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A New General Class of Fuzzy Flip-Flop Based on Türkşen's Interval Valued Fuzzy Sets

Rita Lovassy

Institute of Microelectronics and Technology, Kandó Kálmán Faculty of Electrical Engineering, Budapest Tech, Hungary
lovassy.rita@kvk.bmf.hu

László T. Kóczy

Institute of Information Technology, Mechanical and Electrical Engineering, Faculty of Engineering Sciences, Széchenyi István University, Győr, Hungary
Department of Telecommunication and Media Informatics, Faculty of Electrical Engineering and Informatics, Budapest University of Technology and Economics, Hungary
koczy@sze.hu, koczy@tmit.bme.hu

Abstract: The extension of binary J-K flip-flops to fuzzy generalizations (F^3) was proposed originally by Hirota and Ozawa. They introduced the concept of reset type and set type F^3. Several alternative F^3-s were later proposed, such as algebraic, symmetrical set-reset type and non-associative connective based F^3-s, as well. This paper continues the investigation of the possible range of fuzzy flip-flops by examining the IVFS expression given by Türkşen and its midpoint values, and also by introducing the minimized IVFS (MIVFS), more fitting Hirota's original definitions, and the midpoint of MIVFS. This way a set of new definitions of F^3-s are obtained with diverse behavior. The standard and algebraic F^3-s based on these new definitions are graphically examined and briefly analyzed.

Keywords: Fuzzy logic, J-K flip-flop, DNF, CNF, IVFS, MIVFS

1 Introduction

In 1965 L. A. Zadeh [11] proposed fuzzy set theory. Since then, fuzzy sets, logics, and systems theory have gone through many theoretical developments and its applications to real life problems have been demonstrated with many applications.

Examples of such successful applications can be categorized under two main headings:

(1) Mathematical developments in set and logic theories.

(2) Real life applications in control technology, robotics, and decision support systems.

As a possible starting point for practical applicable systems, but also undoubtedly as part of the further development of the basics of fuzzy logic, Hirota and Ozawa [7] introduced the notion of the fuzzy J-K flip-flop, considering it an essential and basic component for effective processing of fuzzy information.

They introduced two types of fundamental characteristic equations of fuzzy flip-flops (F^3). They are the so called reset type and set type equations, both of which are fuzzy extensions of the characteristic equation of J-K flip-flop. Fuzzy negation, t-norm and s-norm (t-conorm) operations were used to establish the reset and set type fuzzy flip-flop. Both the reset and set type equations might be expressed e.g. by complementation, min and max operations (standard F^3), or complementation, algebraic product and sum operations (algebraic F^3).

While the above cited authors proposed standard operations, the algebraic fuzzy flip-flop was introduced later, when Ozawa, Hirota and Kóczy [6] analyzed this new type of F^3 as an example of the general F^3 concept, which has been also defined as an extension of the binary flip-flop.

2 Binary (Boolean) and Fuzzy Flip-Flops

2.1 A Binary Logic J-K Flip-Flop

All types of traditional binary flip-flop circuits, such as the most general J-K flip-flop store a single bit of information. These elementary circuits are also the basic components of every synchronous sequential digital circuit. The next state $Q(t+1)$ of a J-K flip-flop is characterized as a function of both the present state $Q(t)$ and the two present inputs $J(t)$ and $K(t)$. In the next, *J, K* and *Q* will be used instead of $J(t)$, $K(t)$ and $Q(t)$, respectively, as simplified notations. The minterm expression (disjunctive normal form - DNF) of $Q(t+1)$ can be written as

$$Q(t+1) = \overline{J}\,\overline{K}Q + J\overline{K}\,\overline{Q} + J\overline{K}Q + JK\overline{Q}\,, \tag{1}$$

this can be simplified to the minimal disjunctive form

$$Q(t+1) = J\overline{Q} + \overline{K}Q \tag{2}$$

This latter is well-known as the characteristic equation of J-K flip-flops. On the other hand, the equivalent maxterm expression (conjunctive normal form - CNF) can be given by

$$Q(t+1) = (J+K+Q)(J+\bar{K}+Q)\left(J+\bar{K}+\bar{Q}\right)\left(\bar{J}+\bar{K}+\bar{Q}\right), \tag{3}$$

in a similar way the minimized conjunctive form can be given by

$$Q(t+1) = (J+Q)(\overline{K}+\overline{Q})\,. \tag{4}$$

These four expressions, (1), (2), (3) and (4), are equivalent in Boolean logic, however there exist no such fuzzy operation triplets where these two forms are necessarily equivalent. It is a rather obvious question, which of these four, or any other equivalent, should be considered as the most proper fuzzy extension of the definitive equation of the very fundamental concept of fuzzy J-K flip-flop. There is no justifiable argumentation that prefers any of these four to the other. Thus, there is no unambiguous way to introduce the concept of fuzzy J-K flip-flop. While normal forms are especially important for theoretical reasons both minimal forms are equally important in the practice. This is why Hirota and Ozawa [2, 3] proposed two dual definitions of fuzzy flip-flops. They interpreted (2) as the definition for what they called 'reset type fuzzy flip-flop':

$$Q_R(t+1) = (J \wedge \neg Q) \vee (\neg K \wedge Q)\,, \tag{5}$$

where the denotations for logic operations stand this time for Zadeh type fuzzy conjunction, disjunction and negation. In a similar way the definition of 'set type fuzzy flip-flop' was obtained by re-interpreting (4) with fuzzy operations:

$$Q_S(t+1) = (J \vee Q) \wedge (\neg K \vee \neg Q)\,. \tag{6}$$

As a mater of course, it is possible to substitute the Zadeh operations by any other reasonable fuzzy operation triplet (e.g. De Morgan triplet), this way obtaining a multitude of various fuzzy flip-flop (F^3) pairs, such as the algebraic F^3 introduced in [6] and [7].

2.2 Standard (min-max) J-K Flip-Flop

Schweizer and Sklar [8] state that there is a one-to-one correspondence between t-norms and their dual t-conorms. Most definitions of F^3's in the literature use such corresponding pairs. Often they also form a De Morgan triplet with standard complementation. Supposing that standard complementation (7), conjunction (8), and disjunction (9) are selected as fuzzy negation, t-norm, and s-norm. For $A, B \in [0,1]$,

Complementation $\qquad \neg A = 1 - A\,, \qquad$ (7)

Logical product $A \wedge B = \min(A, B)$, (8)

Logical sum $A \vee B = \max(A, B)$. (9)

The expressions (5) and (6) are re-written as follows:

$$Q_R(t+1) = \max(\min(J, 1-Q), \min(1-K, Q)) \text{ and } \quad (10)$$

$$Q_S(t+1) = \min(\max(J, Q), \max(1-K, 1-Q)). \quad (11)$$

The fuzzy flip-flop characterized by (10) is thus called a standard or min-max reset type F^3, and the one given by (11) a standard or min-max set type F^3.

2.3 Algebraic J-K Flip-Flop

In case of another special conjugate pair of t-norm and s-norm with the complementation (12), namely the algebraic product (13) and algebraic sum (14) we obtain

Complementation $\neg A = 1 - A$, (12)

Algebraic product $A \cdot B = AB$, (13)

Algebraic sum $A \dot{+} B = A + B - AB$. (14)

Replacing 'and', 'or', and 'not' by these three operations the expressions for the two corresponding F^3-s the following two equations are obtained:

$$Q_R(t+1) = J + Q - 2JQ - KQ + JQ^2 + JQK - JKQ^2 \text{ and } \quad (15)$$

$$Q_S(t+1) = J + Q - JQ - JKQ - KQ^2 + JKQ^2. \quad (16)$$

The fuzzy flip-flop characterized by (15) is called algebraic reset type F^3, and the one given by (16) algebraic set type F^3.

The main problem with this approach is that different representations of the original logical operation lead to different results - even if we use the simplest possible operations.

3 The DNF-CNF Interval

DNF and CNF play very special roles in classical Boolean logic. They represent those standard forms, which do not contain any redundancy in the sense that they cannot be further reduced by applying the idempotence law, however they consist of complete members containing all variables in question, thus usually they can be simplified by merging and eliminating - this way obtaining the corresponding minimal forms. In fuzzy logic there are no standard forms in this sense as idempotence itself does usually not hold. Any repeated member would change the value of the expression.

Despite this latter fact several authors consider the fuzzy extensions of DNF and CNF as having special significance. Especially, Türkşen [10] examined the special properties of the extended normal forms and came to the somewhat surprising and very convenient conclusion that in fuzzy logic the equivalents of the CNF and DNF forms represent the extremes of all possible expressions that correspond to forms being equivalent in binary logic. Thus he showed that for any fuzzy connective the value of every other form lies in the interval formed by the CNF and DNF values. In [9], this result was proven for logical operations with an arbitrary number of variables.

3.1 Interval Valued Fuzzy Sets

While the theory of the interval valued fuzzy sets (*IVFS*) is much more general and can be considered as a special case of L-fuzzy sets [1] Türkşen proposed the interval determined by the disjunctive and conjunctive normal forms as the interval associated with the value belonging to an expression obtained by the fuzzy extension of some classic binary concept. Indeed, any theoretically possible formulation of the same concept would result in a value lying within the interval thus proposed.

According to Türkşen's statement the DNF is always included in the corresponding CNF, i.e., $DNF(\cdot) \subseteq CNF(\cdot)$ where $(\cdot)$ represents a particular expression [9]. The fundamental result that every DNF is contained in its corresponding CNF is true for min-max operators and for algebraic triplets as well. Türkşen [9] proposed to define the interval-valued fuzzy set representing a Boolean expression as follows:

$$IVFS(\cdot) = \left[DNF(\cdot), CNF(\cdot)\right] \tag{17}$$

3.2 Fuzzy Flip-Flops based on Türkşen's IVFS

The DNF and CNF forms for J-K flip-flops are expressed in 2.1. By applying the usual denotations for fuzzy negation, t-norm and t-conorm, (1) is re-written as follows

$$Q_{DNF}(t+1) = \big((1-J) \wedge (1-K) \wedge Q\big) \vee \big(J \wedge (1-K) \wedge (1-Q)\big) \vee \big(J \wedge (1-K) \wedge Q\big) \vee \\ \big(J \wedge K \wedge (1-Q)\big), \tag{18}$$

further, in the same way, (3) becomes

$$Q_{CNF}(t+1) = (J \vee K \vee Q) \wedge (J \vee (1-K) \vee Q) \wedge (J \vee (1-K) \vee (1-Q)) \wedge$$
$$((1-J) \vee (1-K) \vee (1-Q)). \tag{19}$$

This way we obtained two more definitions of min-max fuzzy flip-flops. It is questionable, of course, whether these new definitions play any more important role in the practice than the previous set type and reset type equations. These latter ones may be called 'normalized reset type and set type' F^3-s.

Using the algebraic operations, a similar pair of normalized flip-flops may be obtained. Here we omitted the explicit formulas as it is rather easy to determine them but they are somewhat lengthy.

Although the examination of these new F^3-s may be interesting itself, in this paper we go another way. Reset and set type behaviors are generally different and none of them possess the 'nice' symmetrical behavior of the original J-K flip-flop. This is why a symmetrical F^3 was proposed earlier by combining the two minimal forms in the equilibrium point [6]. Only one exceptional combination of operations has been found as far where the two types completely coincided [5].

Another possibility to combine two different extensions of the original definition is to choose a representative point of each interval corresponding to the *IVFS* obtained from the two normal forms. As the most obvious representative, the midpoint is proposed here. Figure 1 depicts the DNF and CNF flip-flops and the graphs belonging to the midpoints of the intervals delimited by them. The 25 small graphs present the behavior of the F^3 for different combinations of *J* and *K* (in steps of 0.25 from 0 to 1).

Because, however, in all earlier publications it was the minterm and maxterm expressions that played important roles rather then the normal forms we propose here a new interval the one limited by the minterm and the maxterm:

$$MIVFS(\cdot) = [DMF(\cdot), CMF(\cdot)] \tag{20}$$

MIVFS is the narrower interval given by the inequality $DNF(\cdot) \subseteq DMF(\cdot) \subseteq CMF(\cdot) \subseteq CNF(\cdot)$. The Figure shows the graphs corresponding to *MIVFS,* as well. In order to obtain a single fuzzy flip-flop we propose that the midpoint of *MIVFS* is considered as the definition of the new type standard F^3.

Figure 2 presents the six graphs corresponding to DNF, DMF, CMF, CNF, further (DNF+CNF)/2 and (DMF+CMF)/2 for algebraic F^3-s.

Studying the behavior of the min-max J-K flip-flop, we remark that in the case of *J = K*, the values of the next state *Q (t+1)* are equal in every 6 focused case. If *Q* *(t) = 0*, the starting values of the piecewise linear characteristics are equal to the value of *J*. If *Q (t) = 1* the endpoint values, for all 6 graphs are identical with the negated values of *K*. If we draw an imaginary diagonal line, it is easy to observe, that the diagrams are symmetrical. In these mentioned cases the values of *J* and *K* are complementary. The characteristics belonging to the intermediate cases show the obvious situation, when the reset type curves always go below the set type ones. In these cases the graphs belonging to the DNF and CNF values represent the extreme cases. This, so called Türkşen interval is the largest when *J = 1,K = 0*, and 'vice versa'. The characteristics representing the (DNF+CNF)/2 and (DMF+CMF)/2 curves are naturally in the middle.

Comparing the behavior of the algebraic J-K flip-flop (Figure 2), with the standard one, we came to the result, that the algebraic operations produced smooth (differentiable) curves and surfaces with no breakpoints or lines at all. The border characteristics of the Türkşen interval are in this case also the graphs corresponding to DNF and CNF. This interval represents a larger surface than in the case discussed first. It is clearly demonstrated that Türkşen's statement discussed in section 3.1 holds. Completely new is the fact that in case of $J, K \in (0,1)$, the values of *Q (t+1)* for different values of $Q(t) \in [0,1]$, are not equal. The appearance of this interval in the real fuzzy cases for *J* and *K* introduces a new concept, a truly interval valued fuzzy flip-flop.

Generally, it may be remarked, that *MIVFS* reflects the typical behavior of fuzzy flip-flop more obviously in both cases and closes around the average behavior in a narrower stripe, while the original *IVFS* opens up wider in the areas where real fuzziness occurs intensively. It remains an open question which of the two new average types F^3-s will be more applicable for practical purposes.

Conclusions

In this paper, interval valued fuzzy sets are defined as an extension of the disjunctive and conjunctive normal forms in Boolean logic. Such *IVFS's* exist for some cases of certain families of the conjugate pairs of t-norms and t-conorms. Considering that the flip-flop thus defined is described by an opening pair of graphs, we suggested that the midpoint of the border values of the intervals be considered as the definitive function for this new type of (point valued) F^3.

In the future we intend to investigate the behavior of complex fuzzy sequential circuits based on interval valued F^3. It would be very interesting to examine the behavior of such fuzzy networks, to determine the possible divergence behavior and to find the proper, ideal functional interval, or point.

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References

[1] Goguen, J. A.: L-Fuzzy Sets, *J. of Math. Analysis ands Applications,* 18, pp. 145-174

[2] K. Hirota, K. Ozawa: Concept of Fuzzy Flip-Flop, *Preprints of 2nd IFSA Congress*, Tokyo, 1987, pp. 556-559

[3] K. Hirota, K. Ozawa: Fuzzy Flip-Flop as a Basis of Fuzzy Memory Modules, In: M. M. Gupta *et al*., eds., *Fuzzy Computing. Theory, Hardware and Applications,* North Holland, Amsterdam, 1988, pp. 173-183

[4] G. J. Klir, T. A. Folger: Fuzzy Sets, Uncertainty, and Information, Prentice Hall, Upper Saddle River, NJ, 1988, pp. 37-63

[5] R. Lovassy, L. T. Kóczy: Non-Associative Fuzzy Flip-Flop with Dual Set-Reset Feature, *4th Serbian-Hungarian Joint Symposium on Intelligent Systems*, Subotica, 2006, pp. 289-299

[6] K. Ozawa, K. Hirota, L. T. Kóczy, K. Ohmori: Algebraic Fuzzy Flip-Flop Circuits, *Fuzzy Sets and Systems 39/2*, 1991, pp. 215-226

[7] K. Ozawa, K. Hirota, L. T. Kóczy: Fuzzy Flip-Flop, In: M. J. Patyra, D. M. Mlynek, eds., *Fuzzy Logic. Implementation and Applications*, Wiley, Chichester, 1996, pp. 197-236

[8] B. Schweizer, A. Sklar: Associative Functions and Statistical Triangle Inequalities, *Publ. Math. 8* Debrecen, 1961, pp. 169-186

[9] I. B. Türkşen: Interval-valued Fuzzy Sets Based on Normal Forms, *Fuzzy Sets and Systems 20,* 1986, pp. 191-210

[10] I. B. Türkşen, A. Esper, K. Patel, S. A. Starks, V. Kreinovich: Selecting a Fuzzy Logic Operation from the DNF-CNF Interval: How Practical Are the Resulting Operations?, *Proceedings of the 21st International Conference of the North American Fuzzy Information Processing Society NAFIPS'2002,* New Orleans, 2002, pp. 28-33

[11] L. A. Zadeh: Fuzzy Sets, *Information and Control 8*, 1965, pp. 338-353

Figure 1
Characteristics of min-max type fuzzy flip-flops for various values of K

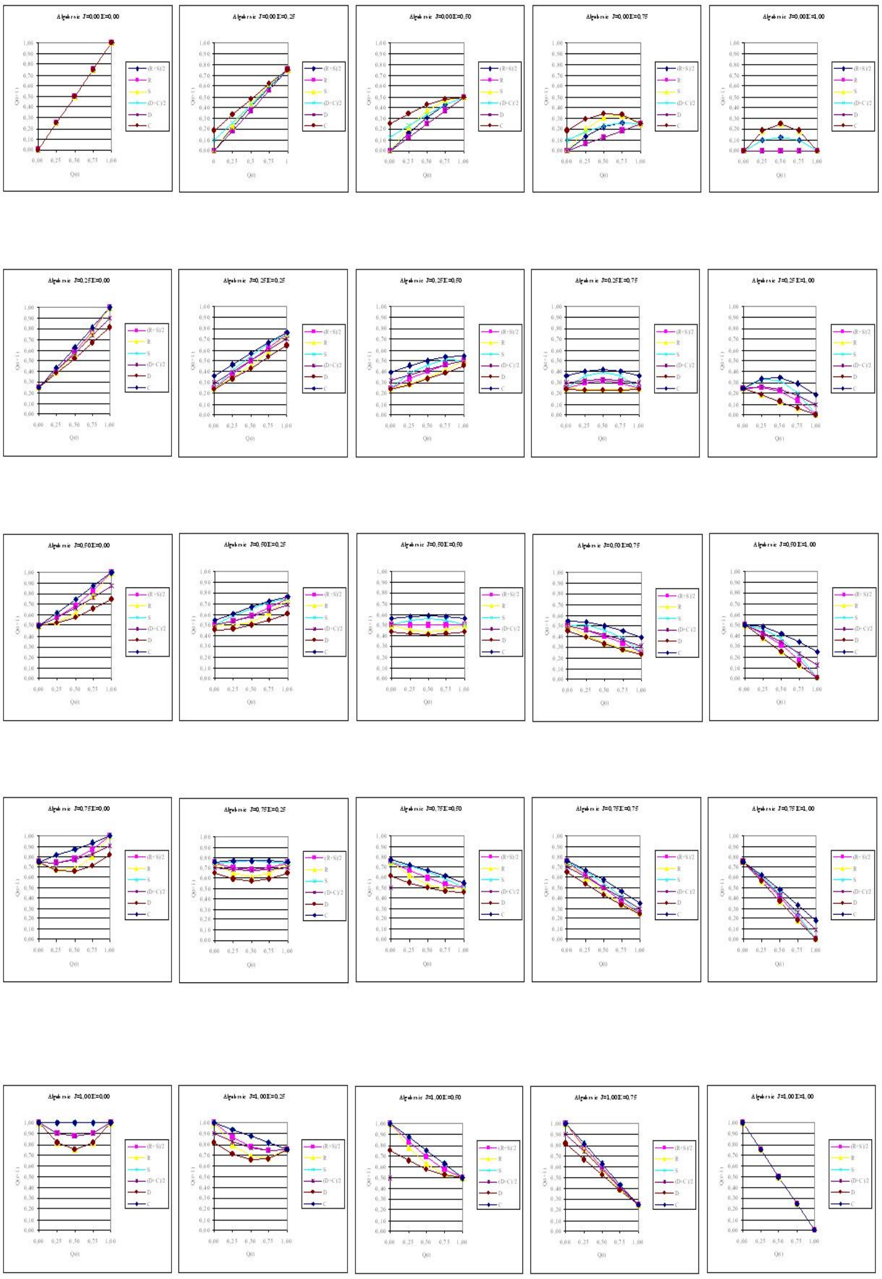

Figure 2
Characteristics of algebraic type fuzzy flip-flops for various values of K