Stochastic Active Observers: Active State Analysis - Theory and A Robotic Force Control Application

Rui Cortesão,† Ralf Koepp,‡ Urbano Nunes,† and Gerd Hirzinger ‡

†University of Coimbra
Institute of Systems and Robotics - ISR
3030 Coimbra, Portugal
Fax: +351 239 406672
Email: cortesao@isr.uc.pt

‡German Aerospace Center - DLR
Institute of Robotics and Mechatronics
82230 Wessling, Germany
Fax: +49 8153 281134
E-mail: Ralf.Koepp@dlr.de

Abstract

The paper introduces the Active Observer (AOB) concept in the framework of stochastic theory. The overall AOB control structure is discussed on a conceptual level, enabling its implementation in any control system, coming or not from the robotics field. The AOB concept was initially discussed in [3]. The paper extends the analysis of the AOB with respect to the active state. Simulation results comparing the force tracking capabilities of the AOB and the Classical Kalman Filter (CKF) are presented, showing the importance of the active state.

1 Introduction

In robotic control systems, unknown disturbances, higher order dynamics, nonlinearities and noise are always present. Particularly, if the robot is doing compliant motion tasks that require contact with unknown environments, "rigid" model-based approaches are seldom efficient, and the robustness of the task execution can be seriously deteriorated if unmodeled disturbances are not handled in a proper way. Recently, several methods have been presented to deal with disturbances. In [8], an extended deterministic observer is constructed to estimate the motion parameters of a moving object in a force control task. In [1], model uncertainties, nonlinearities, and external disturbances are merged to one term, and then compensated with a non-linear disturbance observer, based on the variable structure system (VSS) theory. Several drawbacks of previous methods, like the approximate differentiator and the $H_\infty$ type formulations are also pointed out in [1]. In [5], a neural network approach is used as a compensator to cancel out all the uncertainties occurred in force control, such as robot model mismatches, unknown environment stiffness and location. A manipulator control method using a disturbance observer with no inverse dynamics is addressed in [7].

The proposed AOB structure uses a self-adjusting discrete probabilistic approach to estimate disturbances. The method has a systematic formulation, is mathematically elegant, and uses the Kalman theory with new interpretations to optimise its performance function of system and measurement noises.

2 AOB Concept

Given a linear system represented in state-space form by

$$x_n = \Phi x_{n-1} + \Gamma u_{n-1},$$

(1)

any linear controller can be achieved with state feedback (optimal control, adaptive control, deadbeat control, "pure" pole placement control, etc.). In practice, the main problem of this approach is that the model of the system in Equation (1) does not represent exactly the real system. In fact, higher order dynamics, noise, unknown disturbances, etc., are not addressed in the plant model. Therefore, it is necessary to develop a control structure that can deal with unmodelled terms, so that the overall system has the desired closed-loop dynamics. The AOB state-space control design satisfies these requirements. The main goal of the AOB is to impose to the overall system a desired closed-loop behaviour, regardless the imperfect model of the plant. An active state $p_k$ (extra-state) is introduced to compensate unmodeled terms, providing a feedforward compensation action. A stochastic equation is used to describe the active state,

$$p_k - p_{k-1} = w_k.$$

(2)

Equation (2) only says that the discrete derivative of $p_k$ is randomly distributed. It gives no explicit information about the $p_k$ characteristics. Hence, arbitrary disturbances can be estimated (model-free equation). In fact, the general form of Equation (2) is

$$p_k = \sum_{j=1}^{R} (-1)^{j+1} \frac{R!}{j!(R-j)!} p_{k-j} + ^R w_k,$$

(3)

where $^R w_k$ describes the $R^\text{th}$ derivative of the process to estimate [2]. From Equation (2), the disturbance $p_{k-1}$ can go
2.2 The AOB Algorithm

The AOB algorithm is described in this section. Detailed analysis can be seen in [4] and [3]. The a priori state estimate $\hat{x}_k$ is

$$\hat{x}_k = \Phi_{ac} \hat{x}_{k-1} + \Gamma r_{k-1}. \quad (7)$$

Giving the measure $y_k$, the a priori estimation error $e_k^a$ is

$$e_k^a = y_k - C \hat{x}_k^a. \quad (8)$$

Finally, the corrected estimation,

$$\hat{x}_k = \hat{x}_k^a + K_k e_k^a, \quad (9)$$

where $K_k$ is the Kalman gain,

$$K_k = P_{k-1} C^T (C P_{k-1} C^T + R_k)^{-1}, \quad (10)$$

and

$$P_k = P_{k-1} - K_k C P_{k-1}, \quad (11)$$

$Q_k$ is the system noise matrix, and its interpretation is discussed in Sections 3.2 and 3.3. $R_k$ is the measurement noise matrix and is function of the measurement noise $\eta_k$, $R_k = E(\eta_k \eta_k^T)$. The open loop and the closed-loop system matrices are given by $\Phi_{o}$ and $\Phi_{ac} = (\Phi_{o} - \Gamma L)$ respectively, and include already the active state, i.e.

$$\Phi_{o} = \begin{bmatrix} \Phi & \Gamma & \Gamma_0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \quad (13)$$

$$\Phi_{ac} = \begin{bmatrix} \Phi & \Gamma_1 & \Gamma_0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ -L_1 & -L_2 & -L_3 & \cdots & -L_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \quad (14)$$

$$\Gamma = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}^T. \quad (15)$$

The non-augmented open-loop $\Phi_{oa}$ and command $\Gamma_r$ matrices are

$$\Phi_{oa} = \begin{bmatrix} \Phi & \Gamma_1 & \Gamma_0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (16)$$

and

$$\Gamma_r = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^T. \quad (17)$$
The $L$ components are obtained by Ackermann’s formula for the non-augmented system, given a desired closed-loop behaviour. The state $x_k$ is

$$x_k = [x_k', u_{k-d} \ldots u_{k-2} \ u_{k-1} \ p_k]^T,$$  \hfill (18)

where $x_k'$ is the state of the system considering no dead-time, $u_{k-m}$ are the delayed command efforts, and $p_k$ is the active state.

The CKF can be obtained from the AOB with small changes. Only the active state is eliminated from the estimation, i.e. $\dot{p}_k = 0$, and its corresponding value in the $Q_k$ matrix is set to zero. Re-designing the $Q_k$ matrix is needed in some cases. The explanation is beyond the scope of this paper.

3 Active State: A Probabilistic Approach

To cope with parameter variations, unmodelled nonlinear terms and unpredicted disturbances, lumped in the $p_k$ variable, an active state $\dot{p}_k$ is created. Several possibilities can be considered. For a Deterministic Active Observer, a model for the error dynamics of $p_k$ is needed. The simplest case is for a constant error, giving a state equation of the form $p_k = p_{k-1}$. For a time varying error, the state equation changes. Clearly there are obvious limitations in the deterministic approach. It becomes natural to use stochastic concepts to quantify disturbances, enabling a wide class of signals to be described with the same equation. A stochastic model was developed to deal with unknown disturbances [2]. Looking to Equation (2), the stochastic process $\{w_k\}$ can be seen as the first order active state evolution. Let’s define $w_k$ as one realisation of the process at time $k$. All the random variables $w_k$ have a Gaussian distribution with zero mean and variance $\sigma_{w_k}^2$. $w_k$ represents the evolution (change) in $p_k$ from time $k - 1$ to time $k$. Defining the estimation strategy as:

The probability of the $p_k$ evolution (in module) be greater than its previous evolution (in module), should be equal to $\alpha_k$, the variance of $w_k$ is computed in a straightforward way. The value $\alpha_k$ can change on-line as a function of the task state. Qualitatively, if $\alpha_k$ is big, it means that $p_k$ is able to follow high frequency signals. On the other hand, if $\alpha_k$ is low, only low frequency signals are estimated. In formal terms,

$$w_k \sim N(0, \sigma_{w_k}^2).$$  \hfill (19)

The strategy is

$$P(|w_k| > |w_{k-1}|) = \alpha_k, \ \text{with} \ 0 < \alpha_k < 1.$$  \hfill (20)

Defining a new random variable

$$\chi_k \sim N(0, 1),$$  \hfill (21)

Equation (20) is written as

$$P(|\chi_k| > \frac{|w_{k-1}|}{\sigma_{w_k}}) = \alpha_k,$$  \hfill (22)

which has a well known solution. Defining the function $G$ as

$$G(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt,$$  \hfill (23)

the solution of Equation (22) is given by

$$\sigma_{w_k} = \frac{|w_{k-1}|}{G^{-1}(\alpha_k/2)},$$  \hfill (24)

where $G^{-1}$ is the inverse function of $G$. The function $G(x)$ cannot be obtained explicitly, but many software libraries provide a related function, the error function erf, defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$  \hfill (25)

Hence,

$$G(x) = \frac{1 - \text{erf}(x/\sqrt{2})}{2},$$  \hfill (26)

and Equation (24) can be written as

$$\sigma_{w_k} = \frac{|w_{k-1}|}{\sqrt{2} \text{erf}^{-1}(1 - \alpha_k)}.$$  \hfill (27)

The variance of $w_k$, $\sigma_{w_k}$, is computed from Equation (27), which is in the Kalman notation the $Q_k$ value for the active state. Thus, the $p_k$ estimation is done in the framework of probabilistic estimation. If $w_k$ is a narrow-band process, good results are achieved for a constant $\alpha_k$. However, for wide-band processes, a dynamic assignment of $\alpha_k$ is needed. When $w_k^0 \to 0$, a minimum $\sigma_{w_k}$ should be imposed, so that the $p_k$ estimation is able to follow abrupt changes, with an acceptable time-lag. The price payed for this, is that around low frequency values, the noise sensitivity is increased.

3.1 Computation of $\alpha_k$ for the Desired Minimum $\sigma_{w_k}$

The function $e^{-t^2}$ can be written in Taylor Series around zero, giving

$$e^{-t^2} = 1 - t^2 + \frac{t^4}{2!} \frac{t^6}{3!} + \cdots,$$  \hfill (28)

and,

$$\int_0^{\infty} e^{-t^2} dt = \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \cdots.$$  \hfill (29)

From Equations (25) and (29),

$$\lim_{z \to 0} \text{erf}(z) = \frac{2z}{\sqrt{\pi}}.$$  \hfill (30)

Using Equation (30), straightforward analysis gives,

$$\lim_{z \to 0} \frac{\text{erf}^{-1}(bz)}{z} = \frac{b \sqrt{\pi}}{2}.$$  \hfill (31)

Looking to Equations (27) and (31), a minimum value of $\sigma_{w_k}$, $\sigma_{w_k}$, is achieved for very weak $p_k$ evolutions, $|w_{k-1}| \ll 1$, only if

$$\alpha_k = 1 - \frac{\sqrt{2/\pi}}{\sigma_{w_k}} |w_{k-1}|.$$  \hfill (32)
3.2 Qualitative Meaning of the $Q_k$ Matrix for the Active State $p_k$

The $Q_k$ matrix is written as

$$Q_k = \sigma_{w_k}^2,$$  \hspace{1cm} (33)

and its interpretation is qualitatively defined by the estimation strategy, i.e. the value of $\alpha_k$ at each time step. It is the responsibility of the control designer to input proper values of $\alpha_k$, in order to have good performance in the $p_k$ estimation. For a given $Q_k$ value, Equation (27) says that the strategy is

$$\alpha_k = 1 - \text{erf}\left(\frac{w_{k-1}^2}{\sqrt{2Q_k}}\right).$$  \hspace{1cm} (34)

It can be inferred from Equation (34), that even for a constant $Q_k$ value, the strategy changes at each time step, lying in the framework of stochastic adaptive estimation. To keep always the same strategy, it is necessary that

$$Q_k = \frac{|w_{k-1}^2|}{c^2},$$  \hspace{1cm} (35)

where $c$ is a constant, giving then

$$\alpha_k = 1 - \text{erf}\left(\frac{c}{\sqrt{2}}\right).$$  \hspace{1cm} (36)

Figure 2 presents a schematic overview of the estimation strategy for the active state. The variance $\sigma_{w_k}^2$ is computed at each time step, function of $\alpha_k$ and $|w_{k-1}^2|$. Using this information, the $p_k$ estimate is computed. The stochastic process $\{w_n \}$ is represented in Figure 2.a. At each time step, there is a random variable $w_k$, with a Gaussian distribution of zero mean and variance $\sigma_{w_k}^2$. One realization of the process $\{w_n \}$ is shown in Figure 2.b, obtained from the $p_n$ estimation, displayed in Figure 2.c. Of course, other strategies can be defined, giving different meanings for the $Q_k$ values. This is one of the rich characteristics of stochastic analysis. The $Q_k$ values can be interpreted according to some context. The designer should define the strategy in an intuitive way for a given problem in order to input proper numerical values. Another strategy is defined for the other state variables in Section 3.3, with some interesting "symmetric" properties.

3.3 Qualitative Meaning of the $Q_k$ Matrix for the State $x_k$

A generic system represented in state-space form is given by

$$x_k = \Phi_x x_{k-1} + \Gamma u_{k-1} + \xi_k,$$  \hspace{1cm} (37)

where $u_{k-1}$ is the command input, and $\xi_k$ is a random vector (sometimes called system noise vector) associated with the state $x_k$. Equation (37) says that $x_k$ has a deterministic term function of $x_{k-1}$ and $u_{k-1}$, and a random term function of the statistical properties of $\xi_k$. This section analyses only the role of the random term in $x_k$. Then, since the system is linear, the superposition theorem can be applied to get the full solution of Equation (37). For the random term,

$$x_k = \Phi x_{k-1} + \xi_k.$$  \hspace{1cm} (38)

If the process $\{\xi_k \}$ is a zero mean white Gaussian sequence independent of the initial condition $x_0$, then given $x_{k-1}$, $x_k$ depends only on $\xi_k$, which is independent of the previous states. Hence, the process $\{x_k \}$ is a Gauss-Markov process (GMP). The probability density functions of $x_k$ can be calculated according to the GMP properties. The inverse function of Equation (38) is

$$\xi_k = x_k - \Phi x_{k-1}.$$  \hspace{1cm} (39)

The Jacobian determinant is one, and assuming that,

$$x_k = [x_{k1} \ x_{k2} \ \ldots \ x_{kN_k}]^T,$$

$$\xi_k = [\xi_{k1} \ \xi_{k2} \ \ldots \ \xi_{kN_k}]^T,$$

the conditional probability density function is

$$p_{x_{k1},x_{k2},\ldots,x_{kN_k}|x_{k-1}}(x_{k1},x_{k2},\ldots,x_{kN_k}|x_{k-1}) = p_{\xi_{k1},\xi_{k2},\ldots,\xi_{kN_k}|x_{k-1}}(x_{k1} - \Phi_x x_{k-1},\ldots,x_{kN_k} - \Phi_x x_{k-1}),$$

$$\Phi_x x_{k-1},\ldots,x_{kN_k} - \Phi_x x_{k-1},$$

where $\Phi_x$ is the $i$th row of matrix $\Phi$. In other words, $\Phi_x$ is the $i$th row of matrix $\Phi$. In other words,

$$p_{x_{k1},x_{k2},\ldots,x_{kN_k}|x_{k-1}}(x_{k1},x_{k2},\ldots,x_{kN_k}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p_{\xi_{k1},\xi_{k2},\ldots,\xi_{kN_k}|x_{k-1}}(x_{k1},\ldots,x_{kN_k}) \cdot dx_{k1} \cdots dx_{kN_k}.$$
In most of real applications, the $\xi_k$ ($i = 1, \ldots, N$) variables are not correlated (they are orthogonal). Equation (42) becomes,

$$p_{x_k | x_{k-1}}(x_k | x_{k-1}) = p_{q_k}(x_k - \Phi_k x_{k-1}), \quad i = 1, \ldots, N.$$  
(43)

For a Gaussian distribution of $\xi_k$, Equation (43) is

$$p_{x_k | x_{k-1}}(x_k | x_{k-1}) = \frac{1}{\sqrt{2\pi \sigma_k^2}} e^{-\frac{(x_k - \mu_{x_k})^2}{2\sigma_k^2}}.$$  
(44)

The expected value for $x_k$ given $x_{k-1}$ is $\Phi_k x_{k-1}$, i.e. the "noise-free" situation. The variance around this value is given by $Q_k$. If $Q_k \rightarrow 0$, Equation (44) becomes a Dirac delta function centered around $x_k = \Phi_k x_{k-1}$, which is the deterministic case. The bigger $Q_k$ is, the bigger the uncertainty around the expected value is. Defining now the strategy as: Given $x_{k-1}$, the probability of the deviation (in module) of $x_k$ around the expected value $\mu_{x_k} = \Phi_k x_{k-1}$ be greater than $\gamma_k$ should be equal to $\beta_k$,

$$p_{x_k | x_{k-1}}(x_k - \mu_{x_k} > \gamma_k) = \beta_k,$$  
(45)

the value $\sigma_{x_k} = \sqrt{Q_k}$ can be calculated in a straightforward way, and is given by,

$$\sigma_{x_k} = \frac{\gamma_k}{\sqrt{2} \sqrt{1 - \beta_k}}.$$  
(46)

Equation (46) should be compared with Equation (27). There are multiple solutions for the same strategy $\beta_k$. An interesting one is obtained when $\gamma_k = \sigma_{x_k}$, giving then

$$\beta_k = 1 - \epsilon \left(\sqrt{2}/2\right),$$  
(47)

i.e. $\beta_k = 0.3173$. Therefore, the probability of the signal deviation around the expected value be greater than $\sigma_{x_k}$ is about thirty per cent. This procedure is also valid for the active state ($\Phi = 1$), giving another interpretation for the $Q_k$ value. All these interpretations provide useful guidelines to design the Kalman filter for a given application. However, it should be noted that for the Kalman AOB, the $Q_k$ matrix by itself does not describe the system. The measurement noise matrix $R_k$ has a key role in the estimation process. The balance between model uncertainties given by $Q_k$ and measure uncertainties given by $R_k$ defines the steady-state Kalman gains, that influence the convergence rate of the estimates.

4 AOB vs. CKF: Force Tracking Capabilities

An application of the AOB/CKF in a robotic force tracking task is analysed in this section. In a multi-dimensional compliant motion task, there are couplings between motion and force. Figure 3 illustrates an example where a movement in $y$ direction disturbs the force controller in $x$ direction, since the surface geometry in $x$ changes with time. The velocity disturbance $\dot{x}_{\text{dist}}$ is given by

$$\dot{x}_{\text{dist}} = \frac{\dot{x}_1 - \dot{x}_0}{y_1 - y_0} v_x,$$  
(48)

where $x_0$, $x_1$, $y_0$, $y_1$, and $v_x$ is the velocity in $y$. $x_{\text{dist}}$ enters the system with negative sign (Figure 4). If $x_{\text{dist}}$ has a negative derivative, the velocity disturbance referred to the system input is positive, creating a $\Delta F$ in the force response, if no compensation action is performed. This $\Delta F$ creates a barrier to the maximum $v_x$. In many tasks, velocity scaling is necessary, to prevent high $\Delta F$. In the simulations, each degree of freedom of the the position controlled robot represented in Figure 4 has the transfer function

$$G_r(s) = \frac{1}{1 + T_p s} e^{-T_{\text{delay}} s},$$  
(49)

that is equal to one in steady state. $K_s$ is the system stiffness. In this way, for a disturbance $x_{\text{dist}}, the feedforward velocity should be equal to $\dot{x}_{\text{dist}}$. Several possibilities can be applied to compensate geometric changes by adding position or velocity feedforward information to the system. The feedforward action can be generated [6]: 1) from a priori information, 2) by external geometric sensing, 3) from position and force measurements, and 4) from the output of a skill map, trained from human demonstration data. However, if the AOB is used in the controller, the active state estimates the position disturbance, providing a proper compensation action. On the other hand, if the controller has an observer without active state, like the CKF, the force error is function of the Kalman gains. Simulation tools showed that the force error is

$$\Delta F_{\text{ss}} \approx 0.6839 K_s \dot{x}_r,$$  
(50)

where $K_s$ is the system stiffness and $\dot{x}_r$ is the feedforward compensation error of the velocity. Figure 5 shows the performance of the force controller with AOB and CKF when geometric changes occur during the task. The surface changes at 5 [s] with a negative slope. The active state performs the feedforward compensation action, that converges to the surface derivative (Figures 5.b and 5.d). In this simulation the system stiffness $K_s = 3$. For the CKF there is no external feedforward action, therefore, the error $\dot{x}_r$ is equal to $-\dot{x}_{\text{dist}}$. Using Equation (50), $\Delta F_{\text{ss}} \approx 4.1034$. When the CKF is used, the steady state error disappears, and the transient $\Delta F$ is reduced (Figures 5.a and 5.c). The strategy for the active state is an adaptive one: $Q_k = 10^{-5}$, and $\sigma_k$ is computed from Equation (34), that is function of the on-line data. For the other states, $\gamma_k = 10^{-12}$, and $\beta_k$ is given by Equation (47). In our robotic application, the AOB algorithm described in Section 2.2 was applied with the following values: $d = 5$, $k_1 = K_s/T_p$, $p_1 = 1/T_p$, the feedback gains $L$ were obtained for a critically damped system with a time constant $\tau = 10/T_p$. Finally,

$$\Phi = \phi(h) = \begin{bmatrix} 1 & -e^{-\frac{h}{p_1}} \\ 0 & e^{-\frac{h}{p_1}} \end{bmatrix},$$  
(51)

$$\Gamma_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad \Gamma_1 = \frac{\delta_1}{p_1} \begin{bmatrix} h + e^{-\frac{h}{p_1}} - 1 \\ 1 - e^{-\frac{h}{p_1}} \end{bmatrix}.$$  
(52)
5 Conclusions

The paper introduces the AOB concept, and analyses the active state in the framework of stochastic theory. The AOB algorithm is described. The CKF can be obtained from the AOB with minimal changes. Several strategies can be followed, to input adequate values in the AOB design. This procedure is also extended to the other state variables of the system. The comparison between AOB and CKF is done in a force control application.

Acknowledgements

This work is partially supported by FCT (Portuguese Science and Technology Foundation) project number PCTI/1999/SRI/33594, and by the FCT Ph.D. grant SFRH/BD/2754/2000.

References


