S-Shaped Fuzzy Flip-Flops

Rita Lovassy *, László T. Kóczy **

* Institute of Microelectronics and Technology, Kandó Kálmán Faculty of Electrical Engineering, Budapest Tech
Tavaszmező u. 15-17, H-1084 Budapest, Hungary
lovassy.rita@kvk.bmf.hu

** Institute of Information Technology, Mechanical and Electrical Engineering, Széchenyi István University, Győr, Hungary and Dept. of Telecommunication and Media Informatics, Budapest University of Technology and Economics, Hungary
koczy@sze.hu, koczy@tmit.bme.hu

Abstract: The multilayer perceptron is an artificial neural network that learns nonlinear function mappings. Nonlinear functions can be represented by multilayer perceptrons with units that use nonlinear activation functions. The neurons in the multilayer perceptron networks typically employ sigmoidal activation function. The next state of the J-K fuzzy flip-flops (F3) using Fodor, Yager and Dombi operators present S-shaped characteristics. An interesting aspect of F3-s might be that they have a certain convergent behavior when one of their inputs (e.g. J) is exited repeatedly. If J is considered the equivalent of the traditional input of a neuron (with an adder unit applied before J), K might play a secondary modifier's role, or can just be set fix. The paper proposes the investigation of such possible F3-networks as new alternative types of neural networks.

Keywords: J-K fuzzy flip-flop, Yager, Dombi t-norm, t-conorm, Sigmoid shaped function, ANN

1 Introduction

Neural networks and fuzzy set theory has been the object of intense study and application, especially in the last decade. There are several manners to combine neural networks and fuzzy logic. The main idea is using the high flexibility of neural networks produced by learning, in order to tune the membership functions used in fuzzy control. The aim of this approach is to improve neural network frameworks by bringing some advantages of fuzzy logic. Obviously the introduction of neural networks and fuzzy control into areas in which the analysis and design of control systems is traditionally performed using techniques whose effectiveness is well established, has led to a certain differentiation in the way in which researchers and designers consider these new methodologies. The real problem does not lie in a direct comparison between 'new' and 'traditional' methodologies, but rather in their field of applicability. Both neural networks and fuzzy models were developed to deal with problems which were hard or impossible to solve using traditional techniques. When direct comparisons are made, one inevitably risks going beyond the specific environments in which the methodologies should be considered. The aim of this study is essentially that of showing how neural networks and fuzzy model can be usefully merged and employed to solve nonlinear control and modeling problems, simplifying and automatizing both the process modeling and the controller synthesis phases.

2 Artificial Neural Networks (ANN)

Artificial neural networks are built from simple units, called neurons. The processing ability of the network is stored in weights, obtained by a process of adaptation to, or learning from a set of training patterns. These networks usually organize their units into several layers [8]. The first layer is called input layer – the input signals is feedforward to the network, the last one the output layer – the neurons forward the information in the outside word. The intermediate layer is called hidden layer. Classical artificial neural networks, such as a feedforward network allow signals to flow from the input units to the output units, in a forward direction.

The role of the network is to learn the association between input and output patterns, or to find the structure of the input patterns. The learning process is achieved through the modification of the connection weights between units. The basic neural unit processes the input information into the output information using the computation and the transformation of the activation. A unit collects information provided by the external world (other units) to which it is connected with weighted connections. These weights multiply the input information.

The architecture (i.e. the pattern of connectivity) of the network, along with the transfer function used by the neurons and the synaptic weights, completely specify the behavior of the network.

3 Fuzzy J-K Flip-Flops

J-K flip-flops are elementary digital units providing sequential features/memory functions. Their definitive equation is used both in the minimal disjunctive and conjunctive forms. Fuzzy connectives do not satisfy all Boolean axioms, thus the fuzzy equivalents of these equations result in two non-equivalent definitions, 'reset and set type' fuzzy flip-flops (F^3) by Hirota *& al.* when introducing the concept of F^3 [2, 3]. There are many different norms known from the literature which play important roles in applications or by their mathematical properties. Only very few have been investigated from the point of view of what kind of F^3 is generated, namely the standard, algebraic norms and a pair of operations generated from the standard and the Łukasiewicz operations, later proposed by Fodor [2, 3, 7].

From a practical aspect it is confusing that reset and set type F^3s sometimes do have very different behavior. (The unique exception is the Fodor F^3.)

In [7] we studied the behavior of F^3 based on various fuzzy operations. We evaluated the graphs belonging to the next states of reset and set type fuzzy flip-flops for various values of *J*, where each diagram presented the curves for different values of *Q* and *K*. We noticed, that the characteristics of Fodor type fuzzy flip-flops (F^4), Yager and Dombi class graphs produced smooth (differentiable) curves and surfaces with no breakpoints at all. This paper presents the behavior of these three classes of F^3s, according to the particular cases when one of the input *K* or the present state *Q* has a constant value.

Figure 1 depicts the behavior of F^4 for some typical values of *J, K* with a constant *Q*=0.25 value, and *K*=0.00 for typical *Q* and *J* values. The characteristics belonging to these cases show sigmoidal behavior.

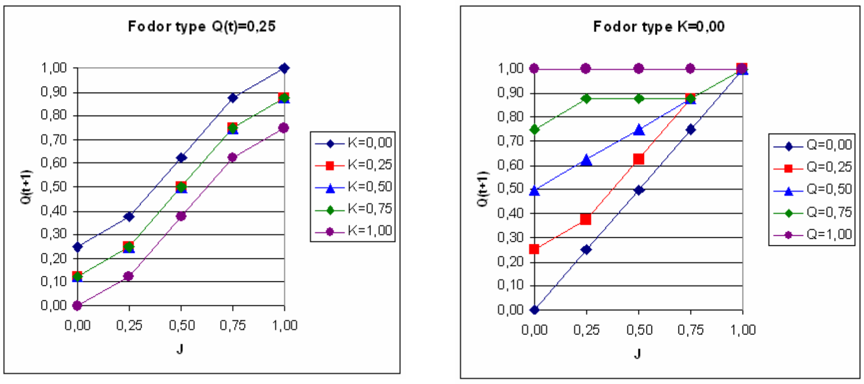

Figure 1
Characteristics of F^4 for various values of *J, K*

3.1 Yager and Dombi Fuzzy Flip-Flops

Several classes of functions had been proposed whose individual members satisfy all the axiomatic requirements for the fuzzy union and intersection. The behavior of the Yager fuzzy flip-flop was shown in [7], for typical cases of parameter *w*.

One of these classes of fuzzy unions is known as the Yager class t-conorm and is defined by the function:

$$u_w(a,b) = \min\left[1,(a^w + b^w)^{1/w}\right], \tag{1}$$

where values of the parameter *w* lie within the open interval $(0, \infty)$.

The family of Yager t-norms, introduced in the early 1980s by Ronald R. Yager, is given by

$$i_w(a,b) = 1 - \min\left[1,((1-a)^w + (1-b)^w)^{1/w}\right]. \tag{2}$$

In the formulas, *a* represents the membership grade for an element in fuzzy set A, and *b* represents the membership grade for an element in fuzzy set B.

The Yager t-norm is nilpotent if and only if $0 < w < +\infty$ (for $w = 1$ it is the Łukasiewicz t-norm). The family is strictly increasing and continuous with respect to *w*.

Using these definitions we can find the solution for:

$$Q_R(t+1) = u_w\left[i_w(J,1-Q), i_w(1-K,Q)\right] \text{ and} \tag{3}$$

$$Q_S(t+1) = i_w\left[u_w(J,Q), u_w(1-K,1-Q)\right]. \tag{4}$$

Figure 2 presents the diagrams for Yager reset and set type F^3, for the typical case Q=0.50 and $w = 2$. The real curves are here also smooth and S-shaped.

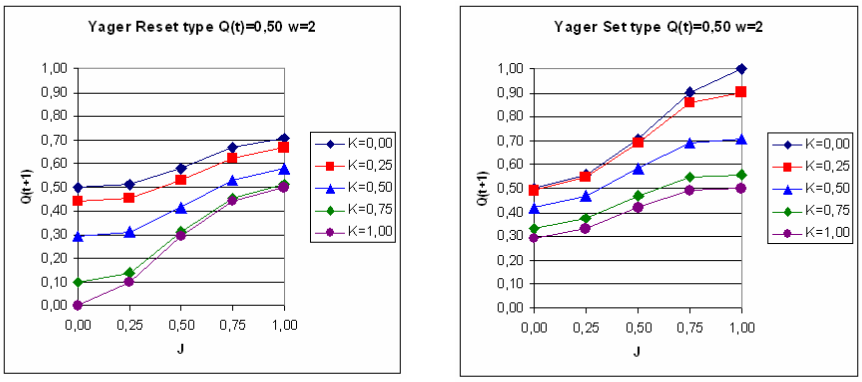

Figure 2
Characteristics of Yager reset and set type F^3 for various values of *J, K*

The main feature of these norm is that as parameter *w* varies among 0 and infinite, the t-norms (and t-conorms) obtained span the space of fuzzy t-norms (and t-conorms) between the drastic product (drastic sum) and the minimum (maximum) [6].

We would like to present the unified equation of reset and set type as it was proposed in [10]. This equation simultaneously involved both set and reset characteristics. Therefore, in order to extend the binary J-K flip-flop to a fuzzy flip-flop smoothly, we must apply the newly defined function

$$Q(t+1)=\begin{cases}(J\vee Q)\wedge\left((1-K)\vee(1-Q)\right) \text{ if } (J\geq K)\\ \left(J\wedge(1-Q)\right)\vee\left((1-K)\wedge Q\right) \text{ if } (J\leq K)\end{cases} \tag{5}$$

The fundamental equation of the binary J-K flip-flops

$$Q(t+1)=\overline{\bar{J}K+\bar{J}\bar{Q}+KQ}=\left(J+\bar{K}\right)\left(J+Q\right)\left(\bar{K}+\bar{Q}\right) \tag{6}$$

Using complementation, Yager-class t-norm and t-conorm, the equation (6) is expressed as

$$Q(t+1)=\left(J\ \ u_w\ \ (1-K)\right)\ \ i_w\ \ \left(J\ \ u_w\ Q\right)\ i_w\left((1-K)\ \ u_w\ \ (1-Q)\right) \tag{7}$$

One can easily confirm that equation (7) is equivalent to equation (5). In a similar way, using complementation and Dombi-class operators, the equation of the next state is expressed as

$$Q(t+1)=\left(J\ \ u_\alpha\ \ (1-K)\right)\ \ i_\alpha\ \ \left(J\ \ u_\alpha\ Q\right)\ i_\alpha\left((1-K)\ \ u_\alpha\ \ (1-Q)\right) \tag{8}$$

where

$$u_\alpha(a,b)=\frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\alpha}+\left(\frac{1}{b}-1\right)^{-\alpha}\right]^{-1/\alpha}} \tag{9}$$

and

$$i_\alpha(a,b)=\frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{\alpha}+\left(\frac{1}{b}-1\right)^{\alpha}\right]^{1/\alpha}} \tag{10}$$

are called the Dombi operators, the parameter α lies within the open interval $(0,\infty)$.

The aim of the unification was the use of the fuzzy J-K flip-flop as a neuron. In order to test the behavior of the curves with regard to the shape and curvature, several characteristics were performed. Several values of the Yager parameter were considered, in an effort to tune the dissimilarity measure. On one hand the equality K=1-Q, and on the other hand the Yager t-norm and t-conorm with different parameters *w* are used. The worst result, the linear characteristics, are obtained when values *w* < 2 are used (Figure 3).

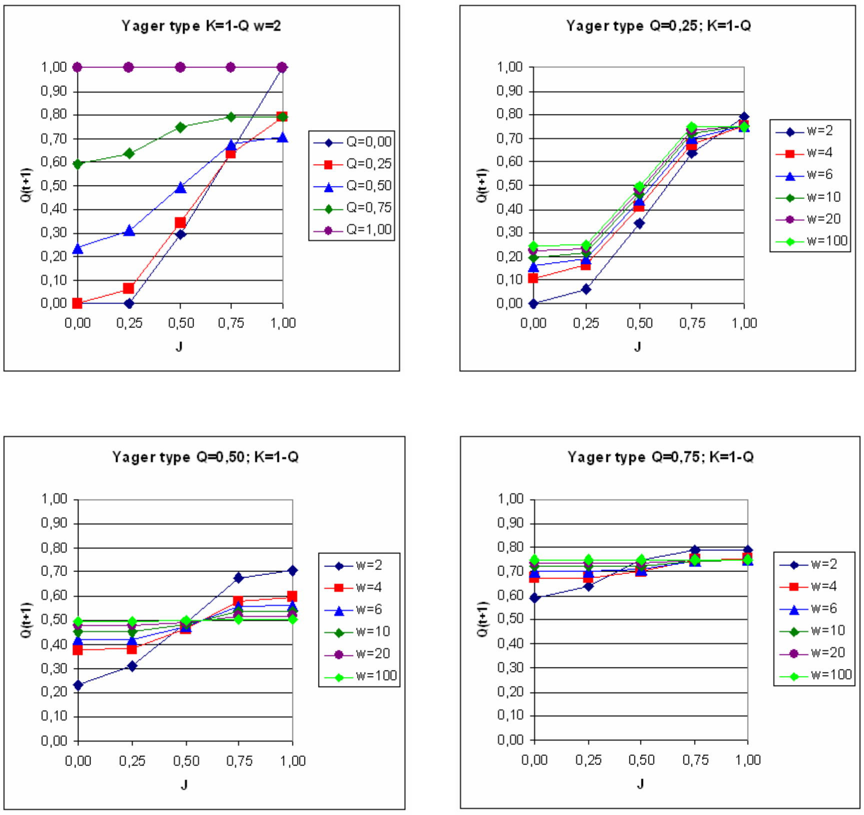

Figure 3
Characteristics of unified Yager type F^3 for various values of *J, K*

Figure 4 depicts typical values in the Dombi operator case for the typical cases when α = 1, 4, 6, 10 and 100.

If *J* = 0, *K* = 0 or *Q* = 0 the values are obviously at the limit. These curves are also smooth S-shaped sigmoidal curves.

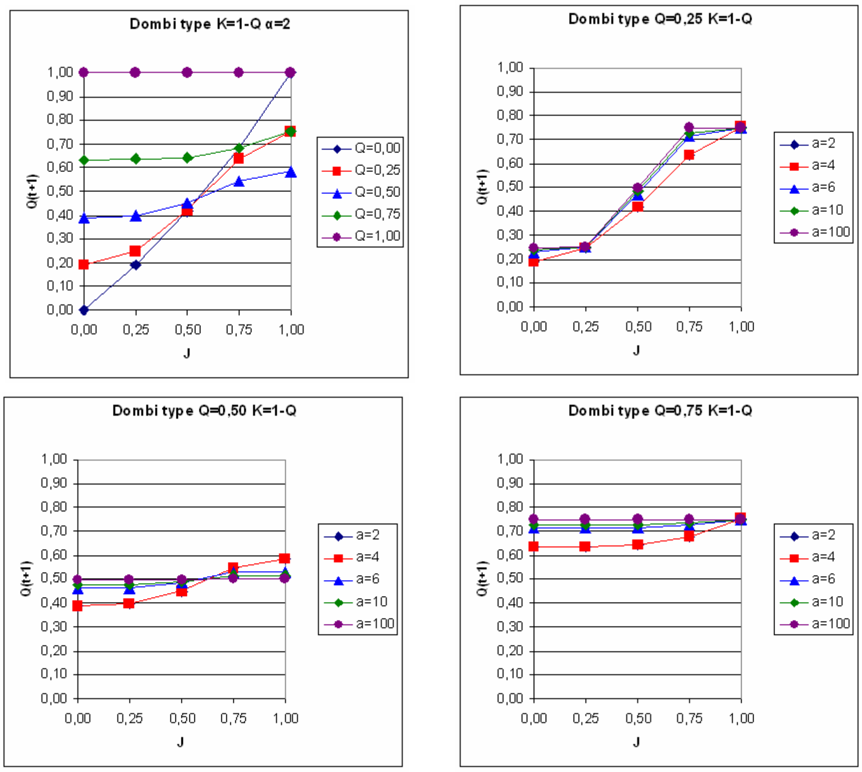

Figure 4

Characteristics of unified Dombi type F^3 for various values of *J*, *K*

4 Fuzzy J-K Flip-Flop as a Neuron

With the use of fuzzy logic techniques, neural computing can be integrated in symbolic reasoning to solve complex real world problems. In fact, artificial neural networks, expert systems, and fuzzy rule systems, in the context of approximate reasoning, share common features and techniques. A model of fuzzy system is proposed, in which an artificial neural network-like approach is designed to construct the knowledge base of an expert system.

We intend to do some comparisons of performed pertaining to the usefulness and realization of several logical connectives, followed by a purposeful fuzzy sequential system design in order to construct a network performing better than those described in the literature. The combinational part of the circuit will be embodied in a single layer neural network in which weights are adjusted on the basis of training situations. Numerical considerations highlight the performance of the design process.

In our concept, K=1-Q is proposed, because if input K of the fuzzy J-K flip-flop is connected with the $\overline{Q}$ output, we receive an elementary fuzzy sequential unit with just one input and one output. The proposed F^3 next state values follow a sigmoidal function such as in an artificial neural network neuron.

It is well known, that the nonlinear characteristics exhibited by neurons are represented by a transfer function such as a binary sigmoidal function. The neuron uses the standard sigmoidal activation function, and a convenient mathematical form for this is given by:

$$\sigma_a^{(\lambda)}(x) = \frac{1}{1+e^{-\lambda(x-a)}} \tag{11}$$

This is the form of the 'S-shaped sigmoid curve' transfer function, where a is the mean value and λ is the slope of the function. The sigmoid also has an easily calculated derivative, which is used when calculating the weight updates in the network. It thus makes the network more easily manipulable mathematically.

An interesting aspect of F^3-s might be that they have a certain convergent behavior when one of their inputs (e.g. J) is exited repeatedly. This is true even if the other input (K) is kept at a constant value. The behavior is more versatile if both inputs are given a series of changing values. If J is considered the equivalent of the traditional input of a neuron (with an adder unit applied before J), K might play a secondary modifier's role, or can just be set fix (Figure 5). The paper proposes the investigation of such possible F^3-networks as new alternative types of neural networks.

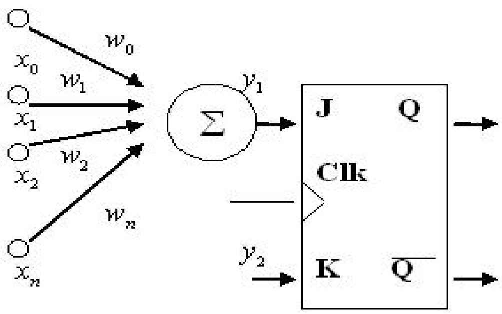

Figure 5
Fuzzy J-K flip-flop as a neuron

Conclusions

In this paper, various special fuzzy J-K flip-flops were defined as elementary units in a new fuzzy sequential digital model. In the future research, of particular interest might be the areas of pattern recognition and computer vision, natural language and text understanding, speech processing, data mining and also general neural computing, machine learning, further fuzzy hardware architectures, software tools, and others for possible applications and further investigations.

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