frac{G_p(\omega)-G(\omega)}{G(\omega)}\right| \qquad (8)$$

The mulitiplicative input uncertainty description for the parametric uncertainties in the ranges specified are shown in figure (10), thus the uncertainty has been modeled as a first order multiplicative input uncertainty as:

$$W_T = 8\times10^{-1}\frac{\left(\frac{s}{22}+1\right)}{\left(\frac{s}{125}+1\right)} \qquad (9)$$

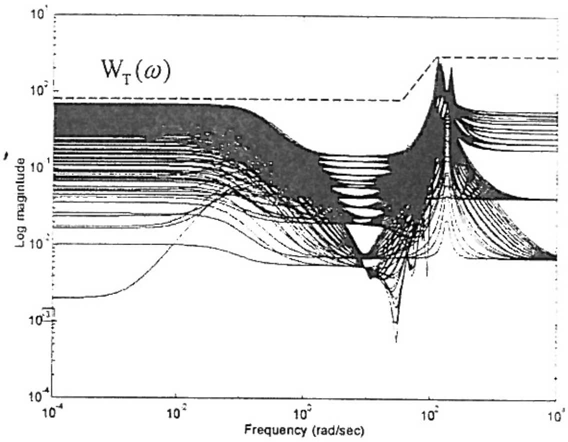

Fig. (10): Uncertainty weight

A solution of the corresponding $\mathcal{H}_\infty$ problem based on Riccati equations is implemented, the solution is documented in a paper by Doyle, et. al.(1989).

The Bode plot of the controller resulting from the solution of the $\mathcal{H}_\infty$ problem is shown in figure (11); seems to be as a lag-lead compensator with the break frequencies at 0.06 and 0.4 (rad/sec) for its pole and zero respectively (rough estimate). The controller expected to improve the steady state error by increasing the low frequency gain, about 7 [mA/mV].

The resulting $\mathcal{H}_\infty$ controller for the proportional lining system have 11 degree, which leads to considerable phase lag plus the complexity with regards to practical implementation. Figure (11) shows the main controller (n=11) and a reduced controllers with n=4 and n=3; the difference appear in the high frequency region with the third order controller. So, it is suitable to deal with the fourth order controller as the best controller for the proportional lining system.

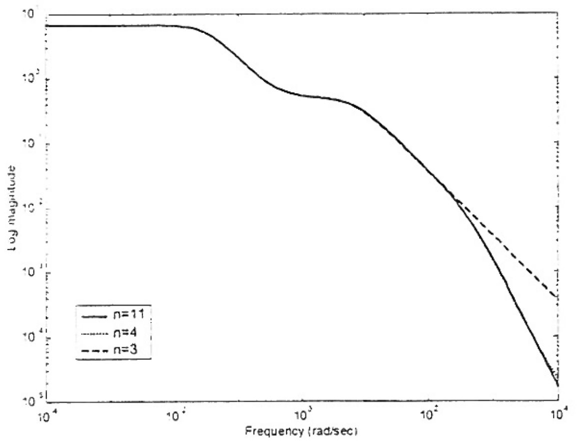

Fig. (11): The model reduction of the resulted $\mathcal{H}_\infty$ controller

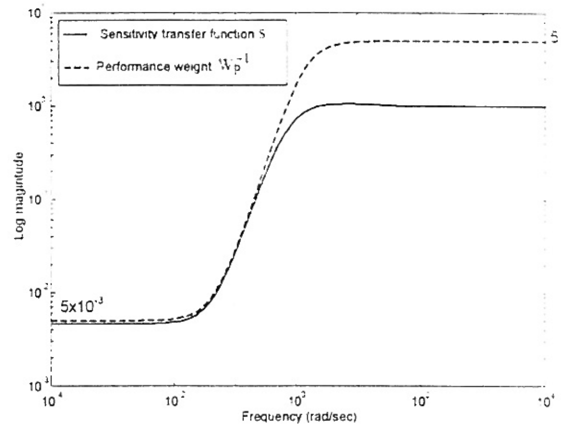

Fig. (12): Checking the Nominal performance

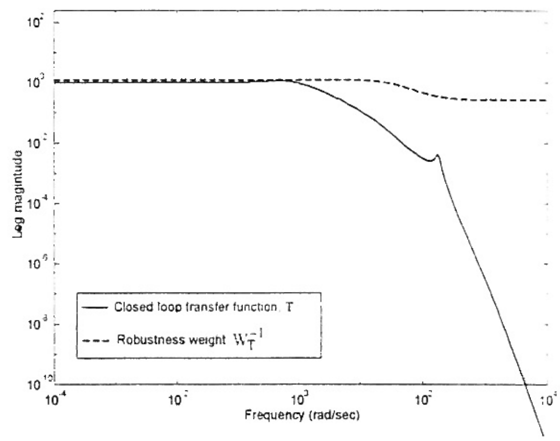

Fig. (13): Checking the robust stability

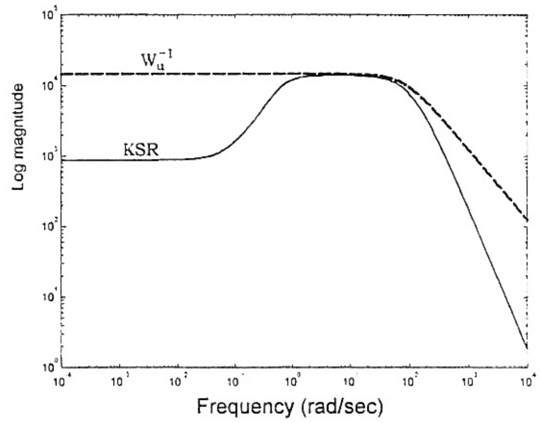

Fig. (14): Checking the control action

The transfer function of the controller should take the form as:

$$K(s) = \frac{7\cdot\left(1+\frac{s}{0.072}\right)\left(1+\frac{s}{0.4}\right)}{\left(1+\frac{s}{0.04}\right)\left(1+\frac{s}{0.054}\right)\left(1+\frac{s}{7.3}\right)\left(1+\frac{s}{402}\right)} \quad (10)$$

5.2. CONTROLLER EVALUATION

To verify the nominal performance and robust stability condition, it is recommended to measure the frequency responses for the reference inputs to the measured position output.

CHECK NOMINAL PERFORMANCE: Figure (12), shows that the sensitivity function S, which obeys the performance weight specified in equation (5) with the assigned parameters. This means that the compensated system is expected to have a steady state error not greater than 5×10^{-3} for a unit step input and a bandwidth not less than 0.64 (rad/sec).

CHECK ROBUST STABILITY: Since the uncertainties, which is modeled as a first order transfer function, equation (9), acts as an upper limits of the complementary sensitivity function T as shown in figure (13); the system is expected to be robust against the system uncertainties specified.

CHECK CONTROL ACTION SIGNAL: as shown in figure (14), while the high frequency current does not exceed the saturation limit (1500 [mA]); the steady state current should be about 9×10^{2} [mA/m], (90mA for 100mm reference value).

The step responses are considered for evaluating the closed loop system performance. This will allow to determine the tracking performance of the feedback loop with the controller, in addition of determining the amount of sensitivity to parameters variation, i.e., indicate the system robustness. For simplicity the step responses will only be evaluated for 100 [mm] reference input, and the effect of leakage, load variation, friction variation and disturbance will be discussed.

Figure (15), shows that, the steady state response is not affected by the presence of disturbance forces up to 100 [KN], with comparison to figure (6), there is a wide difference between the two responses. A very good disturbance attenuation is achieved by applying the designed $\mathcal{H}_\infty$ controller. While the response speed of the system is slightly improved; the settling time achieved is 8 [sec].

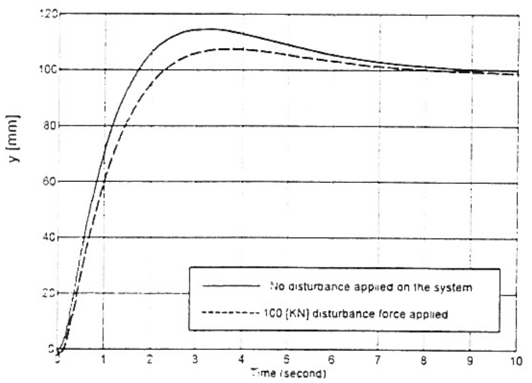

Fig. (15): Effect of disturbance force on the system

Figures (16,17,18), show that the system performance in slightly affected by the parameters uncertainties in the expected ranges. While there is no remarkable difference in the maximum overshoot of the system (12 % to 14 %), the rise time become greater with considerable increase in the inertia load (about 0.6 sec more).

Upon simulating a large valve size of 85 [LPM] rated flow instead of that with 45 [LPM], and by adjusting the control action weight to the corresponding valve saturation (800 [mA]); all performance specifications are satisfied, and the corresponding result with the modified $\mathcal{H}_\infty$ controller is shown in figure (19).

The system becomes faster than the older one, with a rise time of 1.2[sec] and settling time of 3.3 (rad/sec). The maximum overshoot is 13.9 %, and the steady state error is approximately 0.5 %.

6. CONCLUSION

- Based on $\mathcal{H}_\infty$ control algorithm, a robust controller is designed and implemented upon the railroads lining system. While the robust performance is satisfied; with a maximum overshoot not greater that 14% and steady state error not greater than 0.5%; the system bandwidth is limited to 0.1 [Hz] to maintain the system robustness. Otherwise, a larger valve capacity is required (85 LPM, 10 Hz, pilot operated proportional control valve without spool feedback) to achieve a compensated system with 0.32 [Hz].

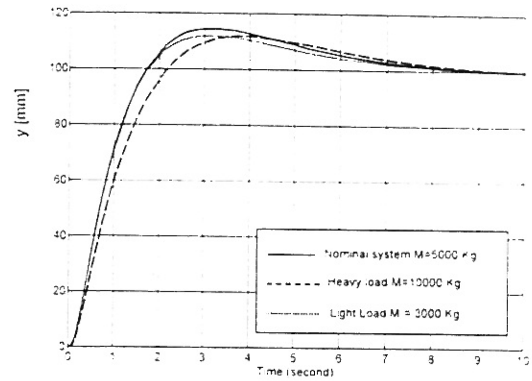

Figure (16): Effect of mass uncertainty on the system

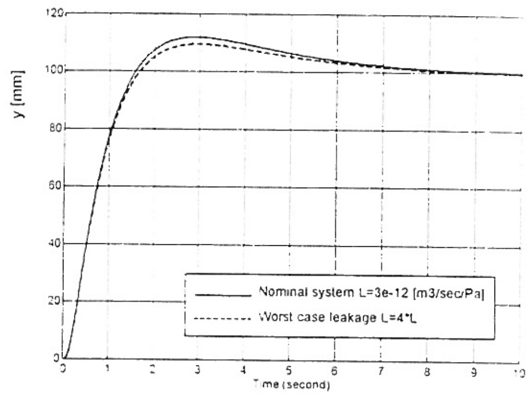

Fig. (17): Effect of leakage on the system performance

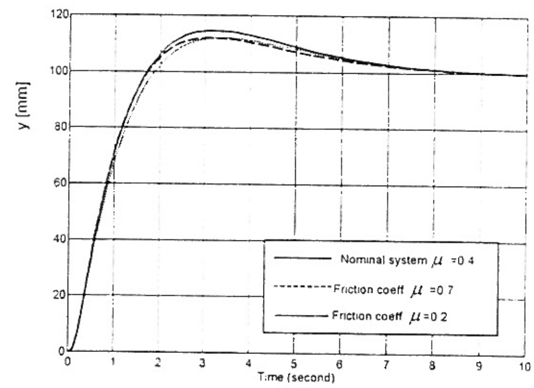

Fig. (18): Effect of friction (according to the ballast type)

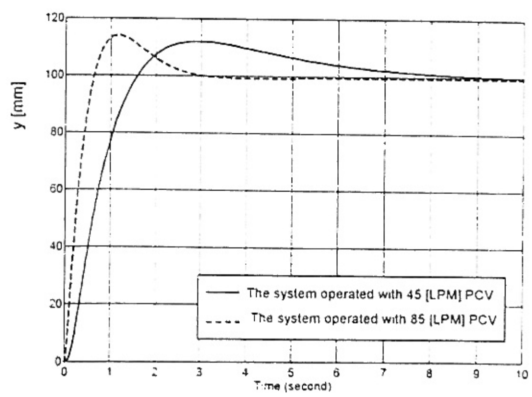

Fig. (19): Time response of the system with large valve

- If the formulation of the structured uncertainty model is tight: the $\mathcal{H}_\infty$ controller will not be very conservative in the sense that robust performance will not be arbitrary poor, **given** nominal performance and robust stability.

- Obtaining a stable $\mathcal{H}_\infty$ based controller depends on the proper choice of the weighting functions. It is seen through the design carried out, that the controller design is very sensitive to the choice of the weighting functions.

- The $\mathcal{H}_\infty$ controller is designed with a computationally demanding method, and the resulting controller is of very high order, which are normally reduced using some model reduction techniques, before implementation. The balanced truncation method is applied: and the resulting controller is stable and the internal stability of the system are satisfied.

NOMENCLATURE

D_h	Damping factor [N.sec/m]
G	Nominal plant transfer function
G_p	Particular perturbed plant models.
k_1, k_2, k_3, k_4	Linearized coefficients of the pressure-flow relationship.
k_f	Valve sizing constant
k_p	No-load pressure gain [m^3/sec/Pa]
K_n	Solenoid gain [mA/V]
k_s	Spring constant [N/m]
k_x	No-Load flow gain [m^3/sec/mm]
L	Laminar leakage coefficient [m^5/N.sec]
M	Actuator load inertia [Kg]
p_1, p_2	Pressures in actuator chamber 1,2 [N/m^2]
S	Sensitivity transfer function
T	Nominal closed loop transfer function.
$\mathcal{T}$	Transfer matrix from w to z, with no uncertainty
T_{sp}	Spool time constant.
$v(t)$	Controller input signal
W_p	Performance weight.
W_T	Robustness weight.
W_u	Control weighting function.
$w(t)$	(Vector of) Exogenous input signal
$z(t)$	(Vector of) desired output signal(s)
μ	Friction coefficient

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Ahmed El-Betar is a lecturer in BENHA Institute of Technology since 1989. He obtained his Msc degree in mechanical engineering in 1989 and his Ph.D. (*Design and vibration control using optimal control theory applied on DAF-vehicle*) from Huddersfiled University Uk in 1993. He has great interests in modeling and system dynamics, vibration and noise control, and finite element technique mainly vehicle design application.

Ayman Nada is a mechanical engineer, working as a demonstrator and lecturer assistant in BHIT since 1998, has a great experience in railroads constructions since he works in the Luxer-Aswan duplication project 1996-1998. He has great interests in fluid power control including classical, PLC, and modern control techniques. He obtained his master degree (*robust $\mathcal{H}_\infty$ control of railroads lining system*) in Dec. 1999.