

Detection of load change in motor-generator plant with interval fuzzy model

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Abstract—In the paper a method of interval fuzzy model identification is presented. It enables defining a family of functions by approximating the confidence band in which all of the functions can be found with certainty of 1. In an uncertain system where parameters can be found in a certain interval, it makes possible to cover the complete area of normal responses by l_∞ -norm optimization of parameters of 2 fuzzy models by using linear programming. The so-called Infumo modelling indicates on possible use in fields of identification, robust control and robust fault diagnosis. In the latter case, faulty activity can be called every time when system response crosses one of the Infumo boundary model responses. The application of Infumo model in detection of load change in motor-generator system with varying parameters is presented to demonstrate the benefits of proposed method in simplicity and possibility to use arbitrary excitation signals.

I. INTRODUCTION

Fuzzy modelling [15] has been widely used in recent years for building models of nonlinear processes. Fuzzy regression analysis is used to evaluate the relationship between dependent and independent variables of a fuzzy model. Usually, due to simplicity and use of linear programming, fuzzy linear regression [9] is applied. It influenced the investigation of uncertainties problem in recent years. In seeking means of describing a domain of functions that come as a result of uncertain system responses, one way to define a fuzzy model is to fix certain membership functions and establish the consequence parameters that vary in a certain interval. The main idea of our approach is to obtain intervals of parameters through fuzzy partial linear programming.

When process parameters vary in a certain tolerance band, it is important to define a confidence band over a finite set of input and output measurements in which the complete measurement set can be found. The interval fuzzy model (Infumo) identification is a methodology to obtain interval of fuzzy parameters by approximating nonlinear functions of a confidence band using l_∞ -norm approach. In the paper it is shown that using linear programming technique, we obtain upper and lower fuzzy model which define the confidence interval of the observed data. That way, the whole data set belongs to given band with certainty of 1. This is especially useful in fault detection, as its purpose is to identify crossing

of fault tolerant system response area.

Fault detection (FD) field has received much attention in the last two decades. The most powerful approaches in recent years appear to be observer-based methods [3], [2]. When considering systems with uncertain parameters, a lot of different methods evolved that deal with robustness and sensitivity to modelling errors and disturbances, for example LMI approach [8], [6], robust \mathcal{H}_∞ -filtering [7] and integral quadratic constraint (IQC) approach [4]. In all of the above mentioned methods, process model defines a normal system response and the uncertainties have to be included separately. Our approach, however, enables us to model a whole set of parameter varying functions that include all of the system responses, resulting from a given normal band of uncertainties.

The main objective of this work is to present a case, where using the proposed method we can develop a simple solution to detecting load change in a motor-generator plant. One of the major advantages of Infumo identification that has to be pointed out is that arbitrary excitation signals can be used. Optimization convergence problems that might arise either from too many parameters or from vast amount of data were solved by low-pass filtering [12] and implying a simple data reduction method, respectively.

The paper is organized in following way. In Section 2 the background of fuzzy modelling is given. In Section 3 the main idea of fuzzy model identification using l_∞ norm is described. Section 4 introduces the interval fuzzy model identification in sense of linear programming and Section 5 presents an application to load change detection in a motor-generator process.

II. NONLINEAR MODEL DESCRIBED IN FUZZY FORM

Typical fuzzy model in [15] is given in the form of rules

$$\begin{aligned} \mathbf{R}_j : & \text{if } x_{p1} \text{ is } \mathbf{A}_{1,k_1} \text{ and } \dots \text{ and } x_{pq} \text{ is } \mathbf{A}_{q,k_q} \\ & \text{then } y = \phi_j(\mathbf{x}), j = 1, \dots, m, \quad k_1 = 1, \dots, f_1, \quad (1) \\ & \quad k_2 = 1, \dots, f_2 \quad \dots \quad k_q = 1, \dots, f_q \end{aligned}$$

The q -element vector $\mathbf{x}_p^T = [x_{p1}, \dots, x_{pq}]$ denotes the input or variables in premise, and variable y is the output of the model. With each variable in premise x_{pi} ($i = 1, \dots, q$), f_i

fuzzy sets $(\mathbf{A}_{i,1}, \dots, \mathbf{A}_{i,f_i})$ are connected, and each fuzzy set \mathbf{A}_{i,k_i} ($k_i = 1, \dots, f_i$) is associated with a real-valued function $\mu_{A_{i,k_i}}(x_{pi}) : \mathbb{R} \rightarrow [0, 1]$, that produces membership grade of the variable x_{pi} with respect to the fuzzy set \mathbf{A}_{i,k_i} . To make the list of fuzzy rules complete, all possible variations of fuzzy sets are given in Eq. (1), yielding the number of fuzzy rules $m = f_1 \times f_2 \times \dots \times f_q$. The variables x_{pi} are not the only inputs of the fuzzy system. Implicitly, the n -element vector $\mathbf{x}^T = [x_1, \dots, x_n]$ also represents the input to the system. It is usually referred to as the consequence vector. The functions $\phi_j(\cdot)$ can be arbitrary smooth functions in general, although linear or affine functions are usually used.

The system in Eq. (1) can be described in closed form if the intersection of fuzzy sets is previously defined. The generalized form of the intersection is the so-called *triangular norm* (T-norm). In our case, the latter was chosen as algebraic product yielding the output of the fuzzy system

$$y = \frac{\sum_{k_1=1}^{f_1} \dots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \dots \mu_{A_{q,k_q}}(x_{pq}) \phi_j(\mathbf{x})}{\sum_{k_1=1}^{f_1} \dots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \dots \mu_{A_{q,k_q}}(x_{pq})} \quad (2)$$

It has to be noted that a slight abuse of notation is used in Eq. (2) since j is not explicitly defined as running index. From Eq. (1) is evident that each j corresponds to the specific variation of indexes k_i , $i = 1, \dots, q$.

To simplify Eq. (2), a partition of unity is considered where functions $\beta_j(\mathbf{x}_p)$ defined by

$$\beta_j(\mathbf{x}_p) = \frac{\mu_{A_{1,k_1}}(x_{p1}) \dots \mu_{A_{q,k_q}}(x_{pq})}{\sum_{k_1=1}^{f_1} \dots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \dots \mu_{A_{q,k_q}}(x_{pq})} \quad (3)$$

$j = 1, \dots, m$

give information about the fulfilment of the respective fuzzy rule in the normalized form. It is obvious that $\sum_{j=1}^m \beta_j(\mathbf{x}_p) = 1$ irrespective of \mathbf{x}_p as long as the denominator of $\beta_j(\mathbf{x}_p)$ is not equal to zero (that can be easily prevented by stretching the membership functions over the whole potential area of \mathbf{x}_p). Combining Eqs. (2) and (3) and changing summation over k_i by summation over j we arrive to the following equation:

$$y = \sum_{j=1}^m \beta_j(\mathbf{x}_p) \phi_j(\mathbf{x}) \quad (4)$$

From Eq. (4) it is evident that the output of a fuzzy system is a function of the premise vector \mathbf{x}_p (q -dimensional) and the consequence vector \mathbf{x} (n -dimensional). The dimension of the input space may be lower than $(q + n)$ since it is very usual to have the same variables present in vectors \mathbf{x}_p and \mathbf{x} . Vector \mathbf{z} (d -dimensional) comprises of the elements of \mathbf{x}_p and those of \mathbf{x} that are not present in \mathbf{x}_p .

Very often, the output value is defined as a linear combination of consequence states

$$\phi_j(\mathbf{x}) = \boldsymbol{\theta}_j^T \mathbf{x}, \quad j = 1, \dots, m, \quad \boldsymbol{\theta}_j^T = [\theta_{j1}, \dots, \theta_{jn}] \quad (5)$$

If the matrix of the coefficients for the whole set of rules is written as $\boldsymbol{\Theta}^T = [\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_m]$ and the vector of membership

values as $\boldsymbol{\beta}^T(\mathbf{x}_p) = [\beta^1(\mathbf{x}_p), \dots, \beta^m(\mathbf{x}_p)]$, then Eq. (4) can be rewritten in the matrix form

$$y = \boldsymbol{\beta}^T(\mathbf{x}_p) \boldsymbol{\Theta} \mathbf{x} \quad (6)$$

The fuzzy model in the form given in Eq. (6) is referred to as affine Takagi-Sugeno model and can be used to approximate any arbitrary function that maps the compact set $C \subset \mathbb{R}^d$ to \mathbb{R} with any desired degree of accuracy in [13], [16] and [17]. The generality can be proven by Stone-Weierstrass in [10] theorem which indicates that any continuous function can be approximated by fuzzy basis function expansion in [14].

III. FUZZY MODEL IDENTIFICATION USING l_∞ -NORM

In this section we discuss an approach to the model parameter estimation where the l_∞ -norm is used as the criterion for the measure of the modelling error. We assume a set of premise vectors $\mathbf{X}_p = \{\mathbf{x}_{p1}, \mathbf{x}_{p2}, \dots, \mathbf{x}_{pN}\}$ and a set of antecedent (or consequence) vectors $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$, from which a set $\mathbf{Z} = \{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_N\}$ can be constructed that represents the input measurement data, collected from the compact set $S \subset \mathbb{R}^d$. A set of corresponding outputs is also defined as $\mathbf{Y} = \{y_1, y_2, \dots, y_N\}$. The measurements satisfy the nonlinear equation of the system

$$y_i = g(\mathbf{z}_i), \quad i = 1, \dots, N \quad (7)$$

According to Stone-Weierstrass Theorem, for any given real continuous function g on a compact set $U \subset \mathbb{R}^d$ and arbitrary $\epsilon > 0$, there exist a fuzzy system f such that

$$\max_{\mathbf{z}_i \in \mathbf{Z}} |f(\mathbf{z}_i) - g(\mathbf{z}_i)| < \epsilon, \quad \forall i \quad (8)$$

This implies the approximation of any given real continuous function with fuzzy function from class \mathcal{F}^d defined in Eq. (6). However, it has to be pointed out that lower values of ϵ imply higher values of m (i.e. number of fuzzy subsets) that satisfy Eq. (8). In the case of approximation, the error between measured values and fuzzy function outputs can be defined as

$$e_i = y_i - f(\mathbf{z}_i), \quad i = 1, \dots, N \quad (9)$$

To estimate the optimal parameters of the proposed fuzzy function the minimization of the maximum modelling error

$$\max_{\mathbf{z}_i \in \mathbf{Z}} |y_i - f(\mathbf{z}_i)| \quad (10)$$

over the whole input set \mathbf{Z} is performed. This implies the *min-max* optimization method. In the case of the Takagi-Sugeno model in Eq. (6), the minimization of the expression in Eq. (10) can be performed in two steps. The first problem is how to minimize the error with respect to \mathbf{x}_p . The answer lies in the proper arrangement of membership functions. This is a well-known problem in fuzzy systems. It can be coped with cluster analysis in [1], [11] or other approaches. By having membership functions defined, the structure of the model is known and only parameters $\boldsymbol{\Theta}$ are to be defined by the *min-max* optimization

$$\boldsymbol{\Theta} = \arg \min_{\boldsymbol{\Theta}} \max_{\mathbf{z}_i \in \mathbf{Z}} |y_i - \boldsymbol{\beta}^T(\mathbf{x}_{pi}) \boldsymbol{\Theta} \mathbf{x}_i| \quad (11)$$

The idea of approximation can be interpreted as the most representative fuzzy function to describe the domain of outputs Y as a function of inputs Z . This problem can also be viewed as a problem of data reduction which often appears in identification problems with large data sets.

IV. INTERVAL FUZZY MODEL IDENTIFICATION

Let us consider a nonlinear function g which is a member of class $g \in \mathcal{G}$ and corresponding set of measured output values $Y = \{y_1, \dots, y_N\}$ over the set of inputs Z , i.e., $y_i = g(z_i)$, $g \in \mathcal{G}$, $z_i \in S, i = 1, \dots, N$.

The idea of robust interval fuzzy modelling is to find a lower fuzzy function \underline{f} and an upper fuzzy function \bar{f} satisfying

$$\underline{f}(z_i) \leq g(z_i) \leq \bar{f}(z_i), \quad \forall z_i \in S \quad (12)$$

In this sense, a function from class \mathcal{G} can always be found in the band defined by the upper and the lower fuzzy function. The main request in defining the band is that it is as narrow as possible according to the proposed constraints. Our approach using fuzzy function approximation can be viewed as a generalization of piecewise linear approach and gives a better approximation or at least much narrower approximation band.

The upper and the lower fuzzy functions, respectively, can be found by solving the following optimization problems:

$$\min_{\underline{f}} \max_{z_i \in Z} |y_i - \underline{f}(z_i)| \quad \text{subject to } y_i - \underline{f}(z_i) \geq 0, \quad \forall i \quad (13)$$

$$\min_{\bar{f}} \max_{z_i \in Z} |y_i - \bar{f}(z_i)| \quad \text{subject to } y_i - \bar{f}(z_i) \leq 0, \quad \forall i \quad (14)$$

The solutions to both problems can be found by linear programming, because both problems can be viewed as linear programming problems.

V. USING INTERVAL FUZZY MODEL IN DESIGNING FAULT DETECTION SYSTEM OF MOTOR-GENERATOR PILOT PLANT

In this section the application of INFUMO on a case of identification and fault detection (FD) of a laboratory plant will be presented.

Electromechanical process consists of 2 DC motors, mounted facing each other, as shown in Fig. 1. The driving shafts are rigidly coupled. The left motor, marked as "G", is the load of the motor "M" when operating in generator mode. Applying negative voltage value to generator produces mechanical torque and results in shifting of operating conditions. System output is a voltage obtained by a tacho generator, mounted to the shaft, that converts rotary speed to DC voltage output signal. u_m and u_g are input voltages for excitation and load, respectively. Signals are connected through AD/DA converter to a PC. Given plant setup enables us to control shaft speed by changing the motor input voltage. Process parameters are varying and are not well defined. Consecutive open-loop experiments with identical input signals resulted in a band of output responses. Fig. 2 depicts how the process responds

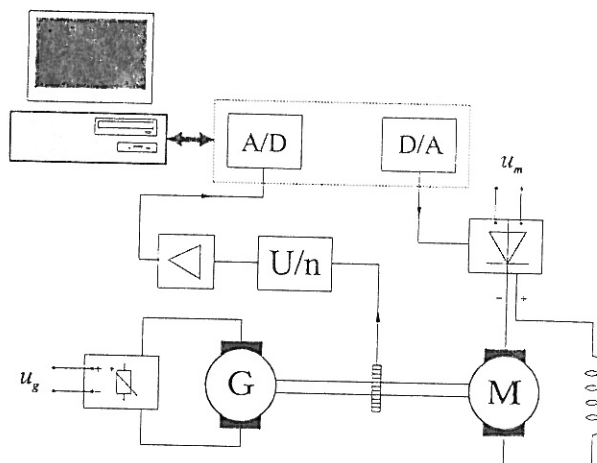


Fig. 1. Schematic representation of the motor-generator plant

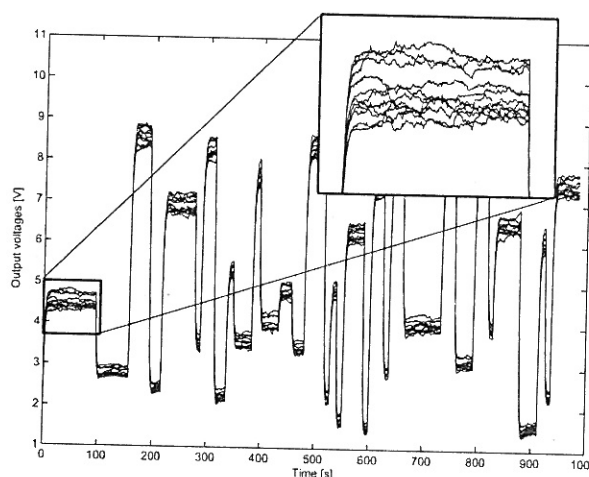


Fig. 2. Consecutive process responses to equal input signals

to equal excitation signal with minor changes in load. System performance depends noticeably on the operating temperature. Its effect on process parameters cannot be determined and is for that reason attributed to unmodelled plant dynamics. Therefore, it is necessary to repeat series of experiments several times to include as much information of working conditions as possible to measurements set.

Boundary load values were defined as 0 and -0.05 V. To test system responses over the complete operating range, a set of 30 experiments was carried out - 5 series of 6 identification signals at load voltage values from the lowest to the highest value in 0.01 V steps. Inputs and belonging output signals are depicted in Fig. 3 and Fig. 4. For the sake of space, only the first, second, and the last set of data is presented. One of the major benefits of interval fuzzy model identification is that input signals can be arbitrary. Normally, to get a confidence band of measurements, it would be expected that experiments are made with identical excitation signals. Our idea, however, was to create a model based on data acquired from experiments on unequal signals. Section 3 deals with optimal tuning of

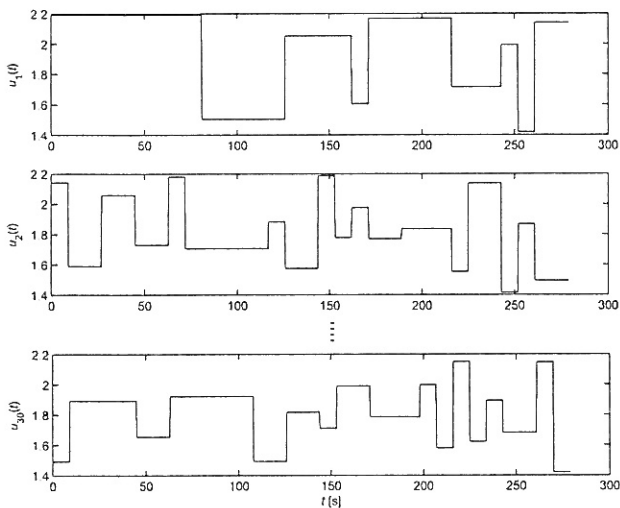


Fig. 3. Inputs: first, second and last experiment

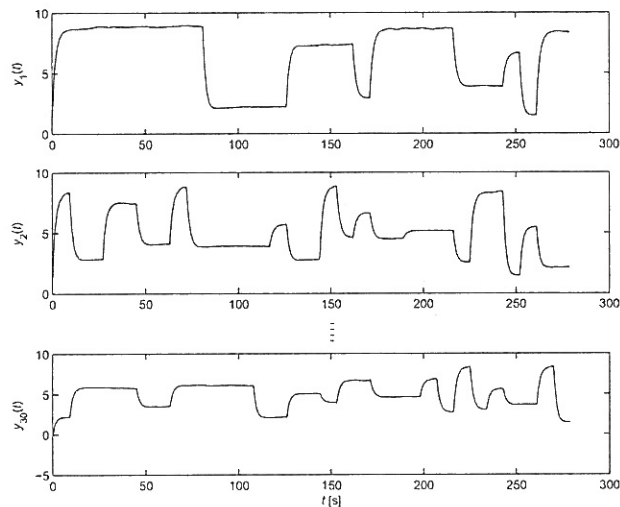


Fig. 4. Responses: first, second and last measurement

Infumo parameters. Two problems arise when seeking optimal function approximation: how to make a representative domain of input-output data and how to reduce the quantity of data needed for identification. To improve the approximation with fuzzy models, the number of parameters has to be increased. Coupled with large number of data, it will result in serious optimization convergence problems. One way to overcome the above mentioned is to subject input and output signals to low-pass filtering [12]. In Fig. 5 a schematic representation of altered FD system is given, showing that measurements are filtered by a low-pass filter (LPF) with transfer function in Eq. (15) and time constant $T_f = 30s$.

$$G_f = \frac{1}{T_f s + 1} \quad (15)$$

As seen from Fig. 6, we obtain a compact set of measure-

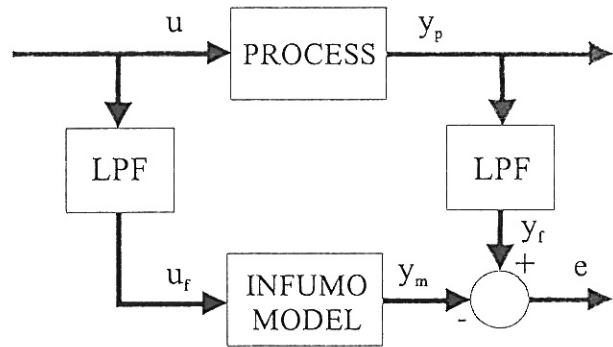


Fig. 5. Fault detection system using static Infumo model

ments that represents system behavior in steady-state. It can be seen as load-dependent input-output mapping area. Total

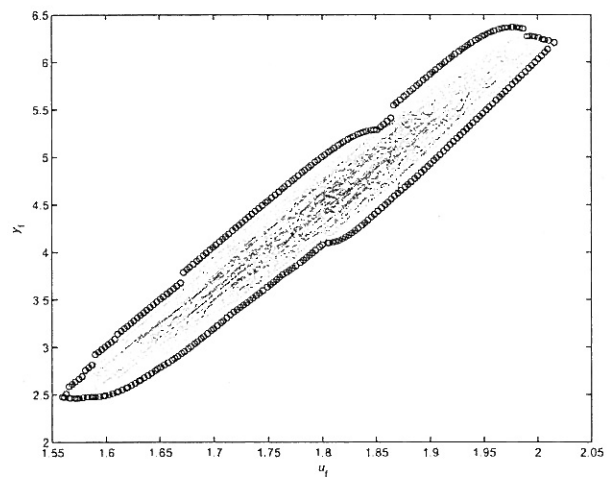


Fig. 6. Set of filtered measurements with boundary points

number of points, gathered from experiments, was 83700. Data reduction is performed by determining the boundary points. They are used as upper and lower bound measurements y_i in Eq. (13) and Eq. (14). First, range of input measurements is divided into equidistant subspaces. In each subspace minimal and maximal value is determined. Resulting set of 302 points is emphasized by circles in Fig. 6 and it represents the training data set for Infumo model identification.

Membership functions of antecedent variables were arranged using Gustafson-Kessel clustering. Because of static input-output data mapping, static modelling was used. Number of fuzzy subsets for upper and lower function was 6 and 5, respectively. Parameters were optimized using fuzzy linear programming, yielding piece-wise linear functions of static tolerance band boundaries, i.e. boundaries of filtered input and output measurement set. Resulting functions can be seen from Fig. 7. It is evident that *min-max* optimization gave satisfactory results in approximating given area. To realize fault detection system we simply connect Infumo model to process model in parallel, filter the signals and compare the filtered outputs; schematic representation is given in Fig. 5.

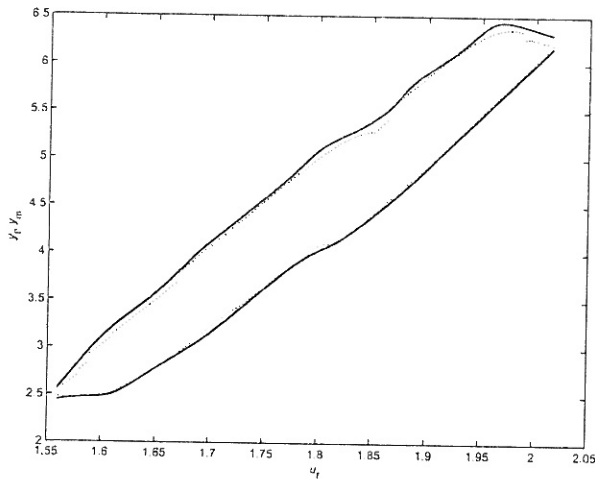


Fig. 7. Infumo functions embracing boundary points

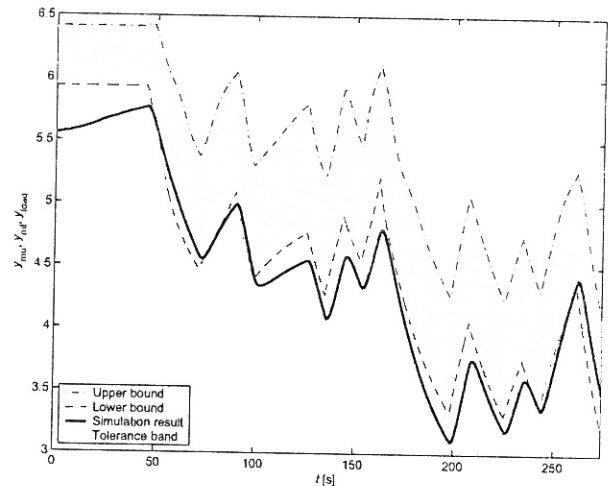


Fig. 9. Results of simulation with variable load

Residual $e(t)$ is then a binary time dependent signal with values 0 and 1 meaning process output being in or out of permitted fault tolerant band, respectively. Upper and lower boundary fuzzy functions confine the permissible area reliably due to the fact that each measurement is located inside the tolerance band with a certainty of 1. Results of simulation with variable load can be seen from Fig. 8. Load signal in test

of filtered process output and the tolerance band. It can be seen that during the transient filtered output was shortly crossing the boundary area. Considering the dependence of system performance on operating temperature, it can be concluded that in those two cases fault prediction was not certain due to the effect of plant unmodelled dynamics. One way of reducing that uncertainty would be to find optimal structure and parameters of the applied filter but that was not considered in this work. The initial violation can be attributed to the initializing period of the filter and is marked with "INIT" in Fig. 8.

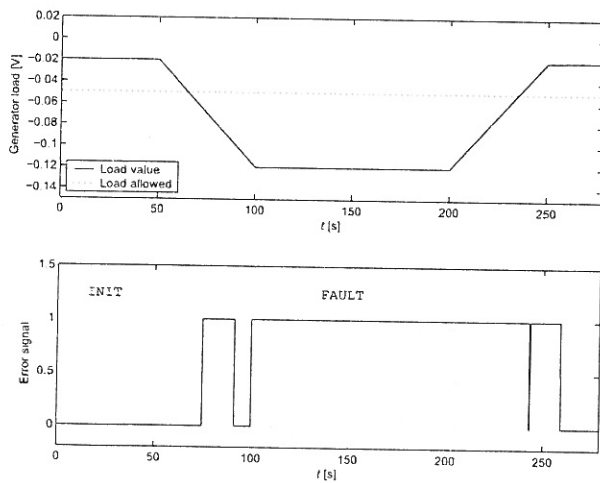


Fig. 8. Results of fault detection system

Fig. 10 shows the actual view of fault detection from the Infumo model perspective. The variable load experiment is

simulation was defined as a combination of ramp signals and is depicted in upper plot of Fig. 8. It is evident that the proposed FD system sets its output to 1 when load is not to be found in the permitted area, marked as "FAULT" in shaded area of lower plot of Fig. 8. Starting time of error signal is not equal to the time when load first crosses lower bound (-0.05 V). The delay depends on the time constant of the proposed low-pass filter - as it takes some time for the filter to reach steady state it is natural that the response of the FD system is not instant. The output of FD successfully tracks "illegal" changes of load, except in two occasions. They will be discussed with reference to Fig. 9 which displays the time-dependent courses

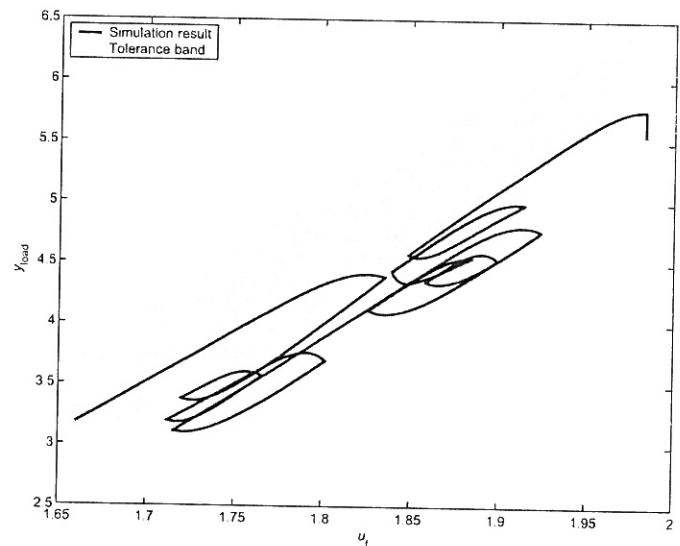


Fig. 10. Results of fault detection system in static conditions

presented as a "static" filtered input-output mapping. During the operation, it is compared with Infumo confidence band and thus giving the information about possible faulty actions. It is evident from Fig. 10 that when load value is below negative limit value, the filtered response abandons the tolerance band.

In addition, the above mentioned two cases of false error estimation can be clearly noted.

VI. CONCLUSION

A novel approach of fuzzy modelling has been presented. Internal fuzzy model (Infumo) was derived using l_∞ -norm function approximation. It was shown that Infumo enables confining an arbitrary confidence band with upper and lower function fuzzy function. It is therefore suitable for identification of systems with uncertain parameters because all the system responses in the given interval of uncertainty can be found in confidence band with certainty of 1. In fault detection, the benefit is to be able to directly model a family of parameter-varying systems which guarantees fault tolerant action. Application to a case of load change detection of motor-generator pilot plant was presented. To get a confidence band of system responses, a large number of experiments were carried out which resulted in immense set of data. To solve the data-reduction problem, input and output signals were filtered by a low-pass filter. That way, major benefits proved to be the possibility of using arbitrary input signals and simplicity of fuzzy static model that was used. Boundary points of gathered data set were determined using a simple algorithm and used as a training data set for identification by linear programming. Connecting the Infumo to process in parallel, it was proven to be successful in detecting unwanted load changes. Future work will concentrate on optimization of filter parameters, investigating the performance due to different choice of filter structure and use of clustering algorithms in assessing Infumo parameters. Current results show great potential of use of presented method in the area of identification, robust control and robust fault detection.

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