Several Case Studies Solved by Plausible Reasoning

Claudiu Pozna

Departament of Product Design and Robotics, University Transilvania of Brasov
Bd. Eroilor 28, 500036 Brasov, Romania
e-mail: cp@unitbv.ro

Abstract: Present paper continues the researches on cognitive system design. The goal of the paper is to illustrate the variety of models which can be constructed using the Bayesian plausible reasoning theory. The first case study develops a classical differential model into a Bayesian model. The second case study solves a geometry problem by plausible reasoning. The third case study models the human reasoning presented by the famous story of Sun Tzu: 'Advance to Chengang by a hidden path'.

Keywords: model, Bayesian theory, plausible reasoning

1 Introduction

Present paper continues the author's researches on cognitive system design. These researches have been started by a phenomenological analysis of AI collocation and have continued by researches on modeling with Bayesian plausible reasoning. The goal of this paper is to illustrate the variety of phenomenon which can be modeled using the mentioned theory. For this reason we have structured our paper in four parts. The presentation starts with a briefly introduction of the plausible reasoning theoretical background. The second part illustrates the transformation of a classical differential model into a Bayesian model. The third part represents a plausible reasoning solution of a geometrical problem. In the end we have tried to model a human reasoning example. More precisely, our intention was to explain (by modeling) the famous story of Sun Tzu: 'Advance to Chengang by a hidden path'.

The principles of Plausible Reasoning:

1 The representation of degree of plausibility is given by the plausibility function:

$$p:\Phi \rightarrow [0 \quad 1];\ p(A\,|\,X) = y \tag{1}$$

where: Θ is a set of sentences; $p(A \mid X)$ is a continuous and monotonic function which associates a particularly degree of truth for the sentence A in the condition that sentence X is true;

2 The consistence of the commune sense requires the following property for the p function

$$p(AB \mid X) = p(A \mid X)p(B \mid AX) \tag{2}$$

$$p(A \mid X) + p(\neg A \mid B) = 1 \tag{3}$$

$$p(A + B \mid X) = p(A \mid X) + p(B \mid X) - p(AB \mid X) \tag{4}$$

$$p(A_i \mid X) = \frac{1}{n} \qquad i = 1...n \tag{5}$$

where $\{A_i\}|_{i=1...n}$ is a complete set of mutual excusive sentence

Some comments are necessary:

- by consistence we mean:
  - every possible way of reasoning a sentence must lead to the same result;
  - the equivalent sentences have an equal degree of plausibility;
- in order to obtain the degree of plausibility for a sentence we must take into account all the evidence available;
  - $p(AB \mid X)$ means the plausibility of sentence A **and** B in the condition that sentence X is true;
  - $\neg A$ means **non A;**
  - $p(A + B \mid X)$ means the plausibility of sentence A **or** B in the condition that sentence X is true;

Theoretical results:

Analyzing the mentioned postulates, theoretical results can be deduced. From the beginning we will mention that because the probability function has the same properties (1…5) it can be accepted that the plausibility function is synonymous with the probability function. This is the only reasons that theoretical results from probability theory can be transferred to the theory of plausible reasoning [7].

We will resume presenting the Bayesian theorem which can easily deduce from (1-5). If we name by d the evidence of an experiment and by $h_{i=1...n}$ a set of mutual exclusive hypotheses the Bayesian theorem tells us that the plausibility of hypothesis h_i in the condition of evidence d is equal with the plausibility of hypothesis h_i multiplied by the plausibility of evidence d in the condition that hypothesis h_i is true and divided by the sum of the same product made for all the hypotheses of the set.

$$p(h_i \mid d) = p(h_i)\frac{p(d \mid h_i)}{\sum_{k=1\ldots n} p(h_k)p(d \mid h_k)} \tag{6}$$

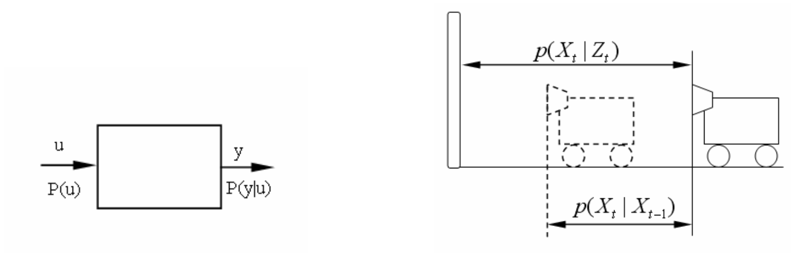
The plausibility of hypothesis h_i in the condition of evidence d is named the a posteriori knowledge, the plausibility of hypothesis h_i is named the a priori knowledge and the plausibility of evidence d in the condition that hypothesis h_i is trough is named the likelihood. The sum from the denominator is named the marginalization sum.

In order to converge to the model construction we will link this theoretical result to the Bayesian filter [2].

2 The First Case Study

The first case study transforms a classical differential model into a Bayesian model. The main achievement of this transformation is the possibility to associate at the computed output the plausibility of this result. In this way we can associate to the output quantity the quality of the degree of truth (see Figure 1a)

In order to exemplify the mentioned theoretical results we will consider the case of a mobile robot which modifies his state (position) and – from time to time – make observations (measure his position), see Figure 1b.



a) b)

Figure 1
The mobile robot

The dynamic model of the robot is very simple (the robot has a constant speed):

$$x_k = x_{k-1} + \Delta \tag{7}$$

where: x_k is the position of the robot.

We know that this model is only on approximation of the reality and from moment two developments – knowledge improvements – are possible:

- developing our model eventual by adaptation: adjust the apropriate value of Δ or introduce new parameters;
- constructing the Bayesian filter over this model.

We have chosen the second possibility which can be mathematical described by the following equations

$$x_k^{est} = x_k + \pi^{est} \tag{8}$$

where: x_k^{est} is the outputs estimations; x_k is the model output; π^{est} is the model perturbations.

We don't know a priori the model perturbation but we can obtain, by experiments, the statistical distribution of π^{es}: $p(\pi^{est})$. This distribution accomplishes (1) so we can define the estimation plausibility like the degree of truth for the following sentence: *'the estimated output k for our model is* x_k^{est} *'*.

From (8) we have:

$$p(\pi^{est}) = p(x_k^{est} - x_k) \tag{9}$$

We must note that using the model we will obtain the state *k* from state *k-1* so we can rewrite (9)

$$p(\pi^{est}) = p(x_k^{est} - x_k) \equiv p(x_k^{est} \mid x_{k-1}) \tag{10}$$

Using the Bayesian rule (6) we can write:

$$p(x_k^{est}) \propto \sum_{x_{k-1}} p(x_{k-1}) p(x_k^{est} \mid x_{k-1}) \tag{11}$$

where: $p(x_k^{est})$ is the plausibility of the output estimation; $p(x_{k-1})$ is the plausibility of state x_{k-1}; $p(x_k^{est} \mid x_{k-1})$ is the plausibility of the estimation when we know the state x_{k-1}; $\propto$ means proportional.

If during locomotion we measure (make observations), we can describe this process in the following mathematical form:

$$x_k^{mes} = x_k^{est} + \pi^{mea} \tag{12}$$

where: x_k^{mes} is the output measurement; π^{mea} is the measurement perturbation.

Once again we don't know a priori the value of the measurement perturbation but if we experiment our sensor we can obtain a statistical distribution of these values. We can write:

$$p(\pi^{mes}) = p(x_k^{mes} - x_k^{est}) \equiv p(x_k^{mes} \mid x_k^{est}) \tag{13}$$

Using (6) we obtain:

$$p(x_k^{mes}) \propto p(x_k^{est})p(x_k^{mes} \mid x_k^{est}) \tag{14}$$

If we use normalized distribution we can transform (11) and (14) in equations.

For the purpose of the Bayesian filter constructing we must define:

- variable definition:

$\{x_k\}_{k\in\{0,\ldots n\}}$ the system states are the position of the robot

$\{x_k^{mes}\}_{k\in\{0,\ldots n\}}$ we will measure the position;

- decomposition

$$p(x_1^{est},\ldots x_n^{est}, x_1^{mea},\ldots x_n^{mea}) = \prod_{i=0}^{t} p(x_k^{est} \mid x_{k-1})p(x_k^{mes} \mid x_k^{est}) \tag{15}$$

- initial knowledge:
  - the initial state distribution, is obtained after experiments, in this case we have chosen the following Gaussian distribution;

$$p(x_0) \propto \exp\left(-\frac{(x-x_0)^2}{2\cdot 0.1^2}\right) \tag{16}$$

  - the transition **model** from state k-1 to state k, is presented in (12), the mathematical form of this distribution can be obtained from experimental measurement, once again we have chosen a Gaussian distribution:

$$p(\pi^{est}) = p(x_k^{est} \mid x_{k-1}) \propto \exp\left(-\frac{(\pi^{est})^2}{2\cdot 0.2^2}\right) \tag{17}$$

  - the sensor **model**: the mathematical form of this distribution can be obtained from experimental measurement, once again we have chosen a Gaussian distribution

$$p(\pi^{mes}) = p(x_k^{mes} \mid x_k^{est}) \propto \exp\left(-\frac{(\pi^{mes})}{2\cdot 0.05^2}\right) \tag{18}$$

- the question is the plausibility of the each state when we know the transition plausibility and the measurement (sensor) plausibility; in order to compute this results we have used (11) and (14):

$$p(x_k^{mes}) \propto \left(\sum_{x_{k-1}} p(x_{k-1})p(x_k^{est} \mid x_{k-1})\right)\cdot p(x_k^{mes} \mid x_k^{est}) \tag{19}$$

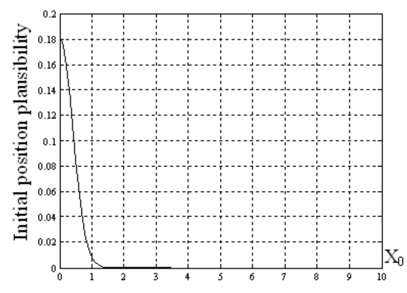


Figure 2
The initial state

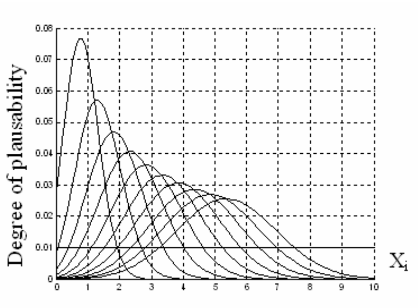


Figure 3
Transitions without observations

The response is a distribution for each *k=0…n*. This distribution has a maximum value which is the most plausibile answer to the question. More precisly, each itteration we obtaine a 2 component information: the most plausible answer (the robot position) and the value of its plausibility. Even the initial data are not crispy because we must admit that we don't know with precision this data (see Figure 2).

The robot has several state transition and no observations are made during this transitions. Simulation results are presented in Figure 2. If we analyze this result the main conclusion is that even the translation value – according to (7) – remains constant, the degree of plausibility has decreased continuously from translation to translation. This means that the degree of truth decrees continuously.This is an obvious situation, because a scientist has already the feeling that using repeatedly a model the degree of confidence will decrease. In this case the benefit is that we can compute this decreasing and of course we can take decisions after these results.

If the robot performs several observations – without performing any transition. In Figure 4 where we have presented the results of this simulation we can see that the degree of plausibility increases continuously and converges to value 1 (absolute trust).

In Figure 5 two particular situations are compared. There are two observations which start from the same state. In the first case, when the observation reproduces the value of the state, we will obtain a bigger rising. At contrary, in the second case there is a difference between the observation and the state. This difference will rule to a smaller degree of truth.

After these results the conclusion is that we can impose a minimum value of truth and perform observations only if we are below of this value. This is a more realistic strategy which is presented in Figure 5. The minimum truth value is 0.1. We have started from 0.18 and after five transitions we are below this value. In this moment we have performed an observation which increased the confidence value to 0.22.

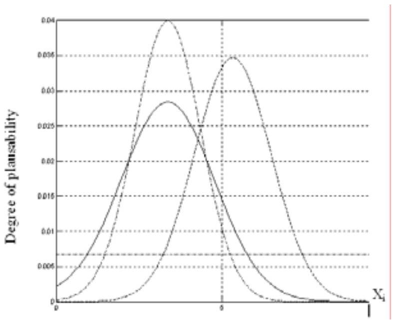


Figure 4
Two observation (.-.- and ---) which starts from the same state (-)

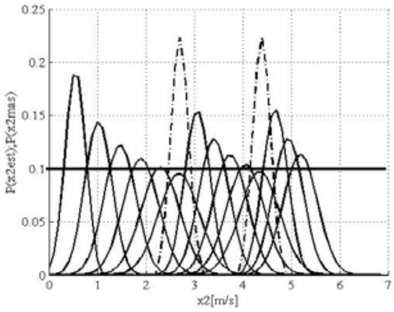


Figure 5
Increasing the plausibility by several observations

3 The Second Case Study

The second case study solves a geometry problem by using plausible reasoning. The problem that we intend to solve is the following:

Problem: If ABC is an isosceles triangle, $M \in AC$ *and* $AM = MC$ *then* $BM \perp AC$, a priori we know that if *ABC is an isosceles triangle and* $BM, \neg\perp, AC$ *then* $AM \neq MC$

If we will rewrite the problem by using the plausibility function we will obtain:

$$p(\perp|\,\Delta,=) = 1 \quad \text{if} \quad p(=|\,\Delta,\neg\perp) = 0 \tag{20}$$

where: $p(\perp|\,\Delta,=)$ is defined like the plausibility that $BM \perp AC$ when we know that ΔABC is isosceles and $AM = MC$; $p(=|\,\Delta,\neg\perp)$ is defined like the plausibility that $AM = MC$ when we know that ΔABC is isosceles and $BM, \neg\perp, AC$

From (2-5) we can write:

$$\begin{aligned} p(\perp|\,\Delta,=) &= \frac{p(=,\Delta,\perp)}{p(=,\Delta)} \\ &= \frac{p(\perp)p(\Delta\,|\perp)p(=|\perp,\Delta)}{p(\perp)p(\Delta\,|\perp)p(=|\perp,\Delta) + p(\neg\perp)p(\Delta\,|\,\neg\perp)p(=|\,\Delta,\neg\perp)} \\ &= \frac{\alpha\beta\delta}{\alpha\beta\delta + (1-\alpha)\varphi\cdot 0} = 1 \end{aligned} \tag{21}$$

Some comments are necessary: usually in the first moment will consider that $\alpha, \beta, \delta, \varphi$ are 50%; but after a more careful examination we will realize that these plausibility are very smalls because there are many possibilities that are also plausible. The solution proves that these values are not important.

4 The Third Case Study

The intention of the third case study is to prouve the ability of Plausible Reasoning in human reasoning modeling. For this pourpouse we will tray to model the famous story of Sun Tzu: 'Advance to Chencang by a hidden path'.

The story that we intend to explain by Bayesian model is the following:

This stratagem took place towards the end of the Qin dynasty. Xiang Yu appointed Liu Bang as king of Hanzhong, effectively making him leave China. To further ensure that Liu Bang does not return to China from the East, Xiang Yu divided Guanzhong into three principilities and put three people in charge, informing them to be alert against Liu Bang.

Liu Bang said, 'In order to placate Xiang Yu and the three kings, we must destroy the mountain plank road to show that we've no intention of returning to China.'

After nine years of preparations, Liu Bang's army became powerful and was ready to march eastwards. Liu Bang ordered his generals to take 10,000 men and horses and repair the plank road within three months.

Meanwhile, his enemies were greatly perturbed. One of the kings even led his forces to block the plank road exit.

Liu Bang then led his generals and several thousand troops to overrun Guanzhong by the old roundabout route through Chencang.

We intend to model this story by using the Bayesian theorem (6). At first sight the victory of Liu Bang is based on his ability to increase the plausibility of the likelihood that he will attack on the plank road.

If we analyze more deeply the story we will find that there are two stage of the conflict: the first when Liu Bang must decide about the reaction concerning the Xiang Yu actions, and the second when Liu Bang shows his attack intention but he must choose the attack direction.

The story scenario is presented in Figure 6. It can be see that in the first stage of the conflict, by destroying the road Liu Bang have increased the peace (non attack) likelihood and in this way manipulate Xiang You. In the second stage of the conflict by restoring the road Liu Bang have increased the mountain direction attack (Am) and manipulate once again his enemy.

From mathematical point of view this scenario can be describe in the following way:

- in the initial moment Xiang You can not decide the intention of Liu Bang:
  - $P(A) = P(\neg A) = 50\%$; where A is the sentence 'Liu Bang will attack'
- after seeing that Liu Bang destroyed the road Xiang You decides that:
  - $P(O_1 \mid \neg A) > P(O_1 \mid A)$; where O_1 is the observation of the destroyed road;
  - in consequence (6) $P(\neg A \mid O_1) > P(A \mid O_1)$;
- in the initial moment Xiang You can not decide the attack direction of Liu Bang:
  - $P(Am) = P(\neg Am) = 50\%$; where Am is the sentence 'Liu Bang will attack from the mountain';
- after seeing that Liu Bang constructs the road Xiang You decides that:
  - $P(O_2 \mid Am) > P(O_2 \mid \neg Am)$; where O_2 is the observation of the constructed road;
  - in consequence (6) $P(Am \mid O_2) > P(\neg Am \mid O_2)$

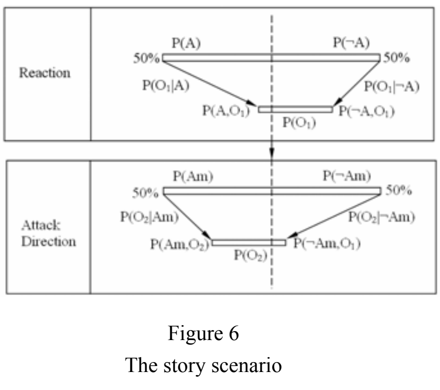


Figure 6

The story scenario

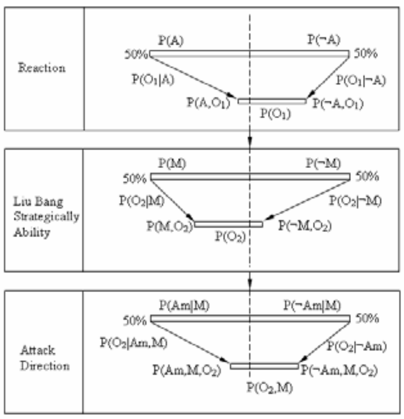


Figure 7

A posible solution

The famous story can be continued with a problem: have had Xiuag You the chance to react at his opponent ability? There are several solutions of this problem the first consist on increasing the number of hypothesis of attack direction and find new observations (spy). The second solution is presented in Figure 7 and is based on changing the causal network by introducing a new decision step. More precisely it can be see that after the second observation O_2 Xiuang You becomes able to decide the tactic that Liu Bang will use. This observation increases the likelihood that his opponent uses his ability to manipulate him.

From mathematical point of view this solution can be described in the following way:

- in the initial moment Xiang You can not decide the intention of Liu Bang:
  - $P(A) = P(\neg A) = 50\%$; where A is the sentence 'Liu Bang will attack'
- after seeing that Liu Bang destroyed the road Xiang You decides that:
  - $P(O_1 \mid \neg A) > P(O_1 \mid A)$; where O_1 is the observation of the destroyed road;
  - in consequence (6) $P(\neg A \mid O_1) > P(A \mid O_1)$;
- in the initial moment Xiang You can not decide about the strategically ability of his opponent:
  - $P(M) = P(\neg M) = 50\%$; where M is the sentence 'Liu Bang is able to manipulete'
- after seeing that Liu Bang intends to attack Xiang You decides that:
  - $P(O_2 \mid M) > P(O_2 \mid \neg M)$; where O_2 is the observation of the constructed road; M is the sentence 'Liu Bang is able to manipulate'
  - in consequence (6) $P(M \mid O_2) > P(\neg M \mid O_2)$;
- in the initial moment Xiang You can not decide the attack direction of Liu Bang:
  - $P(Am) = P(\neg Am) = 50\%$; where Am is the sentence 'Liu Bang will attack from the mountain';
- after seeing that Liu Bang constructs the road, and knowing that his opponent can manipulate Xiang You decide that:
  - $P(O_2 \mid Am, M) < P(O_2 \mid \neg Am, M)$;
  - in consequence (6) $P(Am \mid O_2, M) < P(\neg Am \mid O_2, M)$

Conclusions

Present paper continues the author researches on cognition system design by presenting three different case studies which use the same theory: the Bayesian theory of plausible reasoning. The first case study develops a classical differential model in to a Bayesian model by adding a qualitative description on the output value. The second case study shows the possibilities of plausible reasoning to solve geometrical problems where the degree of truth must be very crispy. In the end we have tray to model by Bayesian theory one of the famous stories of Sun Tzu.

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