Internal Model Representation for Generalized Predictive Control with Constraint Handling

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Abstract – **In the paper constraint handling methods for Generalized Predictive Control are analyzed, based on its polynomial equivalent structure. This, known as RST structure in the literature, has been further modified to obtain a two-degree-of-freedom Internal Model Control structure. From an applicative point of view, three control structures for anti-windup measures have been analyzed, for two benchmark type plants. The best behavior is obtained if the saturation is fed back within the control structure. Simulations have been performed and their results presented in tables and diagrams.**

I. INTRODUCTION

Constraints in Generalized Predictive Control (GPC) can lead to serious problems if they are not treated properly. As far as constraints of the control signal are concerned, they can be incorporated into the cost function, and also dealt with separately. It is well known that every GPC algorithm can be transformed into a polynomial control structure [1] (known as the RST structure). This equivalent structure is transformed into a two-degree-of-freedom Internal Model Control (IMC) structure, which, for stable cases, is equivalent to the Youla parameterization. The IMC structure for GPC is given here and different types of constraint handling are presented, taken into account the GPC parameters, exemplified on benchmark systems.

II. RST AND IMC STRUCTURES OF GPC

A. Derivation of RST and IMC Structures

As presented in [1], the GPC can be easily converted into a polynomial structure, in case if there are no constraints. When defining the GPC algorithm, the SISO plants that are linear or linearized can be described by the following equation (the CARIMA model):

$$A(q^{-1})y(t) = z^{-d}B(q^{-1})u(t-1) + C(q^{-1})\frac{e(t)}{\Delta} \qquad (1)$$

where $u(t)$ is the control sequence and $y(t)$ the output sequence, $e(t)$ is a zero mean white noise, d is the mathematical dead time (physical + 1). Polynomials A, B and C are described in the backward shift operator q^{-1}:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \qquad (2)$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{nb} q^{-nb} \qquad (3)$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \qquad (4)$$

and

$$\Delta = 1 - q^{-1} \qquad (5)$$

The C polynomial is chosen for 1, for simplicity [1]. Defining the cost function that is minimized in order to obtain the control sequence to be applied in the GPC algorithm as follows:

$$J = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j\,|\,t) - r(t+j)]^2 + \sum_{j=1}^{N_u} \lambda_u(j)[\Delta u(t+j-1)]^2 \qquad (6)$$

where N_1 and N_2 are the limits of the prediction horizon, N_u is the control horizon, $\hat{y}(t+j\,|\,t)$ is the j-step ahead prediction of the output, $r(t+j)$ is the future reference trajectory and $\delta(j)$ and $\lambda(j)$ are weighting sequences. By minimizing the cost function, an optimal value for the future control sequence is obtained. If only the first element of the control signal sequence is sent to the process (receding horizon strategy), after the minimization the control law is obtained:

$$\Delta u(t) = \mathbf{K}\,(\mathbf{r} - \mathbf{f}) = \sum_{i=N_1}^{N_2} k_i \left[r(t+i) - f(t+i)\right] \qquad (7)$$

where $\mathbf{K}$ is the first row of the matrix $(\mathbf{G}^T\, \mathbf{G} + \lambda \mathbf{I})^{-1}\mathbf{G}^T$, $\mathbf{f}$ is the free response, $\mathbf{r}$ is the reference signal, see [1].

If the GPC is unconstrained, then a polynomial structure can be obtained (see Fig. 1.), which can be posed in the classical pole-placement structure. **P** stands for the plant transfer function including the dead-time.

The control signal results in form of:

$$R(q^{-1})\Delta u(t) = T(q^{-1})r(t) - S(q^{-1})y(t) \qquad (8)$$

where R, S, T are polynomials in the backward shift operator. If the plant model is given by:

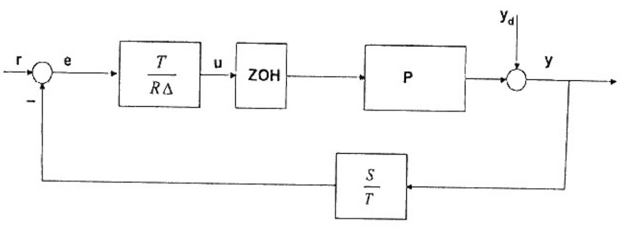

Fig. 1. RST polynomial control structure

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t-1) + T(q^{-1})\frac{e(t)}{\Delta} \quad (9)$$

to determine the controller, a Diophantine equation has to be solved:

$$T(q^{-1}) = E_j(q^{-1})\Delta A(q^{-1}) + q^{-j}F_j(q^{-1}) \quad (10)$$

where j is the length of the prediction horizon.

The T polynomial can be treated as a filter. It plays a role in disturbance rejection, and also it is mentioned [1] that it can influence robust stability. Solving the Diophantine equation and choosing $T(q^{-1})=1$ for simplicity, the final expressions of the R, S and T polynomials will be [1]:

$$R(q^{-1}) = \frac{T(q^{-1}) + q^{-1}\sum_{i=N_1}^{N_2} k_i I_i}{\sum_{i=N_1}^{N_2} k_i} \quad (11)$$

$$S(q^{-1}) = \frac{\sum_{i=N_1}^{N_2} k_i F_i}{\sum_{i=N_1}^{N_2} k_i} \quad , \quad T(q^{-1}) = 1 \quad (12),(13)$$

where I_j are the rows of vector $\mathbf{G}^T$.

The RST polynomial structure can be transformed into a two-degree-of-freedom Internal Model Control (2DOF-IMC) structure, as seen in Fig. 2. Deducing step by step the equivalence between the two structures, the following results are obtained:

$$C(q^{-1}) = \frac{S(q^{-1})A(q^{-1})}{F_W(q^{-1})(R(q^{-1})\Delta A(q^{-1}) + S(q^{-1})B(q^{-1})q^{-d})} \quad (14)$$

$$\frac{T(q^{-1})}{S(q^{-1})} = \frac{F_r(q^{-1})}{F_W(q^{-1})} \quad (15)$$

In equation (15) the two members on both sides must be proportional. In the examples presented, the two have been taken for equal, where F_r insures servo performance and F_w disturbance rejection.

As it is well-known, the IMC structure, in this form (Fig. 2), is valid for stable plants only. In case of unstable plants the Youla parameterization is used.

The IMC structure has many advantages over conventional control. For example, if there is no noise and no mismatch between the plant and its model, then open loop control is obtained [2],[3], and the controller can be designed simply as a realizable quasi-inverse of the model of the plant. The closed loop control structure will work against mismatch and noise rejection.

B. Constraint Handling in Case of RST and IMC Structures

In practice, when there is a control system, they are always subject to different kinds of constraints. In what follows, constraints of the control signal u(t) are dealt with. Starting from the RST structure, one way to handle constraints is presented in Fig. 3. It is not a very fortunate choice since in this case the control signal is simply cutup, which may lead to integrator windup which can produce unwanted behaviors of the system, even lead to instability.

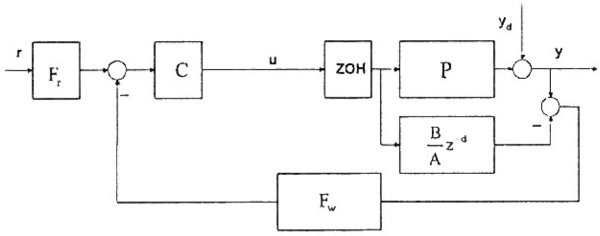

Fig. 2. The 2DOF- IMC structure

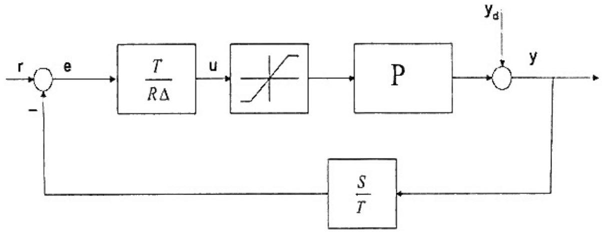

Fig. 3. Constrained RST structure

Constraints of the control signal can be incorporated into the IMC structure, for example see [4]. In Fig. 4 the control system is presented. The advantage of this structure is that the same control signal is applied both to the plant and to the internal model; C itself does not contain integrating effect, which is introduced through the IMC feedback.

In addition, the anti-windup property of this structure can be improved by realizing the IMC controller in a feedback of the saturation [5] (Fig. 5). Its advantage is that it takes into account also the dynamics of the controller.

The limited input of the plant provides the input for the controller in the feedback.

For a given controller C, the C_{Lim} controller that feeds back the saturating element can be calculated according to the following relation:

$$C_{Lim}(q^{-1}) = \frac{C(q^{-1}) - 1}{C(q^{-1})} \quad (16)$$

Special attention must be paid at the implementation of this control structure. An algebraic loop will appear in this case, so measures avoiding it have to be dealt with. One way to handle this is to separate the constant component of the partial fractional representation of C_{Lim} and to incorporate it in the slope of the saturation.

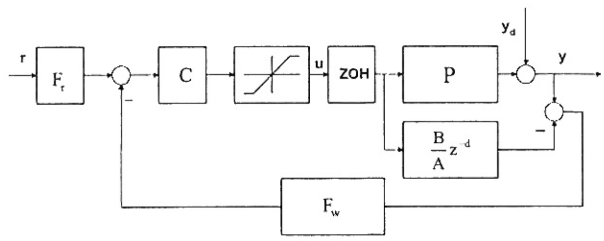

Fig. 4. IMC structure with limitation inside the mode

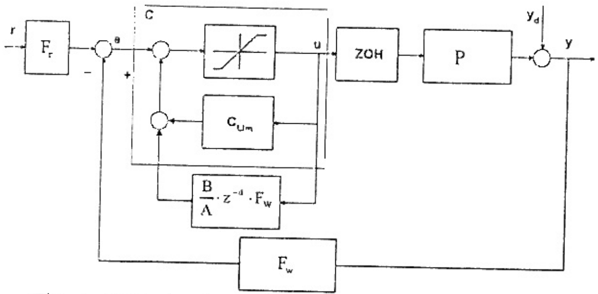

Fig. 5. IMC structure with controller in the feedback of the saturation

III. EXAMPLES

For exemplification, two plant models are taken (a first and a second order one, both without dead-time), and the polynomial structures are derived for different GPC parameters. Moreover, the effects of saturation are presented according to the three schemes (those from Fig. 3, Fig. 4 and Fig. 5).

A. Case of a First Order Plant

Let the plant transfer function be:

$$P(s) = \frac{1}{1+s} \tag{17}$$

Choosing a sampling time of $h=0.1$ sec, the pulse transfer function assuming zero order hold is computed:

$$P(z) = \frac{0.09561}{z-0.9048} \tag{18}$$

Let in the first case the parameters of the predictive controller be:

$$N_1 = 1 \qquad N_2 = 3$$
$$N_u = 1 \qquad \lambda_u = 0 \tag{19}$$

The resulted RST parameters will be:

$$R = 0.2037$$
$$S = 2.9366 - 1.9366\,z^{-1} \tag{20}$$
$$T = 1$$

The parameters of the 2DOF-IMC structure will result as:

$$F_r = 1$$
$$F_W = 2.9366 - 1.9366\,z^{-1} \tag{21}$$
$$C = \frac{4.9098 - 4.443\,z^{-1}}{1 - 0.5328\,z^{-1}}$$

The zeros and poles of the controller are (in z-domain):

$$C(z) = 4.9098\frac{z-0.9048}{z-0.5328} \tag{22}$$

Accordingly, the feedback controller C_{Lm} has the parameters:

$$C_{Lm}(z) = \frac{0.7963z - 0.7963}{z-0.9048} \tag{23}$$

It is also interesting to see how the open loop transfer function looks like:

$$L(z) = C(z)P(z) = \frac{0.46723}{z-5328} \tag{24}$$

Simulation results, when no constraints are given, are presented in Fig. 6. The reference signal is a step, and a step disturbance of amplitude -1 acts on the input of the plant at time point 10 sec.

Suppose there are constraints imposed on the control signal, in such a way that it is limited to $u_{max}=2.5$ and $u_{min}=-2.5$. The three above mentioned ways of constraint handling are presented: simply cut the signal (Fig. 3), IMC inner saturation (Fig. 4) and saturation in feedback (Fig. 5). The simulation results are depicted in Fig. 7. It can be noticed that the output signal in case of feedback saturation proves to have the closest behavior to the unconstrained case. As far as the control signal is concerned, also the same results can be stated.

In case of GPC, the tuning parameters can be modified in order to obtain better performance, from the required point of view. Some further simulations have been performed, and the results are presented. The parameters of the controllers that are obtained are summarized in Table I. It can be noticed that the controller $C(z)$ is always a form of a quasi inverse of the plant's transfer function, canceling the pole of the plant with a zero. Its poles are differing as the predictive parameters are modified.

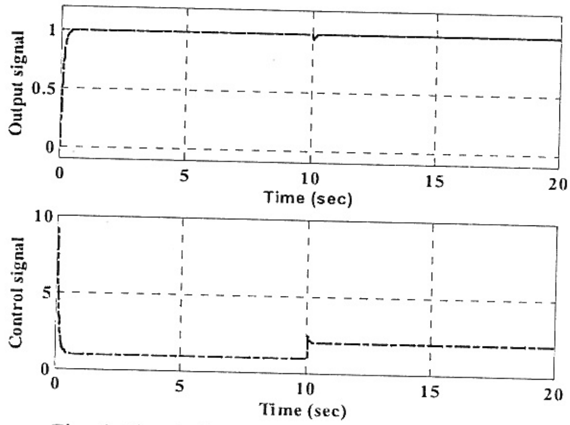

Fig. 6. Simulation results for no constraints

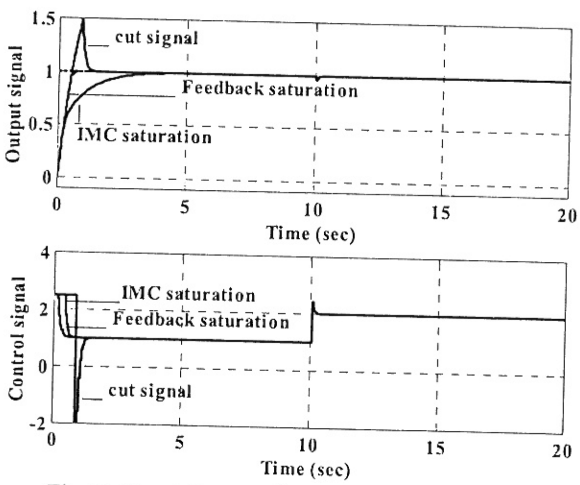

Fig. 7. Simulation results, constrained case

Table I

No	N_1	N_2	N_u	λ_u	R	S	T	C(z)	L(z)=C(z)H(z)
1	1	3	1	0	0.2037	$2.936\text{-}1.936z^{-1}$	1	$4.9098\frac{z-0.9048}{z-0.5328}$	$\frac{0.46723}{z-5328}$
2	1	3	1	0.01	0.3691	$2.62\text{-}1.62z^{-1}$	1	$\frac{2.7093\ z\,(z-0.9048)}{(z-0.4843)\ (z-0.0476)}$	$\frac{0.4911\,z}{z^2-0.5319\,z+0.02306}$
3	1	3	1	0.1	0.4626	$2.62\text{-}1.62z^{-1}$	1	$\frac{2.1615\,z\,(z-0.9048)}{z^2-0.7921z+0.1839}$	$\frac{0.3918\,z}{z^2-0.7921\,z+0.1839}$
4	1	5	1	0.1	0.5753	$3.4401\text{-}2.441^{-1}$	1	$\frac{1.7379\ z\,(z-0.9048)}{(z-0.6591)\,(z-0.07592)}$	$\frac{0.315\,z}{z^2-0.735\,z+0.05004}$
5	1	10	1	0.1	0.7268	$4.2181\text{-}3.2181z^{-1}$	1	$\frac{1.3759\,z\,(z-0.9048)}{(z-0.7451)\,(z-0.02165)}$	$\frac{0.2494\,z}{z^2-0.7667\,z+0.01613}$
6	1	10	3	0.1	0.4844	$2.3883\ \text{-}1.3883z^{-1}$	1	$\frac{2.0646\,z(z-0.9048)}{z^2-0.9249z+0.2992}$	$\frac{0.3742\,z}{z^2-0.9249\,z+0.2992}$
7	1	10	7	0.1	0.5134	$2.4396\text{-}1.4396z^{-1}$	1	$\frac{1.9477\ z\,(z-0.9048)}{z^2-0.9574z+0.3105}$	$\frac{0.3531\,z}{z^2-0.9574\,z+0.3105}$

A. Case of a Second Order Plant

For the second order plant example an oscillating one has been chosen. Let its transfer function be:

$$P(s)=\frac{1}{2s^2+s+1} \tag{25}$$

Choosing a sampling time of $h=0.4$ sec, the pulse transfer function is computed:

$$P(z)=\frac{0.03722z+0.03841}{z^2-1.747z+0.8187} \tag{26}$$

Let the parameters of the predictive controller be:

$$N_1=1 \qquad N_2=3 \qquad N_u=1 \qquad \lambda_u=0 \tag{27}$$

The resulted RST parameters will be:

$$\begin{aligned} R &= 0.2176+0.1386\,z^{-1} \\ S &= 6.3202-8.5801\,z^{-1}+3.2698\,z^{-2} \\ T &= 1 \end{aligned} \tag{28}$$

The parameters of the 2DOF-IMC structure will result as:

$$\begin{aligned} &F_r=1 \qquad F_W=6.3202-8.5801\,z^{-1}+3.2698\,z^{-2} \\ &C=\frac{(4.5942-8.025z^{-1}+3.761z^{-2})}{(1-1.029z^{-1}+0.3603z^{-2})} \end{aligned} \tag{29}$$

The zeros and poles of the controller are (in z-domain):

$$C(z)=4.5942\frac{(z^2-1.747z+0.8187)}{(z^2-1.029z+0.3603)} \tag{30}$$

The controller has a pair of complex conjugate poles and zeros. The zeros cancel the poles of the plant. Accordingly, the feedback controller C_{Lim} has the parameters:

$$C_{Lim}(z)=\frac{0.7823\,z^2-1.523\,z+0.7403}{z^2-1.747\,z+0.8187} \tag{31}$$

The open loop transfer function looks like:

$$L(z)=C(z)P(z)=0.11098\frac{z+0.9354}{z^2-1.029z+0.3603} \tag{32}$$

Simulation results, when no constraints are given, are presented in Fig. 8. The reference signal is a step, and a step disturbance of amplitude -1 acts on the input of the plant at time point 30 sec.

Suppose there are constraints imposed on the control signal, in such a way that it is limited to $u_{max}=2$ and $u_{min}=-2$. The three above mentioned ways of constraint handling are presented: - simply cut the signal (Fig. 3); - IMC inner saturation (Fig. 4); - saturation in feedback (Fig.5).

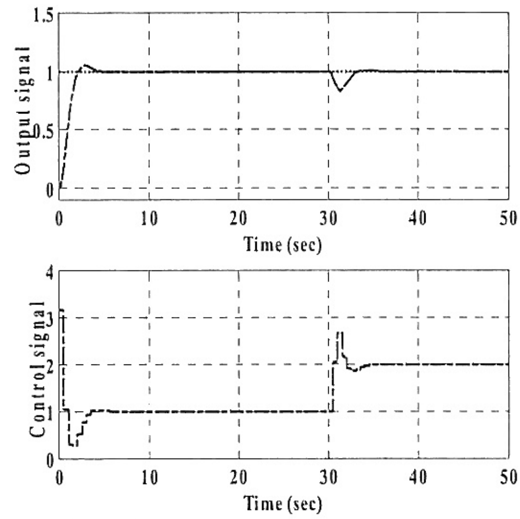

Fig. 8. Simulation results for no constraints, Second order plant

The simulation results are depicted in Fig. 9. It can be noticed that the output signal in case of feedback saturation proves to have the closest behavior to the unconstrained case. As far as the control signal is concerned, also the same results can be stated.

Also in this example of a second order oscillating process, the effect of changes of the GPC tuning parameters are studied. These six obtained controller parameters are summarized in Table II. Also in this case it can be noticed that the controller *C(z)* is always a form of a quasi inverse of the plant's transfer function, canceling the poles of the plant. The introduced dynamics differs as the predictive parameters are modified.

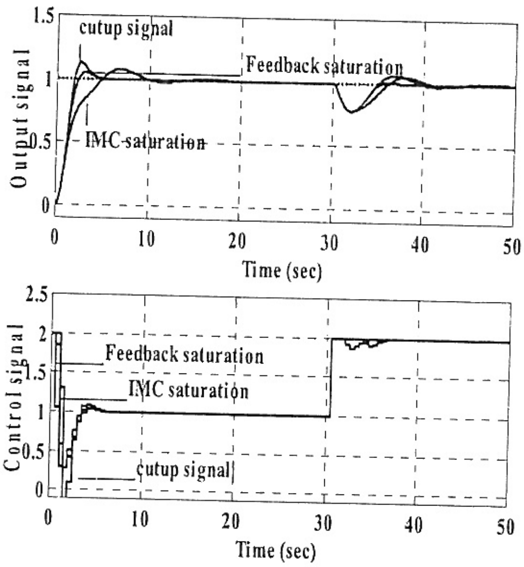

Fig. 9. Simulation results, constrained case, second order plant

III. CONCLUSIONS

In the paper the IMC equivalent of GPC structure has been derived. From an applicative point of view the problem of constraint handling of the control signal with different anti-windup measures with for quick leaving of saturation zone is also dealt with for the GPC and IMC structures. There are analyzed three control structures for two benchmark type plants which can be frequently found in applications of process control.

Table II.

N_1	N_2	N_u	λ_u	R	S	T	$C_{zpk}(z)$	L(z) =C(z) H(z)
1	3	1	0	$0.2176+0.1386z^{-1}$	$6.3202 - 8.5801z^{-1} + 3.2598z^{-2}$	1	$\frac{4.5942(z^2-1.747z+0.8187)}{(z^2-1.029z+0.3603)}$	$\frac{0.17098(z+0.9354)}{(z^2-1.029z+0.3603)}$
1	3	1	.01	$0.3299+0.1966z^{-1}$	$5.754 - 7.674z^{-1} + 2.9195z^{-2}$	1	$\frac{3.031z(z^2-1.747z+0.8187)}{(z-0.1019)(z^2-0.9772z+0.3466)}$	$\frac{0.17281z(z+0.9198)}{(z-0.1019)(z^2-0.9772z+0.3466)}$
1	3	1	0.1	$0.4646+0.1965z^{-1}$	$9.2298 - 14.2173z^{-1} + 5.9875z^{-2}$	1	$\frac{2.1524z(z^2-1.747z+0.8187)}{(z-0.4899)(z^2-1.05z+0.512)}$	$\frac{0.12272z(z+0.9198)}{(z-0.4899)(z^2-1.05z+0.512)}$
1	5	1	0.1	$0.7799+0.4031z^{-1}$	$5.7545 - 7.673z^{-1} + 2.9195z^{-2}$	1	$\frac{1.2822z(z^2-1.747z+0.8187)}{(z-0.05523)(z^2-1.423z+0.571)}$	$\frac{0.073101z(z+0.9198)}{(z-0.05523)(z^2-1.423z+0.571)}$
1	20	3	0.1	$0.6346+0.2941z^{-1}$	$8.0064-11.3746z^{-1} + 4.3682z^{-2}$	1	$\frac{1.5757z(z^2-1.747z+0.8187)}{(z-0.6026)(z^2-0.884z+0.318)}$	$\frac{0.089836z(z+0.9198)}{(z-0.6026)(z^2-0.884z+0.318)}$
1	20	10	0.1	$0.5577+0.2294z^{-1}$	$6.3827 - 8.7899z^{-1} + 3.4072z^{-2}$	1	$\frac{1.7929z(z^2-1.747z+0.8187)}{(z-0.5024)(z^2-1.103z+0.4976)}$	$\frac{0.10222z(z+0.9198)}{(z-0.5024)(z^2-1.103z+0.4976)}$

From the presented anti-windup measures the most efficient proves to be the one that feeds back the saturation. In all cases, the control systems have been tested through simulation.

IV. ACKNOWLEDGMENT

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