Constructing non-statistical association measures on the sets with involution and similarity measure

Ildar Batyrshin

Computing Research Center (CIC),

National Polytechnic Institute (IPN), México, DF

Correlation coefficient

Correlation coefficient is a fundamental concept in data analysis measuring positive and negative relationships between variables.

Sample **Pearson's correlation coefficient**:

$$-1 \le corr(x,y) \le 1$$

$$corr(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \cdot \sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

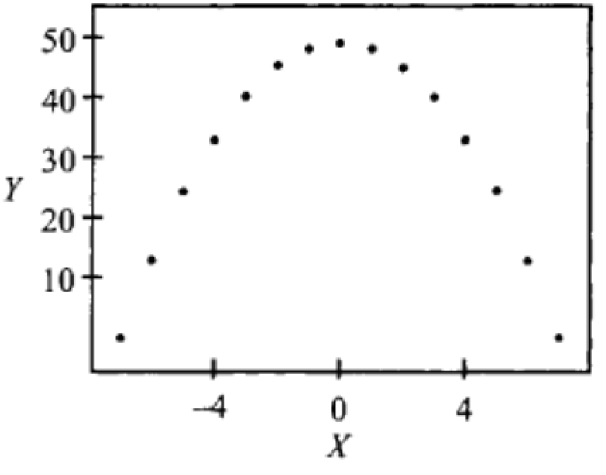
Limitations of the correlation coefficient

Correlation coefficient is **a measure of linear relationship** of variables

Example: For data with perfect nonlinear relationship

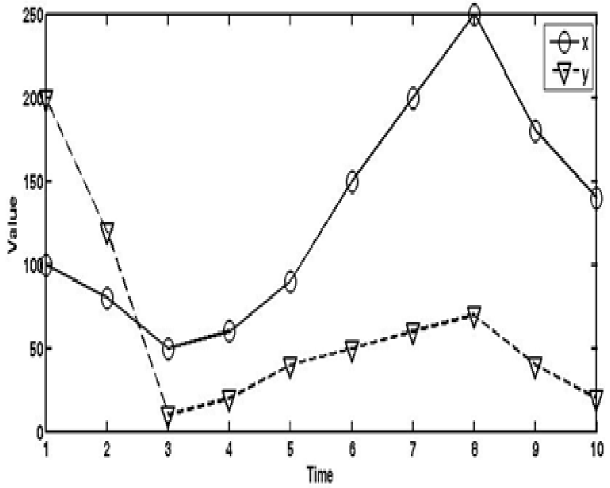
$$y = Ax^2+Bx+C,$$

$$\boldsymbol{corr(x,y) = 0}$$



Chatterjee, S., Hadi, A. S. Regression analysis by example. John Wiley & Sons, 2013. p. 25

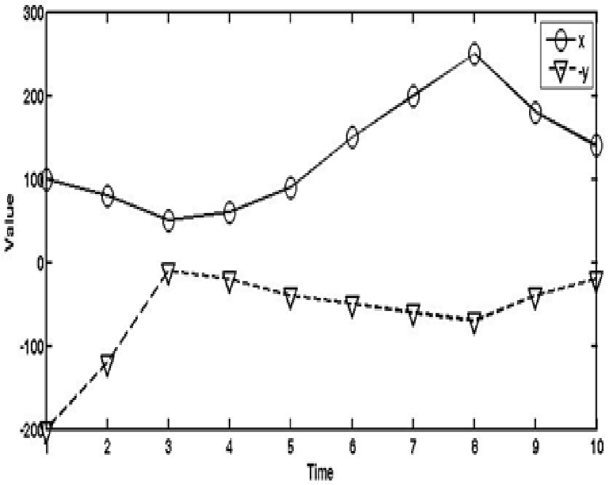
Shortcomings of the correlation coefficient



INVESTOPEDIA:
http://www.investopedia.com/terms/c/correlation.asp

Positive Correlation: A relationship between two variables in which both variables move in tandem. A positive correlation exists when as one variable decreases, the other variable also decreases and vice versa.

For the example above it should be ***A(x,y)* > 0**
but we have ***corr(x,y)* = 0**

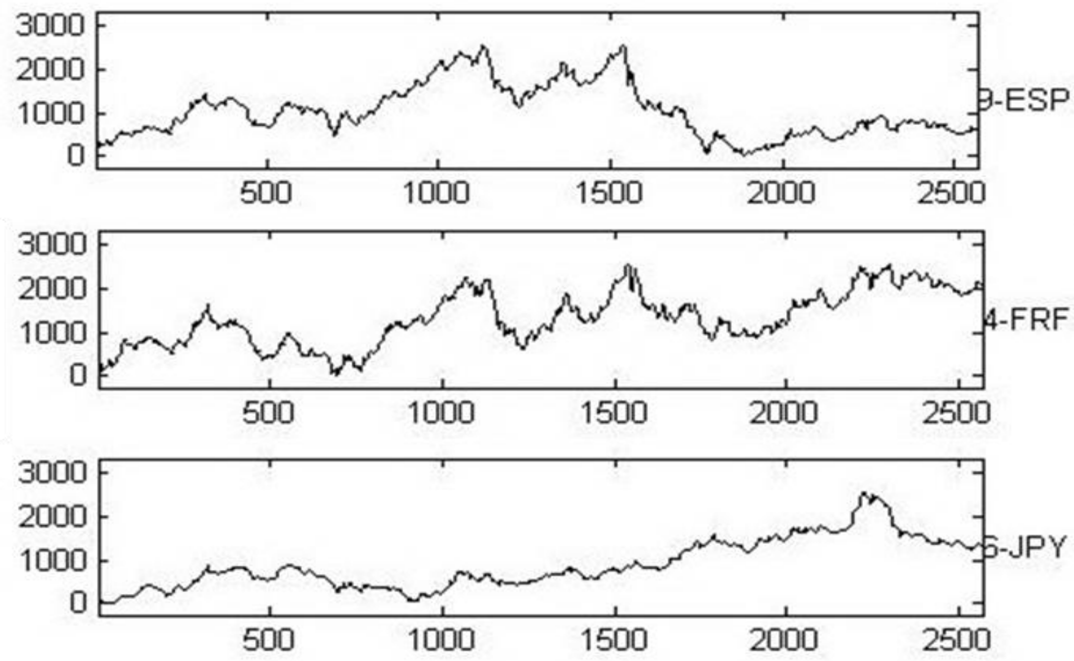


Negative Correlation: A relationship between two variables in which one variable increases as the other decreases, and vice versa.

For the example below it should be ***A(x,y)* < 0**
but we have ***corr(x,-y)* = 0**

The magic of statistics

2567 work-daily spot prices (foreign currency in dollars) over the period 10/9/86 - 8/9/96.



Papadimitriou, S., Sun, J., Yu, P. S. Local correlation tracking in time series. In: ICDM'06. Sixth Intern. Conf. Data Mining:

“The global cross-correlation coefficient of 4-FRF and 9-ESP is **0.30, which is statistically significant, exceeding the 95% confidence interval of ±0.04.** “

corr(4-FRF, 6-JPY) = **0.62 > 0.30** But **it should be inverse relation !!!**

Correlation is not useful for TS shape association analysis.

We need another time series association measure, not the correlation coefficient !

Correlation between membership functions

The measure of correlation of membership functions (in case of finite domain):

$$c(f_1, f_2) = \begin{cases} 1 - \frac{4}{H}\sum_{x \in U}[f_1(x) - f_2(x)]^2, & if\ H \neq 0 \\ 1, & if\ H = 0 \end{cases}$$

$$H = \sum_{x \in U}[2f_1(x) - 1]^2 + \sum_{x \in U}[2f_2(x) - 1]^2.$$

Does it has some statistical meaning?

C. A. Murthy, S. K. Pal, and D. Dutta Majumder. "Correlation between two fuzzy membership functions." *Fuzzy Sets and Systems* vol. 17, 1, pp. 23-38, 1985.

Association measures

- Pearson's correlation coefficient
- Spearman's rank correlation coefficient
- Kendall's rank correlation coefficient (τ)
- Local trend association measure
- Correlation of fuzzy sets
- Association rules

…

1. Is it possible to introduce and analyze the general class of functions that can serve as non-statistical association measures similar to correlation coefficient?
2. How to generate such functions for different classes of objects: time series, elements of [0,1], fuzzy sets, fuzzy sets of type 2, interval valued fuzzy sets etc ?

Properties of Pearson´s correlation coefficient

The properties of the correlation coefficient (defined for *n*-tuples):

corr(x,y) = corr(y,x), **(1)**
***corr(x,x)* = 1,** **(2)**
corr(x, –y) = –corr(x,y), **(3)**

$$corr(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \cdot \sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

corr(–x, –y) = corr(x,y), (4)
corr(x, –x) = –1, (5)
corr(x,x) ≥ corr(x,y). (6)

(4)-(6) follow from (1)-(3).
(5) and (2) are contradictive if $x_i = 0$ for all $i = 1,\ldots,n$
corr(x,y) does not defined for *x = const*, we have 0 in denominator

- We need to avoid such contradictions between desirable properties of association measure.
- We need to define association measure on different domains and can replace –*x* by operation with the similar property
- It is possible to consider additional axioms (requirements) depending on domain

Reflection operation

Definition 1. A function $N:X\to X$, $|X|>1$, satisfying the following properties:

$$N(N(x)) = x \quad \text{for all } x\in X \qquad \text{(involutivity)} \qquad (2)$$

$$N(x) \neq x \text{ for some } x\in X \qquad (3)$$

is called a ***reflection*** on X. An element $x\in X$, such that

$$N(x) = x, \qquad (4)$$

is called a ***fixed point*** of N in X.
The fixed points will be denoted by x_{FP}, hence for any fixed point x_{FP} it is fulfilled:

$$N(x_{FP}) = x_{FP}. \qquad (5)$$

Denote $FP(N,X)$ the set of all fixed points of N in X. This set can be empty.

Examples. 1) $X = R^n$, $x = (x_1,\ldots, x_n)$, $N(x)= -x = (-x_1,\ldots,-x_n)$
2) $X = [0,1]$, $N(x)$ is a strong negation, e.g. $N(x) = 1 - x$

Association measure

Definition 2. Let V be a subset of X, $|V| > 1$, N be a reflection on X and the restriction of N on V be a reflection on V. A function $A: V\times V \to [-1,1]$ satisfying for all $x,y \in V$ the properties:

$$A(x,y) = A(y,x), \qquad \text{(symmetry)} \qquad (6)$$

$$A(x,x) = 1, \qquad \text{(reflexivity)} \qquad (7)$$

$$A(x,N(y)) = -A(x,y), \qquad \text{(inverse relationship)} \qquad (8)$$

is called an ***association measure* on *V*.**

Proposition 1. If A is an association measure on $V \subseteq X$ then $V \subseteq X \setminus FP(N,X)$.

Proposition 2. The association measure A on V satisfies for all $x,y \in V$ the properties:

$$A(x,N(x)) = -1, \qquad (9)$$

$$A(N(x),N(y)) = A(x,y), \qquad \text{(cancellation of reflections)} \qquad (10)$$

$$A(x,N(y)) = A(N(x),y). \qquad \text{(permutation of reflections)} \qquad (11)$$

Definition 4. A function $A: X\times X \to [-1,1]$ will be called ***an association measure of type* 1 on *X*** if (6) and (8) are fulfilled for all $x,y \in X$ and (7) is fulfilled for all $x \notin FP(N,X)$:

$$A(x,x) = 1, \qquad \text{for all } x \notin FP(N,X) \qquad (7a)$$

Association measure of type 1 on X

Definition 4. A function $A: X \times X \to [-1,1]$ will be called **an association measure (of type 1) on *X*** if for all $x,y \in X$ it is fulfilled

$$A(x,y) = A(y,x), \qquad (6)$$

$$A(x,x) = 1, \quad \text{for all } x \notin FP(N,X) \qquad (7a)$$

$$A(x,N(y)) = -A(x,y), \qquad (8)$$

Proposition 4. An association measure A (of type 1) on X satisfies for all $x,y \in X$ the properties

$$A(N(x),N(y)) = A(x,y), \quad \text{(cancellation of reflections)} \qquad (10)$$

$$A(x,N(y)) = A(N(x),y). \quad \text{(permutation of reflections)} \qquad (11)$$

$$A(x,N(x)) = -1 \quad \text{if} \quad x \notin FP(N,X), \qquad (14)$$

$$\mathbf{A(x,x_{FP}) = A(x_{FP},x) = 0, \quad \text{for all } x_{FP} \in FP(X,N),} \qquad \mathbf{(15)}$$

$$\mathbf{A(x_{FP},x_{FP}) = 0.} \qquad \mathbf{(16)}$$

Association measure of type 1 on X

Definition 5. Suppose X is a subset of real values **R**, and N is a reflection on X with a unique fixed point $c = x_{FP}$. An association measure $A{:}X\times X\rightarrow[-1,1]$ on X is called ***c-separable*** if the following properties are fulfilled for all $x,y\in X$:

$$A(x,y) > 0 \quad \text{if} \quad x, y > c \quad \text{or} \quad x, y < c, \qquad (17)$$

$$\mathbf{A(x,y) = 0 \quad \text{if} \quad x = c \quad \text{or} \quad y = c, \qquad (18)}$$

$$A(x,y) < 0 \quad \text{if} \quad x < c < y \quad \text{or} \quad y < c < x. \qquad (19)$$

Example 3. $X = [0,1]$

$N(x)= 1-x$. $c= x_{FP} = 0.5$

$A(x,y) = A(y,x)$

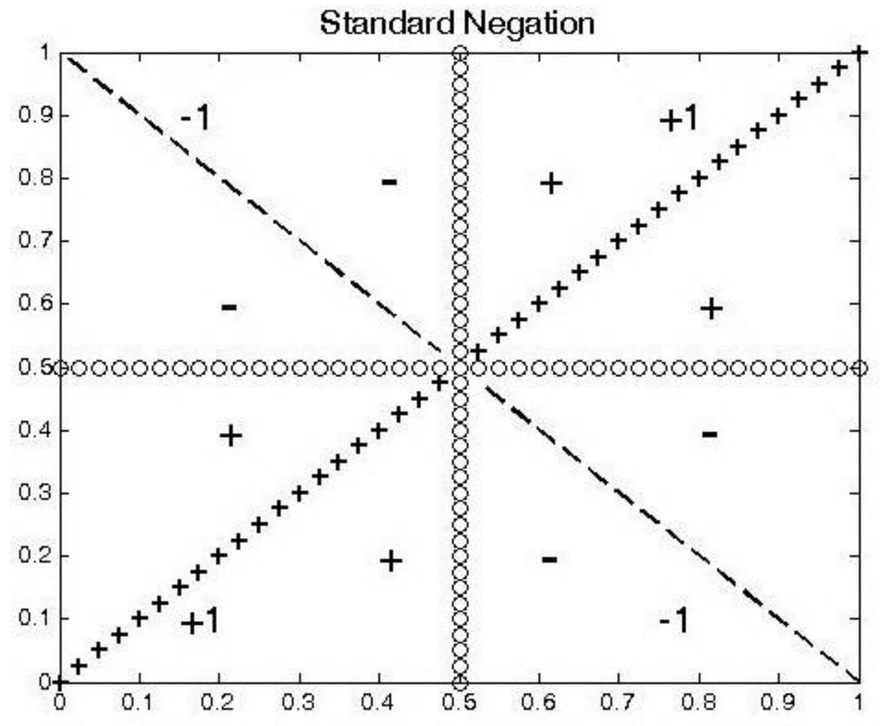
$A(x,x) = 1,$ if $x\neq c$

$A(x,N(x))= -1$ if $x\neq c$

$A(x,c)= A(c,x)= 0$

$A(x,N(y)) = -A(x,y)$

Example. $A(x,y) = sign((x-c)\cdot(y-c))$



Association between fuzzy (plausible) predicates $P(x), Q(y) \in [0,1]$

It is reasonable to have association measure such that:

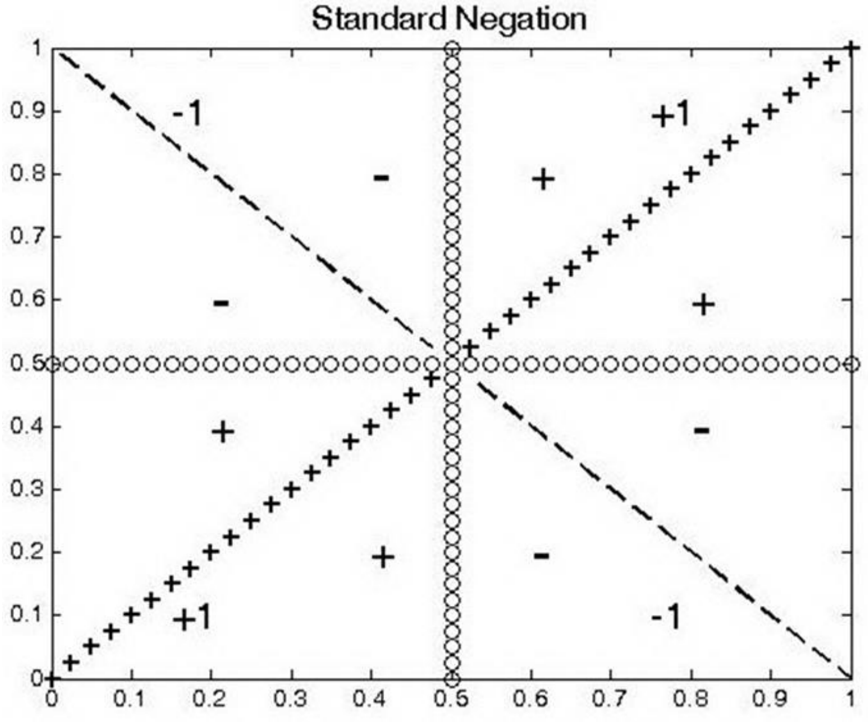
$A(P,Q) > 0$ if $P(x), Q(y) > 0.5$

or $P(x), Q(y) < 0.5$

$A(P,Q) < 0$ if $P(x) < 0.5 < P(y)$

or $P(x) > 0.5 > P(y)$

for all $x,y \in X$



Association measure on the set of time series

X is a set of real n-tuples $x = (x_1,\dots, x_n)$, $(n>1)$,

$q_{(n)} = (q,q,\dots,q)$ is a ***constant*** n-tuple, q is real.

$N(x)= -x = (-x_1,\dots,-x_n)$ is a ***reflection*** with a unique ***fixed point*** $0_{(n)} =(0,0,\dots,0)$.

Define $x+y = (x_1+y_1, \dots, x_n+y_n)$, $py+q= (py_1+q, \dots, py_n+q)$, p,q are real, $p > 0$.

Definition 3. Suppose $V\subseteq X$, $|V| > 1$, such that

from $x\in V$ it follows $-x\in V$, and $x+q\in V$ for all real q.

A function $A: V\times V \to[-1,1]$ satisfying for all $x,y\in V$ and for all real q the properties

$A(x,y) = A(y,x)$,	(symmetry)	(6)
$A(x,x) = 1$,	(reflexivity)	(7)
$A(x,N(y)) = -A(x,y)$,	(inverse relationship)	(8)
$\boldsymbol{A(x+q,y) = A(x,y)}$,	**(translation invariance)**	(12)

is called a ***time series shape association measure*** on V.

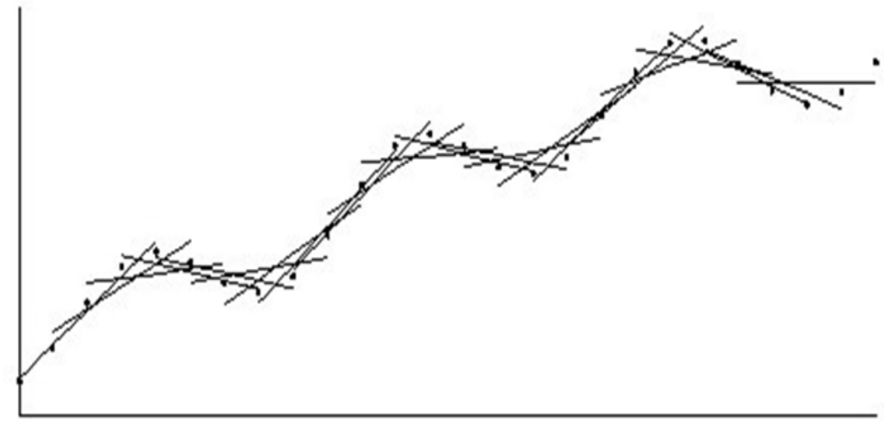
If for all $x\in V$ and for all $p>0$ it is fulfilled $px\in V$ and A satisfies on V the property:

$\boldsymbol{A(px,y) = A(x,y)}$,	**(scale invariance)**	(13)

then A is called a ***scale invariant*** **time series shape association measure**.

Moving Approximation Transform (MAT) and local trend association measure

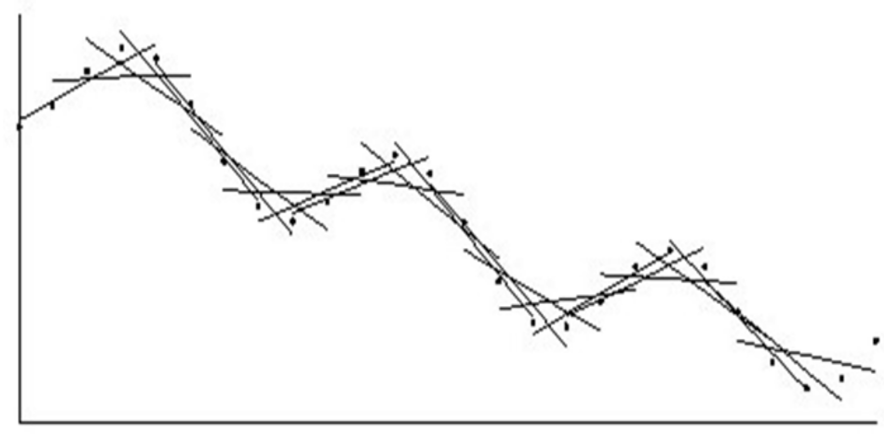
Calculate least squares approximations $f_i = a_i t + b_i$ of time series $x = (x_1, \ldots, x_n)$ in sliding window of size k. Replace x by sequence of local trends:



$$MAT_k(x) = (a_1, \ldots, a_{n-k+1}).$$

Local trends $a_1, \ldots, a_{n-k+1}$ depend on the size of window k.

Example: $k = 5$



Local trend association measure:

$$lta_k(x,y) = \frac{\sum_{i=1}^{m} a_{xi} \cdot a_{yi}}{\sqrt{\sum_{i=1}^{m} a_{xi}^2 \cdot \sum_{j=1}^{m} a_{yj}^2}}$$

Batyrshin I., Sheremetov L. …(2004,2007)

Association measure on the set of time series

Proposition 4. If A is a time series shape association measure on V then $V \subseteq X \backslash X_C$, where X_C is a set of all constant n-tuples.

Example. $corr(x,y)$ is a scale invariant association measure on $V = X \backslash X_C$

Example. Local trend association measure $lta_k(x,y)$ is a scale invariant association measure on $V = X \backslash X_{Mk0}$ where X_{Mk0} is a set of all time series x such that $MAT_k(x) = 0_{(m)}$, i.e. $a_{xi} = 0$ for all $i = 1, \ldots, m$.

Association measures related with similarity measures

A similarity measure $SIM: X \times X \to [0,1]$ satisfies for all x, y from X:

$$SIM(x,y) = SIM(y,x),$$

$$SIM(x,x) = 1.$$

An association measure A related with SIM:

$$A(x,y) > 0, \text{ if } SIM(x,y) > SIM(x,N(y)),$$

$$A(x,y) < 0, \text{ if } SIM(x,y) < SIM(x,N(y)).$$

Can we use (?) the difference between $SIM(x,y)$ and $SIM(x,N(y))$:

$$A(x,y) = SIM(x,y) - SIM(x,N(y))$$

- **What properties should be fulfilled for *SIM* to obtain association measure *A* satisfying the desirable properties?**
- **Can we use here pseudo-difference operation related with t-conorms ?**

t-conorms

***t*-conorm** is a function S:$[0,1]^2 \rightarrow [0,1]$ satisfying

commutativity, associativity, monotonicity

boundary condition: $S(a,0) = a$.

We have: $S(1,a) = S(a,1) = 1$, $S(0,a) = S(a,0) = a$.

$a \in]0,1[$ is **a nilpotent element** of S if there exists $b \in]0,1[$ such that

$S(a,b) = 1$.

t-conorm S **has no nilpotent elements** if and only if

from $S(a,b) = 1$ it follows $a = 1$ or $b = 1$

Examples:

$S_M(a,b) = max\{a,b\}$, (maximum)

$S_L(a,b) = min\{a+b, 1\}$, (Lukasiewicz t-corm)

$S_P(a,b) = a+b-ab$. (probabilistic sum)

S_M and S_P have no nilpotent elements but S_L has.

S-difference and pseudo-difference

Let S be a t-conorm. ***S*-difference** is defined by:

$$a \overset{S}{-} b = inf\{c \in [0,1] | S(b,c) \geq a\} \qquad \text{for any } a,b \text{ in } [0,1].$$

Pseudo-difference associated (related) to S is defined by:

$$a(-)_S b = \begin{cases} a \overset{S}{-} b, & if\ a > b \\ -\left(b \overset{S}{-} a\right), & if\ a < b \\ 0, & if\ a = b \end{cases} \qquad \text{for any } a,b \text{ in } [0,1]^2$$

Equivalently

$$a(-)_S b = sign(a-b)(max(a,b) \overset{S}{-} min(a,b)).$$

Pseudo-differences related with basic t-conorms

$S_M(a,b) = max\{a,b\}$ (maximum)

$S_L(a,b) = min\{a+b, 1\}$ (Lukasiewicz *t*-conorm)

$S_P(a,b) = a+b-ab$ (probabilistic sum)

$$a(-)_M b = \begin{cases} a, & if\ a > b \\ -b, & if\ a < b, \\ 0, & if\ a = b \end{cases}$$

$$a(-)_L b = a - b.$$

$$a(-)_P b = (a - b)/(1 - \min(a, b)).$$

Similarity measures

$SIM: X \times X \rightarrow [0,1]$ is a similarity measure on X if it satisfies the properties:

$SIM(x,y) = SIM(y,x)$ (symmetry)

$SIM(x,x) = 1$ (reflexivity)

$SIM(N(x),N(y)) = SIM(x,y)$ (cancellation of reflections)

$SIM(x,N(y)) = SIM(N(x),y)$ (permutation of reflections)

$SIM(x,y) < SIM(x,x)$ for all $x \neq y$ (strict reflexivity)

$SIM(x,N(x)) < 1$ (weak similarity of reflections)

$SIM(N(x),x) = 0.$ (non-similarity of reflections)

Main result

Theorem 2. Suppose X is a set with a reflection N, $V \subseteq X \backslash FP(N,X)$, $|V| > 1$, V is closed under N and the restriction of N on V is a reflection on V, S is a t-conorm and SIM is a similarity measure on X satisfying the properties of **cancellation of reflection**s and **weak similarity of reflections** then the function

$A_{SIM,S}: V \times V \rightarrow [-1,1]$ defined for all $x,y \in V$ by

$$\boldsymbol{A_{SIM,S}(x,y) = SIM(x,y)(-)_S\, SIM(x,N(y)),}$$

is an association measure on V if one of the following is true

a) the t-conorm S has no nilpotent elements (29)

b) $SIM(x,N(x)) = 0$, for all $x \in V$ (non-similarity of reflections)

Association measure on [0,1]

Theorem 3. Suppose N is a strong negation, S is a t-conorm, SIM is a similarity measure on [0,1] satisfying the properties of cancellation of reflections and weak similarity of reflections then the function $A_{SIM,S}$:[0,1]×[0,1]→[0,1] defined for all $x,y\in[0,1]$ by:

$$A_{SIM,S}(x,y) = SIM(x,y) \ominus_S SIM(x,N(y)), \qquad (37)$$

is an association measure on [0,1] if one of the following is fulfilled:

1) $SIM(x,N(x)) = 0$, for all $x\in[0.1]$, (38)

2) the t-conorm S has no nilpotent elements. (39)

Association measure of type 1 on X

Definition. Suppose $X = [0,1]$, N is a strong negation with a unique fixed point c. An association measure $A: X \times X \to [-1,1]$ on $[0,1]$ is called ***c-separable*** if the following properties are fulfilled for all $x,y \in X$:

$$A(x,y) > 0 \quad \text{if} \quad x, y > c \quad \text{or} \quad x, y < c,$$

$$\mathbf{A(x,y) = 0} \quad \textbf{if} \quad \mathbf{x = c} \quad \textbf{or} \quad \mathbf{y = c,}$$

$$A(x,y) < 0 \quad \text{if} \quad x < c < y \quad \text{or} \quad y < c < x.$$

Definition. A similarity measure SIM is **strict monotonic** if for all $x,y,z \in [0,1]$ it is fulfilled:

$$SIM(x,z) < min(SIM(x,y), SIM(y,z)) \quad \text{if } x < y < z .$$

Theorem 4. Suppose in the conditions of Theorem 3 the t-conorm S is continuous at the point 0 in both arguments and the similarity measure SIM is strict monotonic then the association measure

$$A_{SIM,S}(x,y) = SIM(x,y)(-)_S SIM(x,N(y)),$$

is c-separable.

Constructing *SIM* satisfying the cancellation of reflections property $SIM(N(x),N(y)) = SIM(x,y)$

Proposition 7. Suppose $f,g:[0,1]\to[0,1]$ are automorphisms of [0,1] and g defines the strong negation N on $X=[0,1]$ then the function

$$SIM(x,y) = 1 - f(|g(x) - g(y)|) ,$$

is a similarity measure on $X= [0,1]$ satisfying the properties of the strict monotonicity, the strict reflexivity and hence the weak similarity of reflections, the cancellation of reflections with respect to N, but it does not satisfy the non-similarity of reflections property.

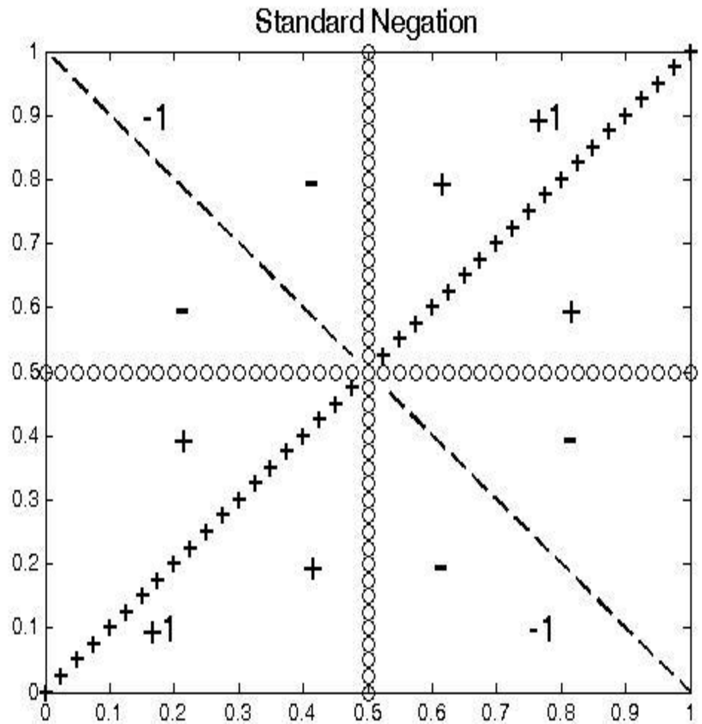
$$SIM(x,y) = 1 - \varphi^{-1}(|\varphi(x) - \varphi(y)|),$$

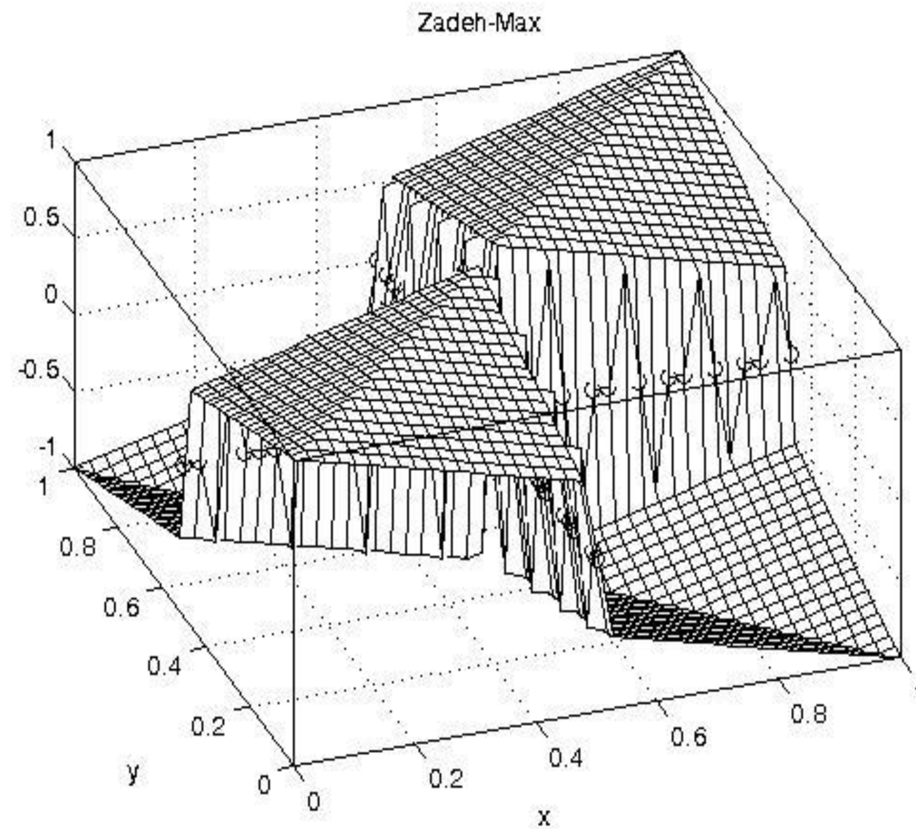
For generator $\varphi(x) = x$ of standard negation $N(x) = 1 - x$, we obtain:

$$SIM(x,y) = 1 - |x - y|.$$

For standard negation $N(x)=1-x$, fixed point c = 0.5, pseudo-difference $(-)_S$ related with $S_M(a,b) = max\{a,b\}$, (maximum), $SIM(x,y) = 1 - |x-y|$:

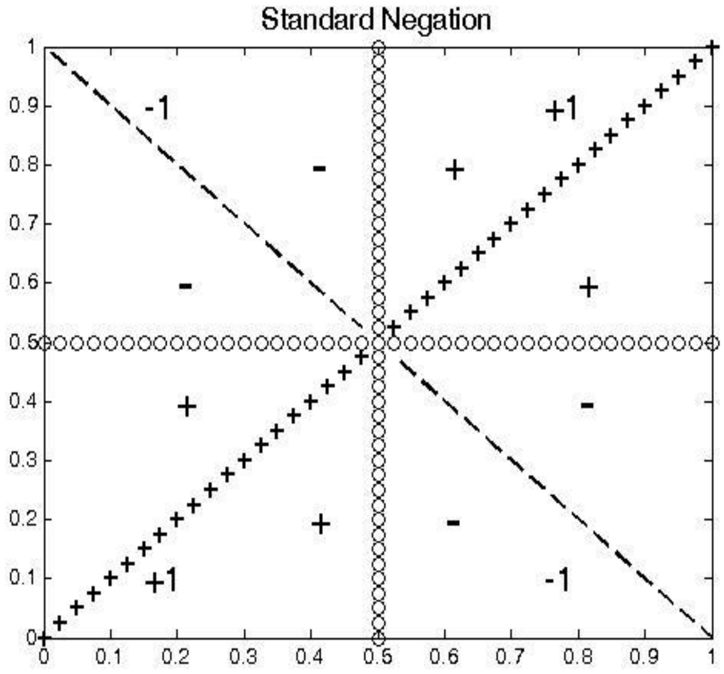
$$A_{SIM,S_M}(x,y) = \begin{cases} 1-|x-y| & if \quad x,y > 0.5 \quad or \quad x,y < 0.5 \\ |x+y-1|-1 & if \;\; x < 0.5 < y \;\; or \;\; y < 0.5 < x \\ 0 & if \quad x = 0.5 \;\; or \;\; y = 0.5 \end{cases}$$

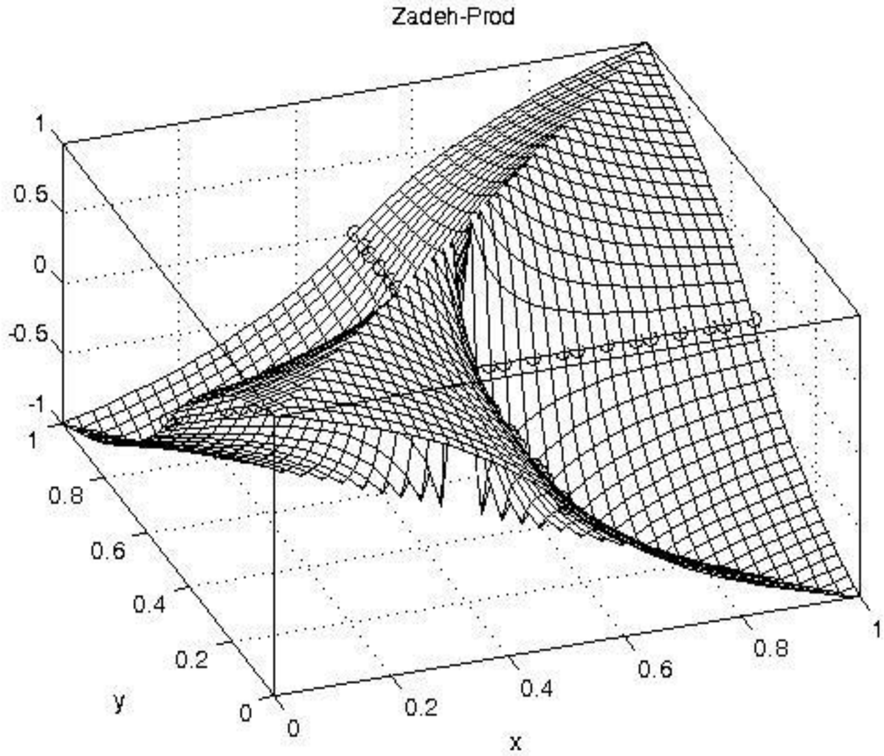




For standard negation $N(x)=1-x$, fixed point c = 0.5, pseudo-difference $(-)_S$ related with $S_P(a,b) = a+b-ab$, (probabilistic sum), $SIM(x,y) = 1 - |x-y|$:

$$A_{SIM,S_P}(x,y) = \begin{cases} \dfrac{|x+y-1| - |x-y|}{max\{|x+y-1|, |x-y|\}} & if \quad x,y \neq 0.5 \\ 0 & if \ \ x = 0.5 \ \ or \ \ y = 0.5 \end{cases}$$

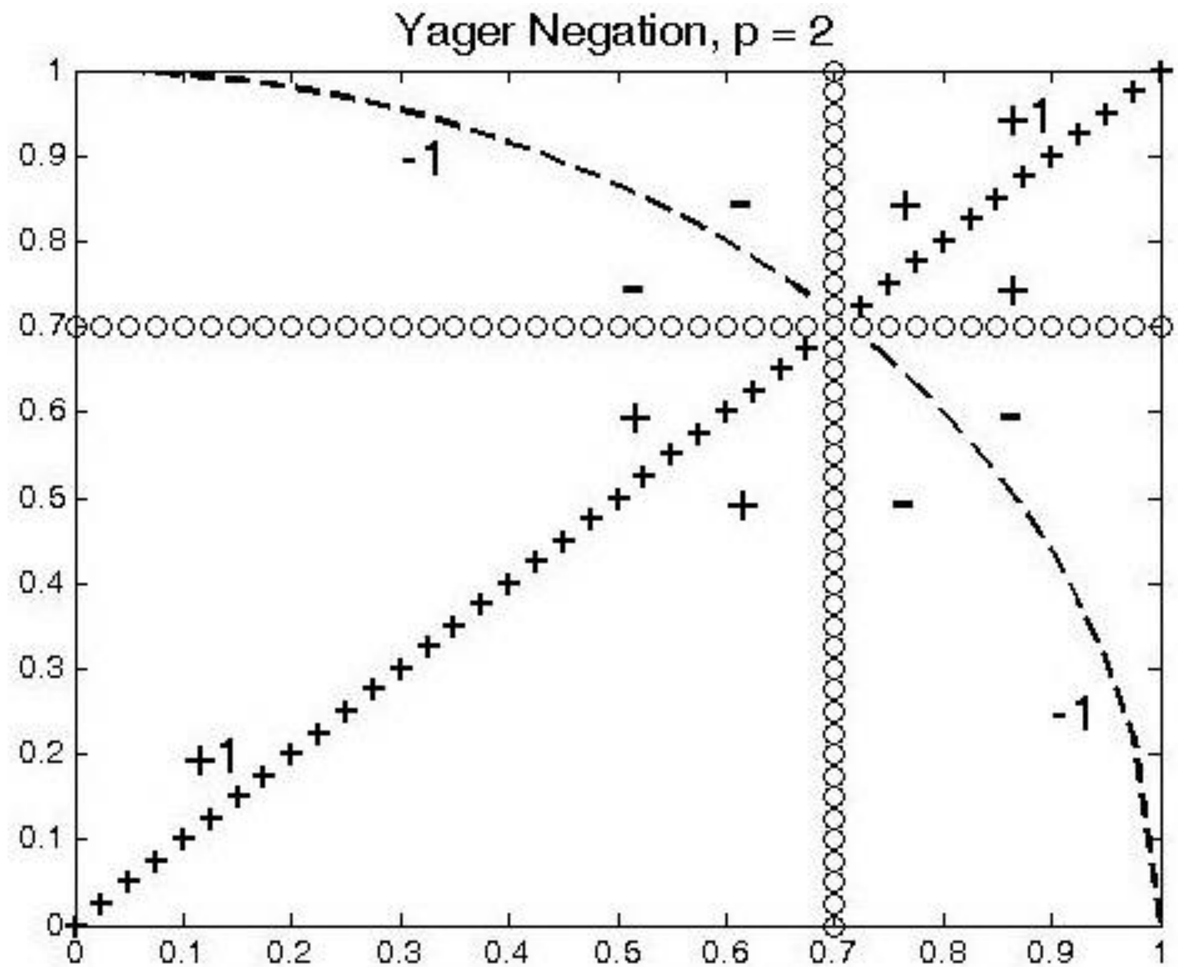




Association measure of type 1 on X

Example 3. X = [0,1], Yager negation: $N_p(x) = \sqrt[p]{1 - x^p}, \;\; p > 0,$

$c = \sqrt[p]{0.5}.$



Example of distance based association measure on fuzzy sets

$SIM(x,y) = 1 - \frac{1}{n}\sum_{i=1}^{n}|g(x_i) - g(y_i)|^2$

where $g(x)$ is a generator of involutive negation on [0,1].

For $A(x,y)=A_{SIM,P}(x,y)$ and Zadeh negation

N(x)=1-x with generator $g(x) = x$ we obtain:

$$A(x,y) = \frac{\sum_{i=1}^{n}(2x_i-1)(2y_i-1)}{max(\sum_{i=1}^{n}|x_i-y_i|^2, \sum_{i=1}^{n}|x_i+y_i-1|^2)}.$$

Constructing *SIM* satisfying the cancellation of reflections property $SIM(N(x),N(y)) = SIM(x,y)$

Definition 14. A function M:[0,1]×[0,1]→[0,1] is an **aggregation function** of two arguments if it is non-decreasing in each arguments and satisfies:

$M(x,y) = M(y,x)$ (symmetry)

$M(0,0) = 0$, $M(1,1) = 1$, (boundary conditions)

Proposition 6. If M is an aggregation function and SIM is a similarity measure then

$$\boldsymbol{SIM_M(x,y) = M(SIM(x,y),SIM(N(x),N(y)))} \quad (32)$$

is the similarity measure satisfying the cancellation of reflections property (26).

SIM_M is strict reflexive if SIM is strict reflexive and M satisfies:

$$M(a,b) < 1 \quad \text{if} \quad min(a,b) < 1. \quad (33)$$

Examples: $SIM_M(x,y) = min(SIM(x,y),SIM(N(x),N(y)))$, (34)

$SIM_M(x,y) = (SIM(x,y)+SIM(N(x),N(y)))/2$. (35)

Similarity and dissimilarity measures

Definition 15. A function $D:X\times X \rightarrow [0,1]$ is a **dissimilarity measure** on X if it satisfies:

$D(x,y) = D(y,x)$,

$D(x,x) = 0$.

If D is a dissimilarity measure and $U:\mathbf{R}\rightarrow\mathbf{R}$ is a strictly decreasing nonnegative real function such that $U(0)= 1$, then the function

$SIMD(x,y)=U(D(x,y))$

is a similarity measure.

If it exists some positive constant H such that $D(x,y) \leq H$ for all x,y and W is a strictly increasing function such that $W(0)= 0$, $W(H) \leq 1$, then a similarity function can be obtained as follows:

$$S_D(x,y)= 1 - W(D(x,y)).$$

Simmilarity and dissimilarity measures

Examples of $S_D(x,y) = U(D(x,y))$

$$S_D(x,y) = \frac{K}{D(x,y)+K}, K > 0,$$

$$S_D(x,y) = \frac{1}{e^{D(x,y)}}.$$

$$D_{r,F}(x,y) = \left(\sum_{i=1}^{n} \left|F(x_i) - F(y_i)\right|^r\right)^{1/r}$$

I. Standardization of time series values

A transformation F of time series x of length n into time series $F(x)$ of the same length is said to be a **standardization** if for all non-constant time series x it is fulfilled:

$F(F(x)) = F(x)$. (idempotency)

Two additional requirements on standardization transformation can be considered:

$F(x) \neq const$ if $x \neq const$,

$F(q_{(n)}) = 0_{(n)}$, for any real value q.

Standardization of time series values

A time series x is said to be **in a standard form wrt a standardization *F*** if $F(x) = x$. As it follows from the definitions, a standardization transforms any time series x into a standard form $F(x)$.

A transformation E of time series x of length n into real value $E(x)$ is said to be an **estimate** of x.

Standardization of time series values

Proposition. *Suppose* $E_1(x)$ *is a translation additive estimate such that* $E(q_{(n)})=q$*, then the transformation*

$F(x)= x–E(x),$

is a translation invariant standardization such that

$E(F(x)) = 0.$

If $E(x)$ *is an odd function, then* $F(x)$ *is an odd function. If* $E(x)$ *is scale proportional then* $F(x)$ *is scale proportional.*

Standardization of time series values

Proposition 3. *Suppose* $E_1(x)$ *is a translation additive and scale proportional estimate such that* $E(q_{(n)})=q$, *and* $E_2(x) \neq 0$ *is a translation invariant and scale proportional estimate then the transformation*

$$F(x)=(x-E_1(x))/E_2(x)$$

is a translation invariant and scale invariant standardization such that

$$E_1(F(x)) = 0.$$

If $E_1(x)$ *is an odd function and* $E_2(x)$ *is an even function, then* $F(x)$ *is an odd function.*

If then $F(x)$ *satisfies r-normality.*

Constructing *SIM* satisfying the cancellation of reflections property $SIM(N(x),N(y)) = SIM(x,y)$

Proposition 8. Let X be the set of time series of the length n and $D(x,y) = D_{r,F}(x,y)$:

$$D_{r,F}(x,y) = \left(\sum_{i=1}^{n} \left|F(x_i) - F(y_i)\right|^r\right)^{1/r} \quad (39)$$

where F:**R**→ **R.** A dissimilarity measure D satisfies on $V \subseteq X \backslash X_C$ the property

$D(N(x),N(y))= D(x,y),$ (cancellation of reflections) (42)

if for all $x \in V$ the function F satisfies the following condition :

$F(N(x))+F(x) = Q,$ (43)

where Q = const.

$D(N(x),x) > 0$ if in (43) $F(x_i) \neq Q/2$ for some $x \in V$.

$D(N(x),x) = 2$ if in (43) $Q= 0$ and for all $x \in V$ it is fulfilled *r-normality*:

$$\sum_{i=1}^{n} \left|F(x_i)\right|^r = 1$$

Minkowski distance → Association

SIMD(x,y)=U(D(x,y))

$$D_{r,F}(x,y) = \left(\sum_{i=1}^{n} \left|F(x_i) - F(y_i)\right|^r\right)^{1/r}$$

If it exists some positive constant H such that $D(x,y) \le H$ for all x,y and W is a strictly increasing function such that $W(0) = 0$, $W(H) \le 1$, then a similarity function can be obtained as follows:

$$S_D(x,y) = 1 - W(D(x,y)).$$

Corollary. *A shape association measure defined in Proposition coincides with a cosine similarity measure:*

$$A_{cos,F}(x,y) = cos(F(x),F(y)),$$

*if r=*2, i.e. *$D(x,y) = D_{2,F}(x,y)$ is Euclidean distance, and if $W(D)$ is defined as follows: .*

$$W(D(x,y)) = \frac{(D(x,y))^2}{4}.$$

Minkowski distance → Association

D → *S* → *A*, *F* → *D* → *S* → *A*

$$D_{r,F}(x,y) = \left(\sum_{i=1}^{n} \left|F(x)_i - F(y)_i\right|^r\right)^{1/r}$$

where *F(x)* is some standardization of time series *x*.

F is an **odd function,** i.e. it satisfies: *F(-x)= –F(x).*

Constructing Pearson's correlation coefficient

Example 5. Let X be a set of real n-tuples $x = (x_1,\dots, x_n)$, $(n>1)$, with the reflection operation $N(x) = -x = (-x_1,\dots,-x_n)$. Define dissimilarity measure by

$$D(x,y) = \sqrt{\sum_{i=1}^{n} |F(x_i) - F(y_i)|^2},$$

where $F(x_i)$ is given by:

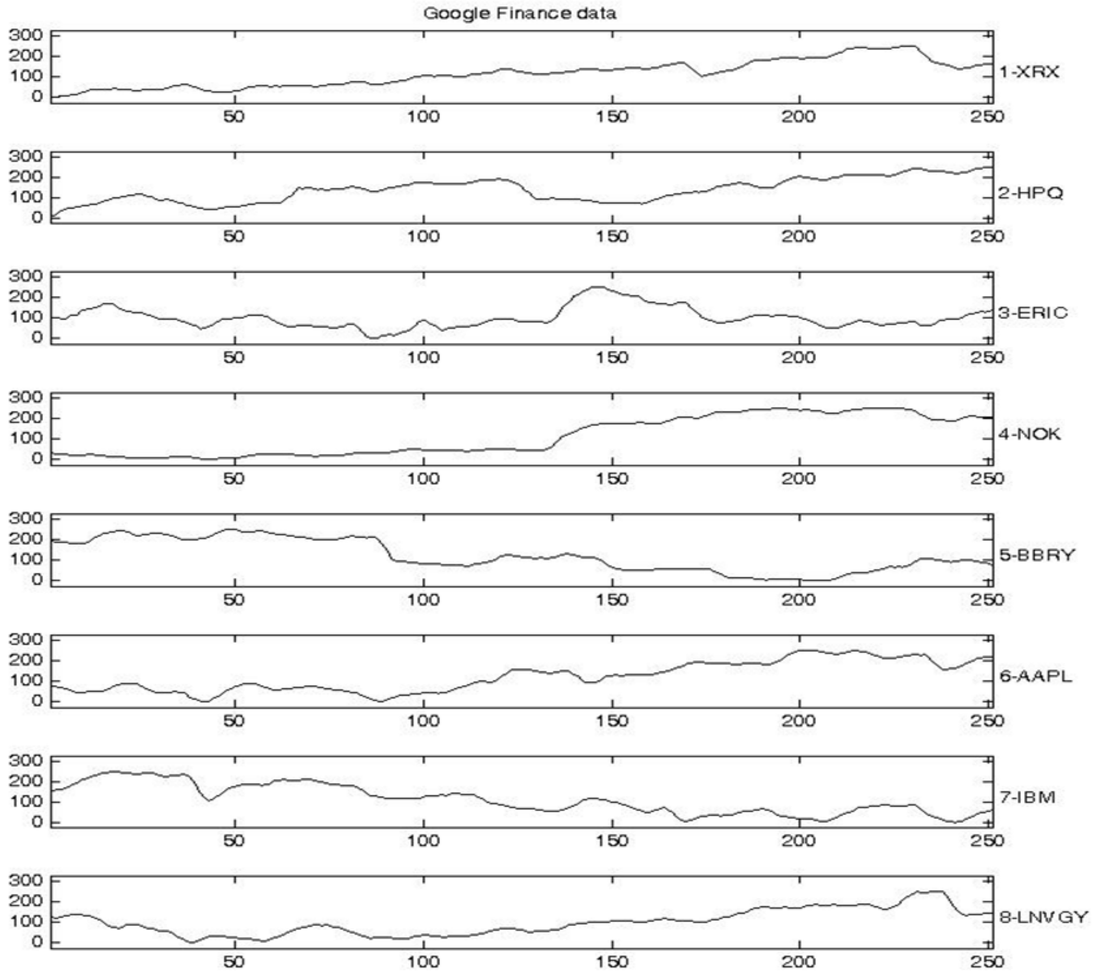
$$F(x_i) = \frac{x_i - \bar{x}}{\sqrt{\sum_{j=1}^{n}(x_j - \bar{x})^2}}, \quad \bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j,$$

and $SIM(x,y) = 1 - \frac{1}{4}D(x,y)^2 = \frac{1}{4}\sum_{i=1}^{n}|F(x_i) - F(y_i)|^2$. Then

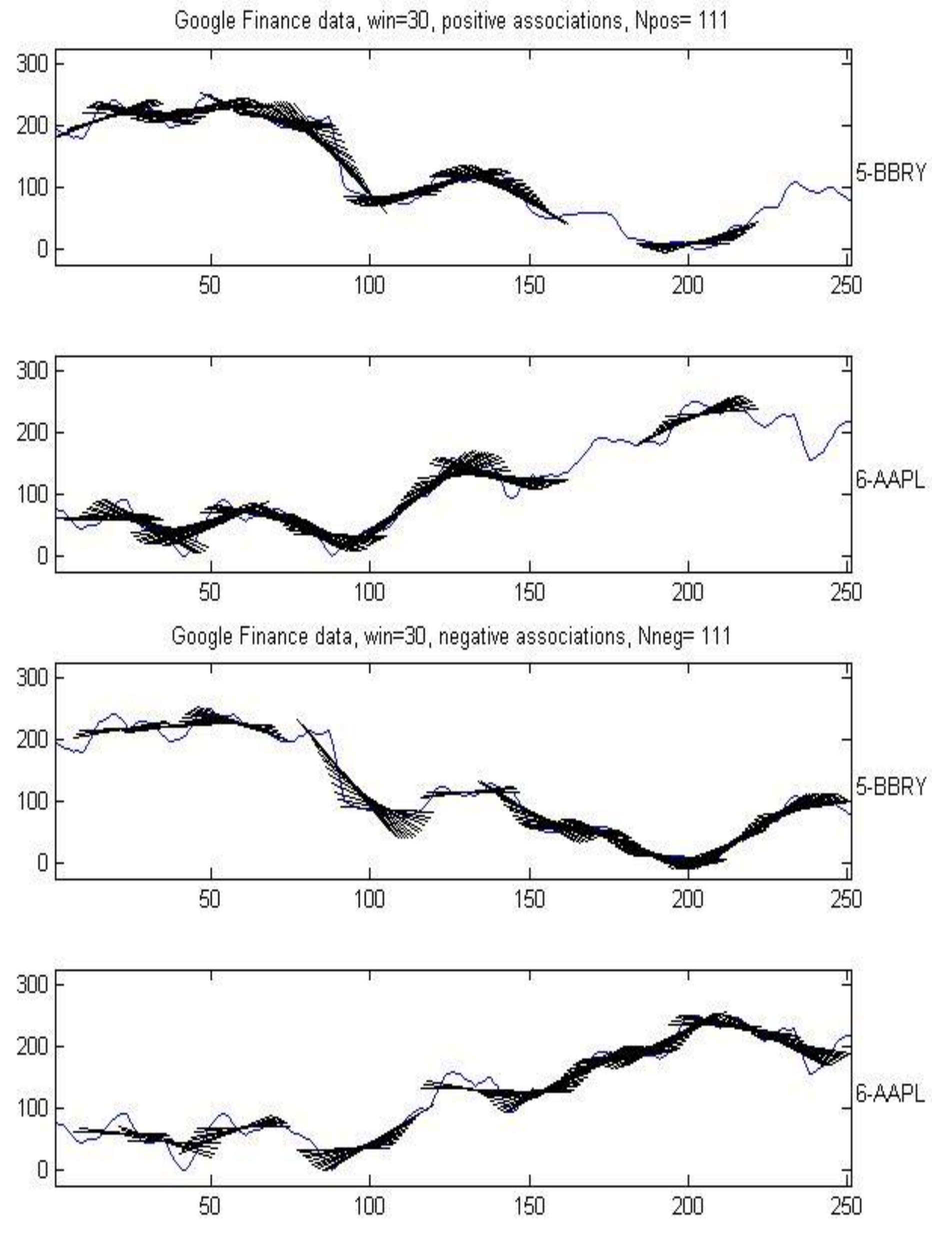
$$corr(x,y) = SIM(x,y)\ (-)_S\ SIM(x,N(y)),$$

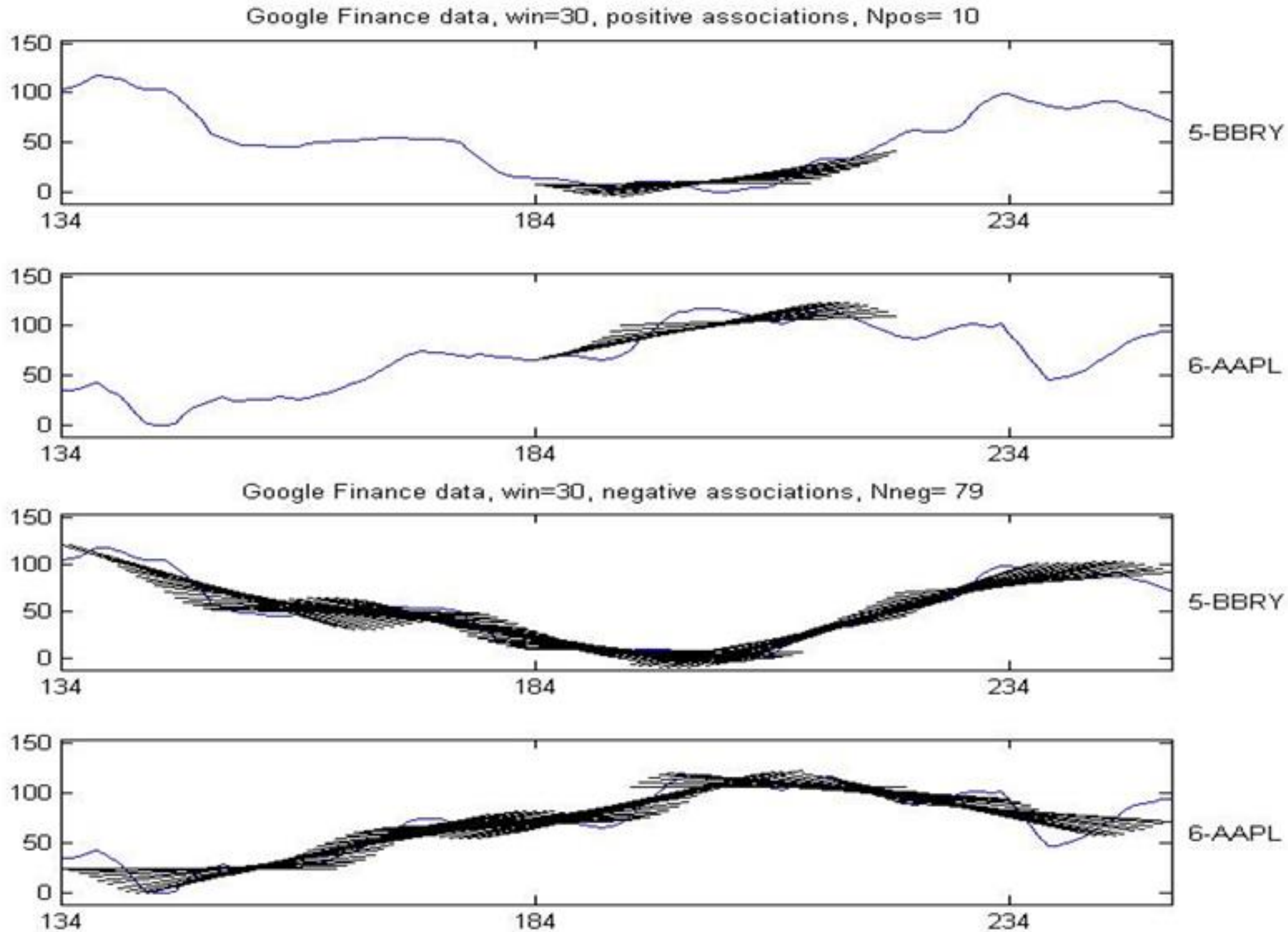
where the pseudo-difference operation $a(-)_S b = a - b$, i.e. associated to Lukasiewizc t-conorm.

Example of normalized Google Finance data after smoothing by moving average (w=5).

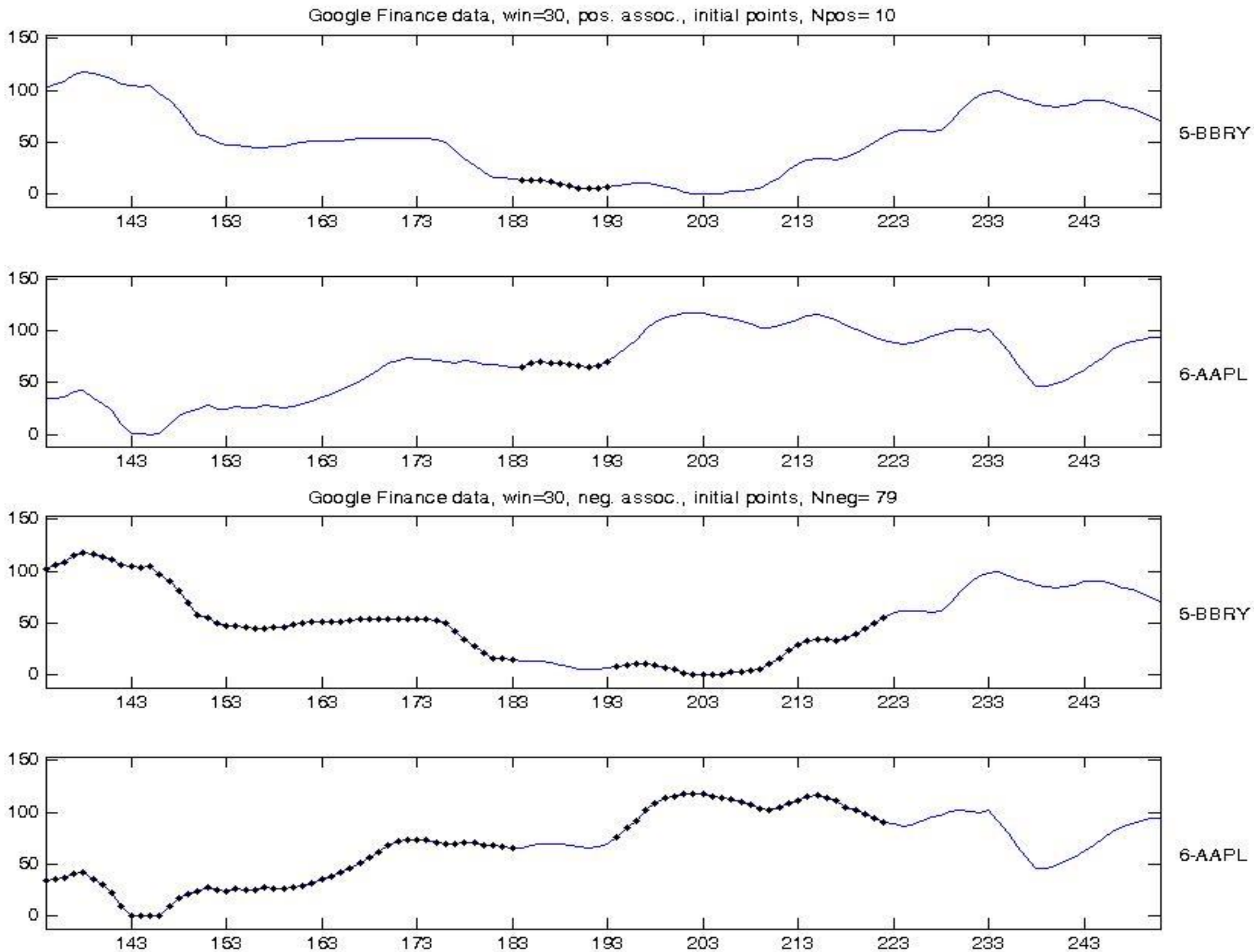


Positively (two charts at the top) and negatively (two charts on the bottom) associated moving approximations of BBRY and AAPL data in sliding window of size k =30.



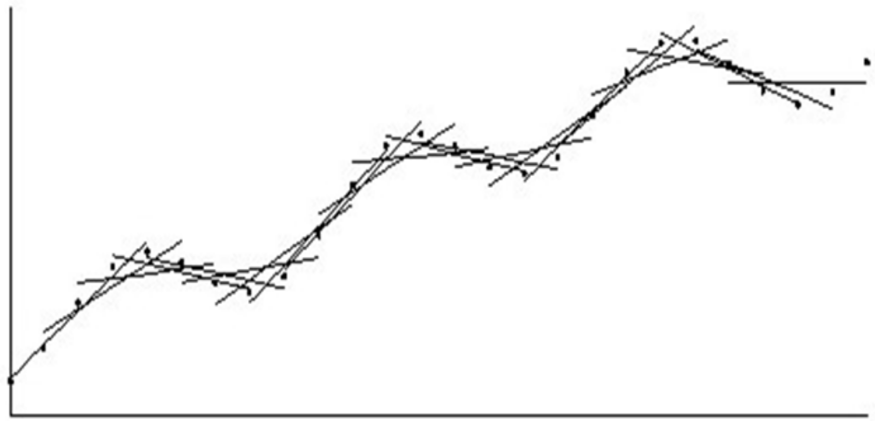


Google Finance data, win=30, positive associations, Npos= 10
5-BBRY
6-AAPL
Google Finance data, win=30, negative associations, Nneg= 79
5-BBRY
6-AAPL
150
100
50
0
134
184
234


Google Finance data, win=30, pos. assoc., initial points, Npos= 10
5-BBRY
6-AAPL
Google Finance data, win=30, neg. assoc., initial points, Nneg= 79
5-BBRY
6-AAPL
150
100
50
0
143
153
163
173
183
193
203
213
223
233
243

Moving Approximation Transform (MAT) and local trend association measure

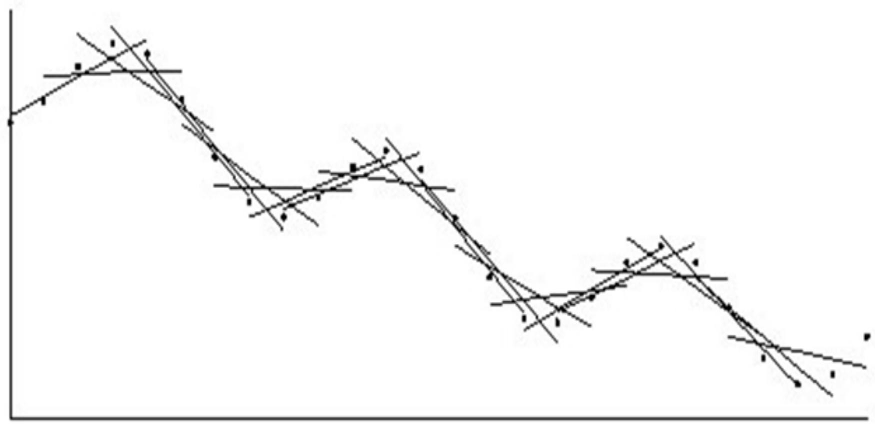
Calculate least squares approximations $f_i = a_i t + b_i$ of time series $x = (x_1, \ldots, x_n)$ in sliding window of size k. Replace x by sequence of local trends:



$$MAT_k(x) = (a_1, \ldots, a_{n-k+1}).$$

Local trends $a_1, \ldots, a_{n-k+1}$ depend on the size of window k.

Example: $k = 5$

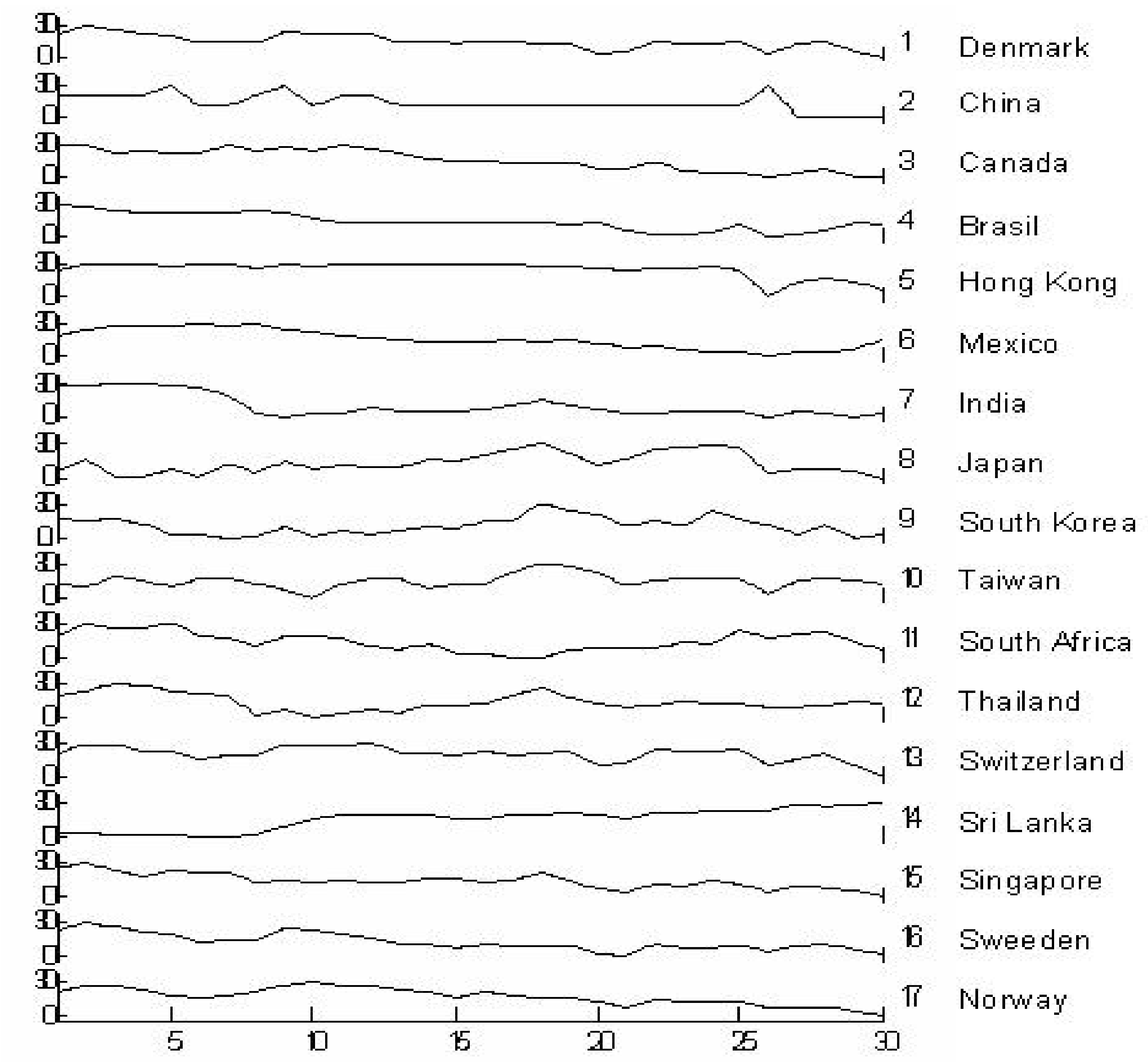


Local trend association measure:

$$lta_k(x,y) = \frac{\sum_{i=1}^{m} a_{xi} \cdot a_{yi}}{\sqrt{\sum_{i=1}^{m} a_{xi}^2 \cdot \sum_{j=1}^{m} a_{yj}^2}}$$

Batyrshin I., Sheremetov L. …(2004,2007)

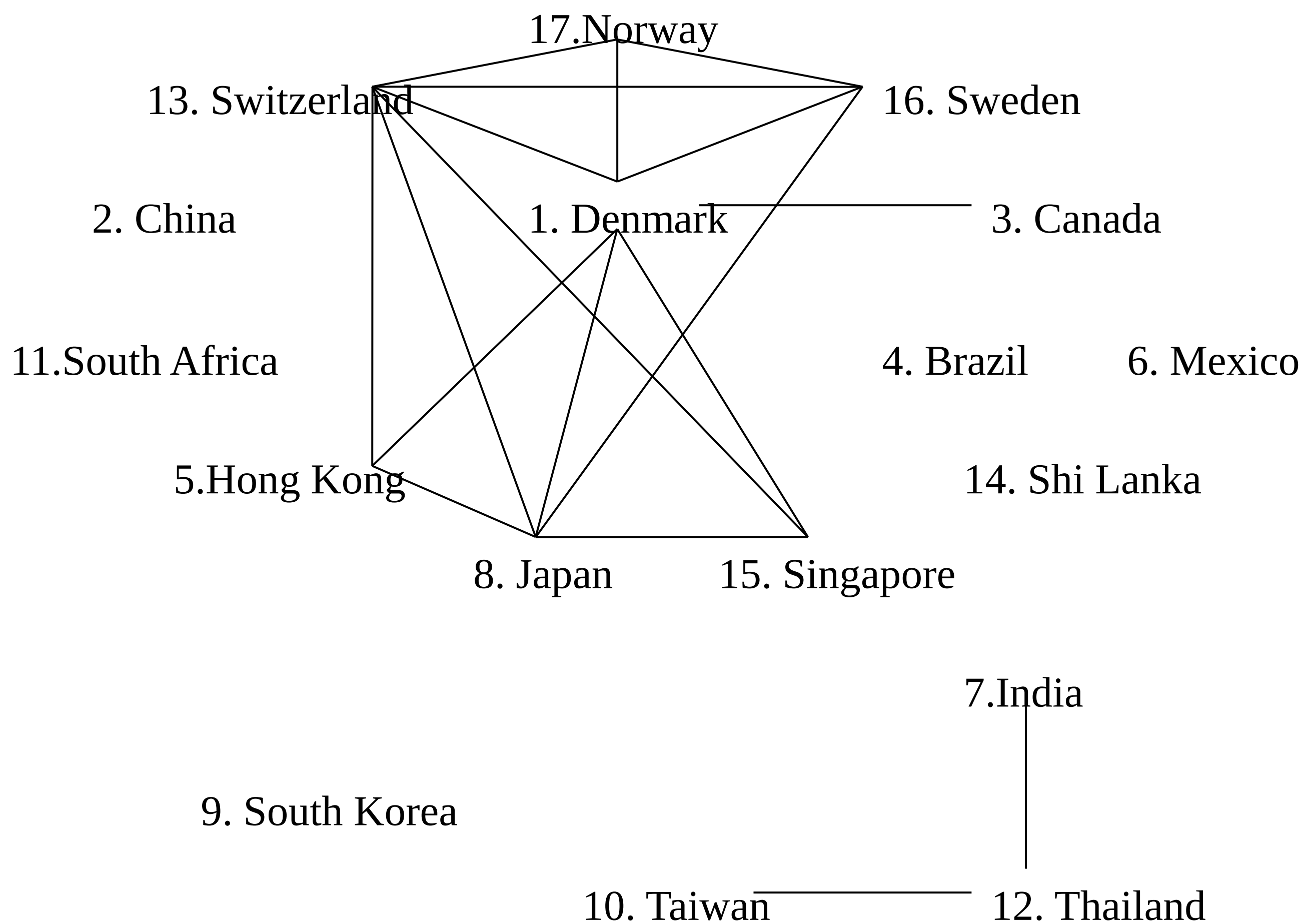
Foreign Exchange Rates



Time series of Foreign Exchange Rates (money of different countries to one U.S. Dollar) measured daily since 2004-09-02 to 2004-10-15

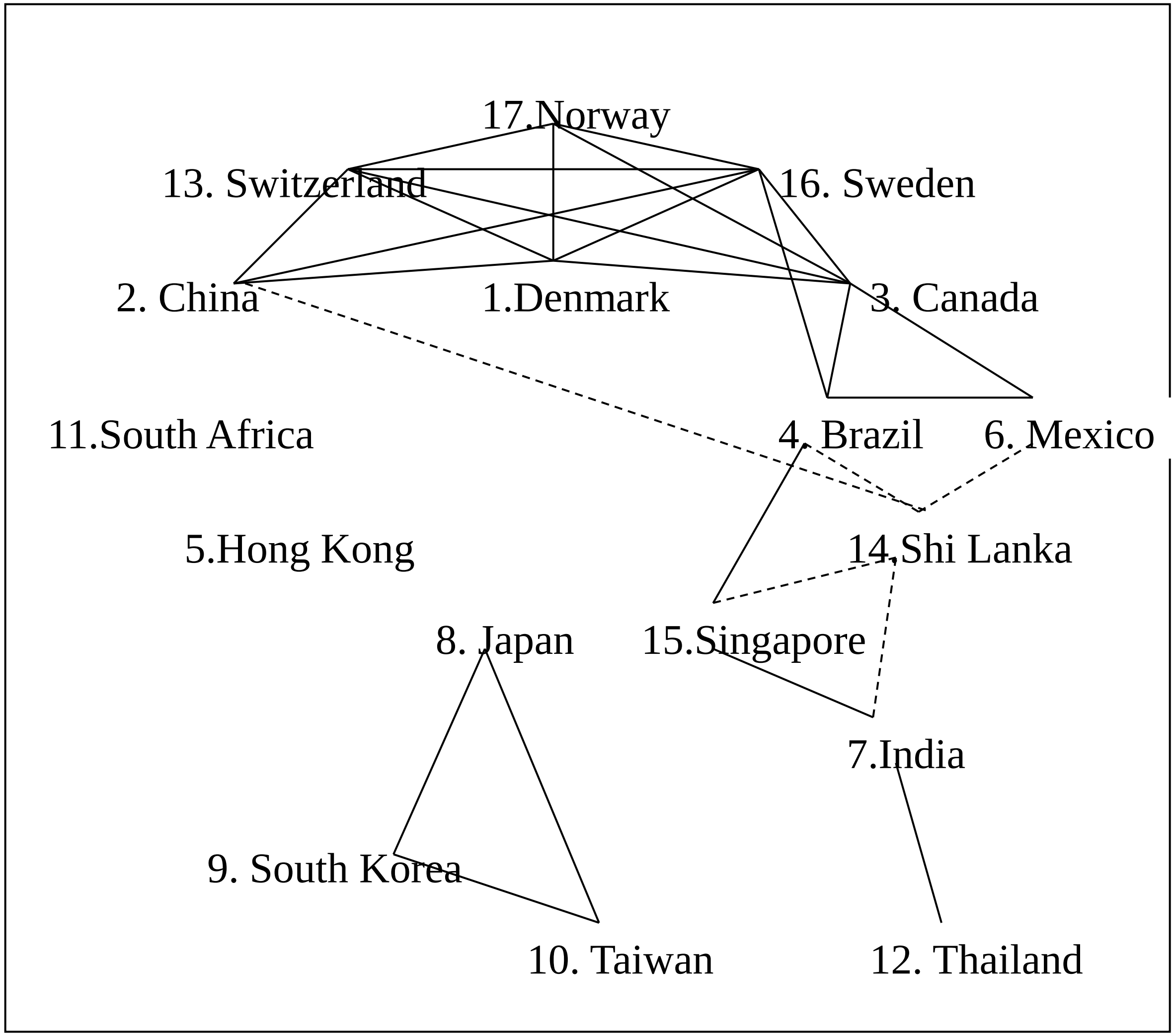
MAT and association networks

Local trend association network of foreign exchange rates obtained for small windows. Only links with high associations are shown.

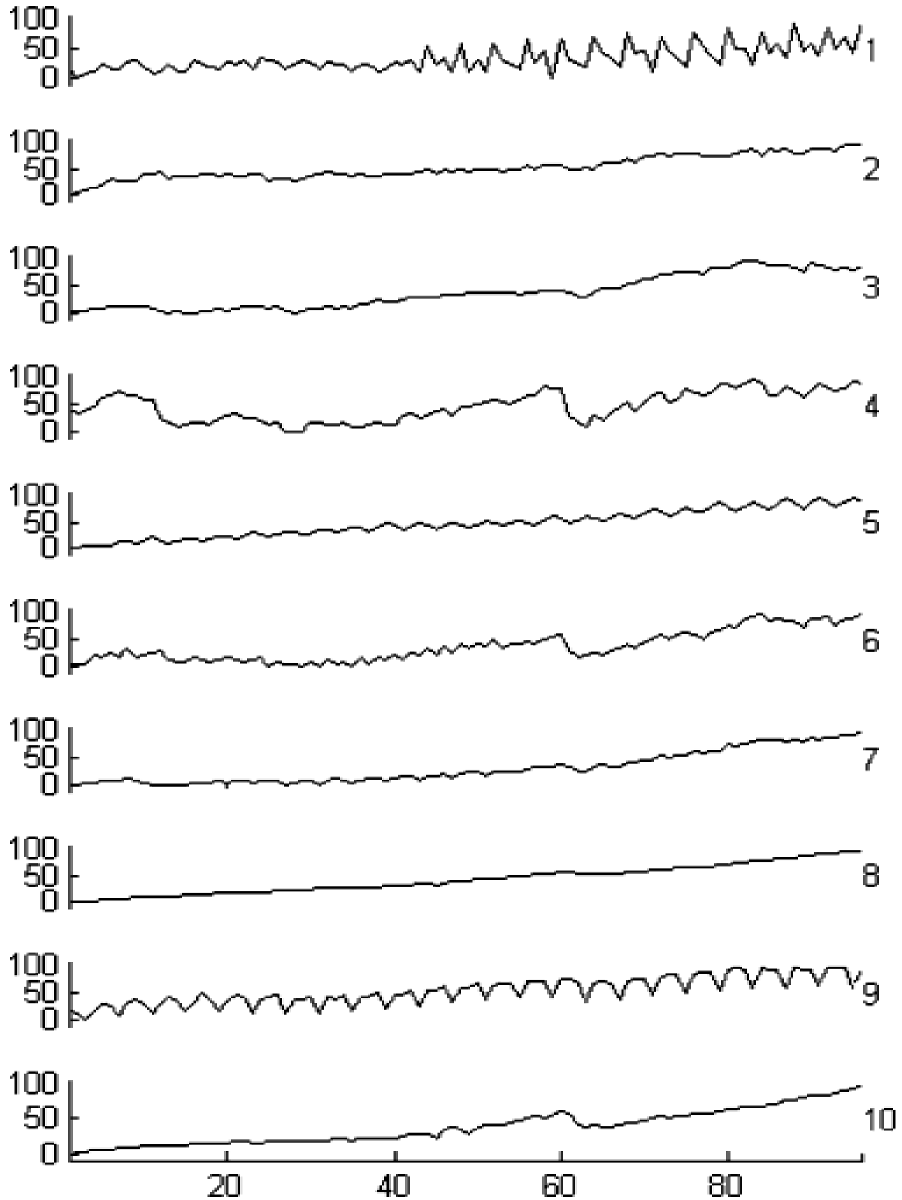


MAT and association networks

Global trend association network of foreign exchange rates obtained for large windows. Only high associations are shown.



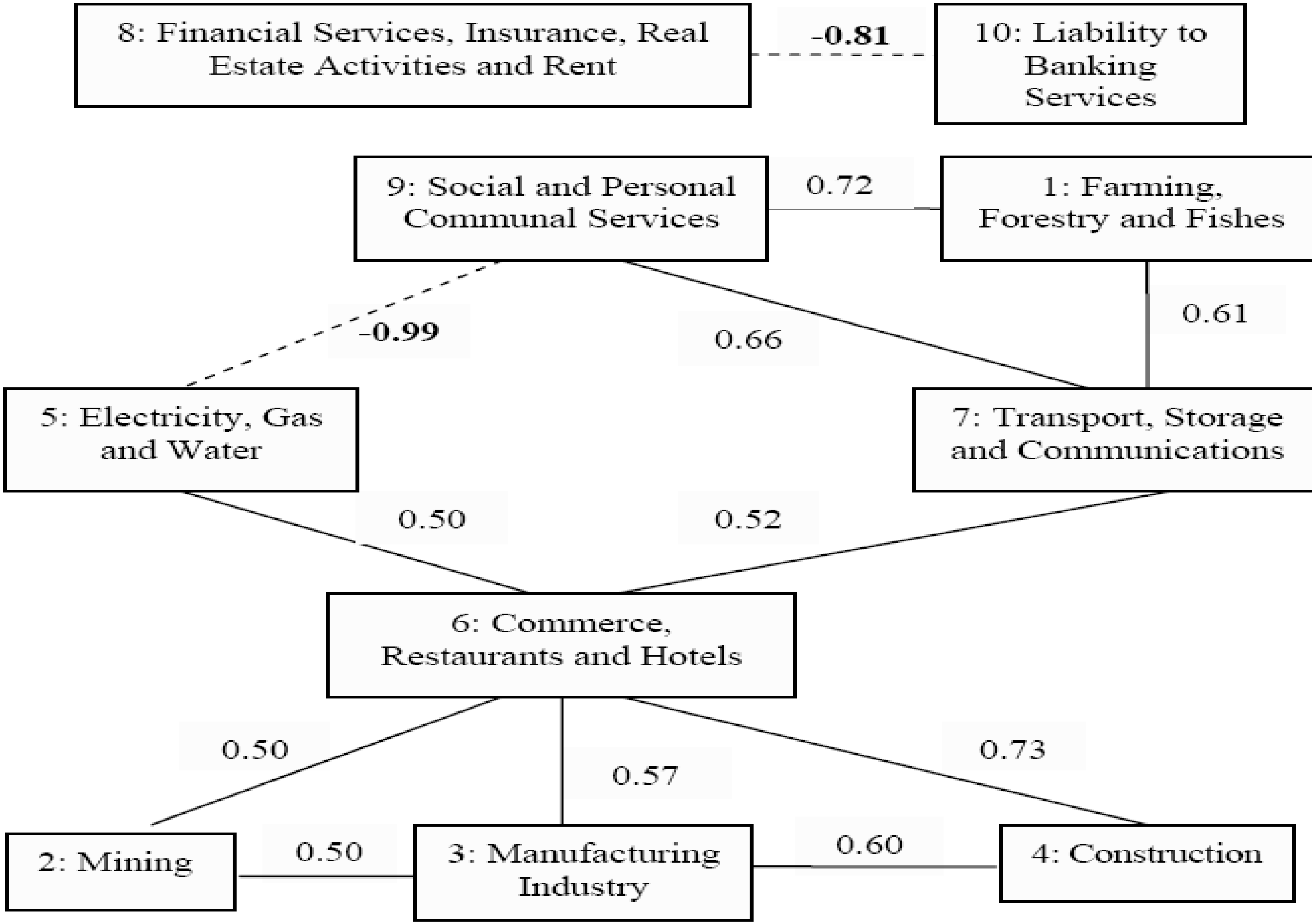
Gross internal product of Mexico



Gross internal product of Mexico checked quarterly over the period 1980 – 2003 (96 data).

1. Farming, Forestry and Fishes;
2. Mining
3. Manufacturing Industry
4. Construction
5. Electricity, Gas and Water
6. Commerce, Restaurants and Hotels
7. Transport, Storage and Communications
8. Financial Services, Insurance, Real Estate Activities and Rent
9. Social and Personal Communal Services
10. Liability to Banking Services Allocate

Association network of Mexican indexes of economics



How to define association measure on the set of subsets of [0,1], on the set of interval valued truth or membership values ???

Association Measures on the Set of Subintervals of [0,1]

Denote $D[0,1]$ the set of all closed subintervals of [0,1], i.e. $D[0,1] = \{a^* = [a^-, a^+] \subseteq [0,1]\}$, where $a^-, a^+ \in [0,1]$, $a^- \leq a^+$.

Definition. Suppose *neg* is an involutive negation of [0,1]. An involution (reflection) $N: D[0,1] \rightarrow D[0,1]$ on $D[0,1]$ is defined for all $a^* \subseteq [0,1]$ as follows:

$$N(a^*) = N([a^-, a^+]) = [neg(a^+), neg(a^-)]$$

It is fulfilled: $N(N(a^*)) = a^*$

Set of fixed points of N on $D[0,1]$

Proposition. The set of fixed points of N on $D[0,1]$ is the following:

$$FP = \{[a, neg(a)] \mid a \in [0,1],\ a \leq a_{FP}\},$$

where a_{FP} is the fixed point of a negation *neg* on [0,1].

Example. For the negation of Zadeh $neg(a)=1-a$, we have

$$FP = \{[a, 1-a] \mid a \in [0,0.5]\},$$

e.g. [0,1], [0.1,0.9], [0.4,0.6], [0.5,0.5] etc.

Association measure on $D[0,1]\backslash FP$ related by S_M

Define the similarity measure on $D[0,1]$:

$$SIM(a^*,b^*) = 1 - 0.5(|a^- - b^-| + |a^+ - b^+|).$$

Association measure on $D[0,1]\backslash \mathrm{FP}$ related by S_M:

$$A_M(a^*,b^*) = 1 - 0.5P, \quad \text{if } Q > P;$$

$$A_M(a^*,b^*) = 0.5Q - 1, \quad \text{if } Q < P;$$

$$A_M(a^*,b^*) = 0, \quad \text{if } Q = P.$$

where

$$P = |a^- - b^-| + |a^+ - b^+|$$

$$\mathrm{Q} = |a^- + b^+ - 1| + |a^+ + b^- - 1|.$$

Association measure on $D[0,1]\backslash FP$ related by S_P

$$A_P(a^*,b^*) = (Q - P)/max(Q,P)$$

where

$$P = |a^- - b^-| + |a^+ - b^+|$$

$$\mathrm{Q} = |a^- + b^+ - 1| + |a^+ + b^- - 1|.$$

where Q and P are defined in (29), (30).

Examples

a^*= [0.1, 0.3], b^*= [0.2, 0.4], c^*= [0.8, 0.9],
d^*= [0.6, 0.8], e^*= [0.2, 0.7]

$A_M(a^*,b^*) = 0.9$, $A_M(a^*,c^*) = -0.95$,
$A_M(b^*,d^*) = -1$, $A_M(a^*,e^*) = 0.75$.

$A_P(a^*,b^*) = 0.8$, $A_P(a^*,c^*) = -0.92$,
$A_P(b^*,d^*) = -1$, $A_P(a^*,e^*) = 0.29$.

Conclusions

The general approach to definition and construction of association measures is developed. It is an interesting and promising task to extend these results on various types of sets with reflection operations, on other types of similarity measures and to use them in various tasks of data analysis and data mining.

Thank you very much!

Ildar Batyrshin

CIC IPN, Mexico

batyr1@gmail.com

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