A NEURO-FUZZY APPROACH TO ENHANCE FEEDBACK CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract: A neuro-fuzzy approach to enhance feedback control is proposed in this paper. In the proposed design, the nonlinear process is first approximated by a composite model consisting of a linear ARX model and a neuro-fuzzy model, which captures the linearization error between the process output and predicted output from ARX model. The output of this neuro-fuzzy model is viewed as the measured disturbance and another neuro-fuzzy controller is designed to eliminate this disturbance influence. Simulation results are given to illustrate the effectiveness of the proposed control scheme. *Copyright©2004 IFAC*

Keywords: nonlinear process control, neuro-fuzzy system, feedforward control, internal model control

1. INTRODUCTION

PID controller are widely used in chemical processes because of their simplicity in architecture and being best understood by process operators. However, demands for tighter environment regulation, better energy utilization, higher product quality and more production flexibility have made process operations more complex within a larger operating range. Consequently, conventional linear control technologies cannot give satisfactory control performance. Many advanced control schemes are thus developed to efficiently control nonlinear chemical processes based on their mathematical model. However, the first-principle models are often unavailable due to the lack of complete physicochemical knowledge of chemical processes. An alternative approach is to use neural network and fuzzy system to build model from process data measured in industrial processes, due to their inherent ability to approximate an arbitrary nonlinear function (Narendra and Parthasarathy, 1990), (Sugeno and Tanaka, 1991), (Wang and Langari, 1996). Several neural network or fuzzy system based nonlinear controller design algorithms have been proposed, for example the work done by (Bhat and McAvoy, 1990), (Nahas *et al.*, 1992), (Loh *et al.*, 1995) and (Xie and Rad, 2000). Among these results, Internal Model Control (IMC) structure is a typical application of neural network to nonlinear control. When the neural network model for the process dynamics and its inverse dynamics are given, the realization of IMC using neural network is straightforward. However, the application of the neural network and fuzzy system in model-based controller design has some fundamental limitations: (1) the non-transparent and black-box nature of the techniques makes it difficult to interpret the final structure meaningfully and to express the control law explicitly; (2) it is very difficult to analyze the closed-loop properties like stability and robustness.

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Kim *et al.* (1997) presented a neural linearizing control scheme by using a radial basis function to linearize the relation between the output of a linear controller and the process output. However, a proper selection of the predefined linear reference model is crucial to the successful application of this method (Gao *et al.*, 2000). Gao *et al.* (2000) proposed an analytical predictive control law, in which a linear model is used to capture the linear dynamics and a multilayered feedforward neural network is employed to predict the linearization errors. However, the resulting predictive control law requires the use of nonlinear programming and some assumptions are necessitated in order to simplify the control law.

Motivated by the previous work, a neuro-fuzzy approach to enhance feedback control is proposed in this paper for improved control of a class of nonlinear processes with modest nonlinearities, namely linear dynamics play a dominant role in the operating range concerned but the associated linearization errors is still significant enough to degrade the feedback control performance. The proposed design consists of two integral components: (1) a neuro-fuzzy based model built to capture the linearization error between the process output and the predicted output from the linear model; (2) a neuro-fuzzy based controller that mimics "feedforward" controller by treating the output from the neuro-fuzzy model as a measured disturbance and in this case aims to remove the nonlinearity from the process dynamics, leading to an improved control performance achieved by feedback controller.

This paper is organized as follows. Section 2 introduces the neuro-fuzzy system used in this paper. The proposed control scheme and its design procedure are described in section 3. Simulation results are presented in Section 4 to illustrate the effectiveness of the proposed control scheme. Finally, concluding remarks are given in Section 5.

2. NEURO-FUZZY SYSTEM

Neuro-fuzzy system (NFS) is a neural network-based fuzzy logic control and decision system, and is suitable for online nonlinear systems identification and control (Lin and Lee, 1991). The NFS is a multilayer feedforward network that integrates the TSK-type fuzzy logic system and RBF neural network into a connectionist structure. The TSK-type fuzzy rule is formulated as:

$$R^l : IF\ x_1\ is\ F_1^l\ AND\ x_2\ is\ F_2^l \cdots x_M\ is\ F_M^l \quad THEN\ y^l = \mathbf{h}_l \bar{\mathbf{x}}^T, \qquad (1)$$

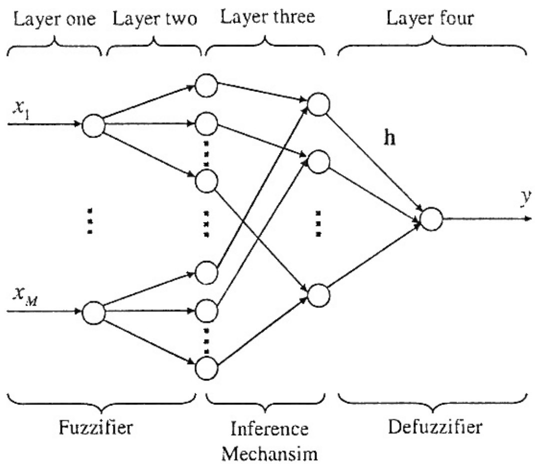

Fig. 1. The structure of NFS

where $R^l(l = 1, 2, \cdots, N)$ denotes the lth fuzzy rule, N is the total number of fuzzy IF-THEN rules, $\bar{\mathbf{x}} = [x_1, x_2, \ldots, x_M, 1]^T$ is the input variables of the system, y^l is the output variable of the lth fuzzy rule. $F_i^l(i = 1, 2, \cdots, M)$ are fuzzy sets defined on the corresponding universe and $\mathbf{h}_l = [h_{l1}, h_{l2}, \ldots, h_{lM}, h_{l(M+1)}]^T$ is the lth consequence of fuzzy rule.

The NFS consists of four layers as shown in figure 1:

Layer 1: This layer is the input layer, whose nodes just transmit the input variables $\mathbf{x}$ to the next layer directly.

Layer 2: This layer is the membership function layer that receives the signals from the input layer and calculates the membership of the input variables. The membership function chosen in this paper is the Gaussian membership function as described by:

$$\mu_{li} = \exp\left(-\frac{(x_i - c_{li})^2}{\sigma_{li}^2}\right) \quad i = 1, 2, \cdots, M; l = 1, 2, \cdots, N \qquad (2)$$

Layer 3: This layer is the rule layer. The number of the nodes in this layer represents the number of fuzzy rules. It computes the fired strength of a rule as following

$$\varphi_l = \prod_{i=1}^{M} \mu_{li} \qquad (3)$$

Layer 4: This layer is the output layer. All consequence weights are fully connected to the output node in which defuzzification of the TSK-type NFS is performed.

The output of the whole NFS is then given by:

$$y = \sum_{l=1}^{N} \Phi_l \mathbf{h}_l \bar{\mathbf{x}}^T \qquad (4)$$

where

$$\Phi_l = \frac{\exp\left(-\sum_{i=1}^{M} \frac{(x_i - c_{li})^2}{\sigma_{li}^2}\right)}{\sum_{l=1}^{N} \exp\left(-\sum_{i=1}^{M} \frac{(x_i - c_{li})^2}{\sigma_{li}^2}\right)} \quad (5)$$
$$(l = 1, 2, \cdots, N)$$

3. PROPOSED NEURO-FUZZY CONTROL SCHEME

In this section, the design procedure of the proposed neuro-fuzzy control scheme will be described in details. In what follows, however, the motivation of the proposed design is discussed first. It is assumed that the nonlinear systems under consideration can be approximated by a composite model consisting of a linear ARX model and a NFS-based model (NFSM), which captures the linearization error between the process output and the predicted output from ARX model. NFS is employed in this paper due to its capability of uniformly approximating any nonlinear function to any degree of accuracy. Next, following the feedforward control design principle, the output of NFSM (i.e. nonlinearity) is treated as measurable "disturbance" and a compensator can be designed to eliminate the disturbance influence. To achieve this goal, a NFS-based controller (NFSC) is designed (to be discussed in section 3.3). Ideally speaking, if the NFSM can completely learn the nonlinear characteristics, the nonlinear process can be linearized. As a result, linear controllers can deliver more effective control of the resulting linearized system. This motivates the use of the proposed enhanced feedback control scheme as depicted in figure 2.

3.1 Nonlinear Process Modelling

In this paper, we consider a class of nonlinear processes with modest nonlinearities, namely the linearized model has sufficient accuracy in the operating range of interest but the associated linearization errors is still significant enough to degrade the feedback control performance. The process dynamics can be described by the following composite model:

$$\hat{y}(k) = \text{ARX}(\mathbf{x}(k-1)) + \text{NFSM}(\mathbf{x}(k-1)) \quad (6)$$

where

$$\mathbf{x}(k-1) = [y(k-1), y(k-2), \cdots, y(k-n_y), u(k-n_d-1), u(k-n_d-2), \cdots, u(k-n_d-n_u)]^T \quad (7)$$

where u is the process input, y is the process output, n_d is the process time delay, n_y and n_u are the integers relating to the order of the process.

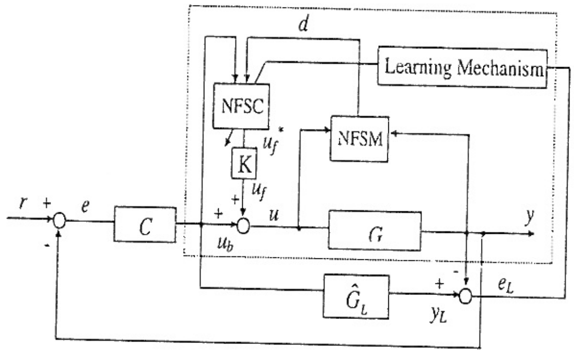

Fig. 2. The enhanced feedback control system

The identification procedure of NFSM can be summarized as follows:

(a) Given the input data $\mathbf{x}(k)(k = 1, 2, \ldots, P)$. The input data point $\mathbf{x}(1)$ is set as the first cluster (fuzzy rule) and its cluster center is $\mathbf{c}_1^M = \mathbf{x}(1)(\mathbf{c}_1^M = [c_{11}^M, c_{12}^M, \ldots, c_{1(n_y+n_u)}^M]^T$. The number of input data point belonging to the first cluster N_1^M and the number of fuzzy clusters N^M at this time are thus respectively given by $N_1^M = 1$ and $N^M = 1$;

(b) For the kth training data point $\mathbf{x}(k)$, compare the similarity of the kth training data point to every cluster centers $\mathbf{c}_l^M (l = 1, 2, \cdots, N^M)$ and find a cluster (fuzzy rule) whose center is the closest to $\mathbf{x}(k)$ as determined by the following criteria:

$$S_L = \max_{1 \le l \le N} (\sqrt{e^{-\|\mathbf{x}(k) - \mathbf{c}_l^M\|_2^2}}) \quad (8)$$

(c) Decide whether a new cluster (fuzzy rule) should be added or not according to the following criteria:

- If $S_L < \beta$, where β is a prespecifed threshold, the kth training data point does not belong to all the existing cluster and a new cluster will be added with its center located at $\mathbf{c}_{N+1}^M = \mathbf{x}(k)$. Set $N^M = N^M + 1$ and $N_N^M = 1$, and keep other cluster unmodified;
- If $S_L \ge \beta$, the kth training data point belong to the Lth cluster and the Lth cluster will be adjusted as follows

$$\begin{aligned} \mathbf{c}_L^M &= \mathbf{c}_L^M + \lambda(\mathbf{x}(k) - \mathbf{c}_L^M) \\ \lambda &= \lambda_0/(N_L^M + 1), \lambda_0 \in [0, 1] \\ N_L^M &= N_L^M + 1 \end{aligned} \quad (9)$$

(d) Let $k = k + 1$ and repeat steps (b) and (c) until all training data points are clustered into the corresponding cluster. At the completion of the first three steps, the number of cluster (fuzzy rule) is fixed as N^M, and the width of fuzzy set can be calculated as:

$$\sigma_{li}^M = \min_{j=1,2,\cdots,N^M; j \ne l} \frac{|c_{li}^M - c_{ji}^M|}{\rho} \quad (10)$$

where ρ is the overlap parameter ($1 \leq \rho \leq 2$) (Lin and Lee, 1991).

The consequent parameters $\mathbf{h}_l^M$ ($l = 1, 2, \cdots, N$) are obtained as (Setnes *et al.*, 1998)

$$\mathbf{h}_l^M = [\mathbf{X}^T \Phi_l \mathbf{X}]^{-1} \mathbf{X}^T \Phi_l (\mathbf{Y} - \mathbf{Y}_{ARX}) \quad (11)$$

where

$$\mathbf{X} = [\bar{\mathbf{x}}(1)^T, \bar{\mathbf{x}}(2)^T, \ldots, \bar{\mathbf{x}}(P)^T]$$
$$\mathbf{Y} = [y(1), y(2), \ldots, y(P)]^T$$
$$\mathbf{Y}_{ARX} = [y_{ARX}(1), y_{ARX}(2), \ldots, y_{ARX}(P)]^T$$

3.2 Neuro-fuzzy Controller Design

Based on the abovementioned composite model, a control system is devised as given in figure 2, where G is the controlled nonlinear process, C is the feedback controller, $\hat{G}_L$ represents the linear ARX model, NFSM is the error model, NFSC is a controller designed by NFS approach that aims to eliminate the process nonlinearity, a measurable "disturbance" obtained from NFSM, i.e.

$$u_f = \text{NFSC}(\tilde{d}), \quad \tilde{d} = [u_b \ \ d]^T \quad (12)$$

where d is the "disturbance" estimated by NFSM constructed off-line by the aforementioned procedure, u_b represents the output of the feedback controller, and u_f is the output of the controller NFSC.

Considering both computational efficiency and ease of adaptation, the zero-order TSK-type fuzzy rule and the structure of $2 - 2n - N - 1$ ($N = n^2$) are chosen for NFSC. The adjustable parameters of NFSC are $\mathbf{c} = [c_{li}]_{n\times 2}$, $\sigma = [\sigma_{li}]_{n\times 2}$ and $\mathbf{h} = [h_v]_{N\times 1}$. We seek to determine a NFSC($\tilde{d}|\mathbf{c}, \sigma, \mathbf{h}$) such that the following quadratic criterion E is minimized with respect to the parameter $\mathbf{c}$, σ, and $\mathbf{h}$.

$$E(k) = \frac{1}{2}((y_L(k) - y(k))^2 \quad (13)$$

where $y_L = \hat{G}_L u_b$.

If NFSC has learned the error characteristics completely, i.e. $E = 0$, we have

$$y = \hat{G}_L u_b \quad (14)$$

Since $u_b = C(r - y)$, y can be expressed as

$$y = \frac{\hat{G}_L C r}{1 + \hat{G}_L C} \quad (15)$$

Obviously, the closed-loop performance can be improved because the nonlinear process to be controlled by linear controller has been linearized, i.e. the part denoted by dotted rectangle in figure 2 is a linear dynamics. As a result, conventional linear controller design can attain improved performance with the aid of NFSC that is designed on-line by updating its network parameters $\mathbf{c}$, σ, and $\mathbf{h}$ as discussed in the next section.

Remark: The aforementioned linearization procedure was also discussed in the previous work (Kim *et al.*, 1997) where a single neural network was employed to achieve this purpose. However, a proper selection of the predefined linear reference model is crucial to the successful application of this method.

3.3 On-line Learning Algorithm for NFSC

Note that the inputs to NFSC are normalized through the hyperbolic tangent function, which ensures the universe of discourse to lie in the range of [−1 1].

$$\begin{aligned} \psi_1 &= (1 - exp(-\xi_1 u_b))/(1 + exp(-\xi_1 u_b)) \\ \psi_2 &= (1 - exp(-\xi_2 d))/(1 + exp(-\xi_2 d)) \end{aligned} \quad (16)$$

The adjustment of these parameters can be divided into two tasks according to the antecedent part and the consequence part of the fuzzy rules. The parameters of the antecedent part, $\mathbf{c}$ and σ, are initialized in the range of [-1 1], and the parameters of the consequence part, $\mathbf{h}$ can be simply set to zero.

In what follows, the gradient-descent algorithm is employed as the parameter updating law of NFSC. The parameter $h_v(k)$ can be updated as follows

$$h_v(k+1) = h_v(k) - \eta \frac{\partial E(k)}{\partial h_v(k)} \quad (17)$$

where $v = 1, 2, \ldots, N$, and

$$\begin{aligned} &\frac{\partial E(k)}{\partial h_v(k)} \\ &= -(y_L(k) - y(k)) \frac{\partial y(k)}{\partial u(k-1)} \frac{\partial u_f(k-1)}{\partial h_v(k)} \\ &\approx -(y_L(k) - y(k)) \frac{\hat{y}(k) - \hat{y}(k-1)}{u(k-1) - u(k-2)} \cdot \\ &\frac{\partial u_f(k-1)}{\partial h_v(k)} \end{aligned} \quad (18)$$

and

$$\hat{y}(k) = \text{ARX}(\mathbf{x}(k-1)) + \text{NFSM}(\mathbf{x}(k-1)) \quad (19)$$

$$\frac{\partial u_f(k-1)}{\partial h_v(k)} = K\Phi_v \quad (20)$$

Similarity, we get

$$\Delta c_{l1}(k) = \eta(y_L(k) - y(k)) \cdot \frac{\hat{y}(k) - \hat{y}(k-1)}{u(k-1) - u(k-2)} \frac{2K(O_{l1}^{(1)} - c_{l1})O_{l1}^{(2)}}{\sigma_{l1}^2(\sum_{j=1}^{N} O_j^{(3)})^2} \cdot \sum_{q=1}^{n} O_{q2}^{(2)}(h_{(l-1)n+q}\sum_{j=1}^{N} O_j^{(3)} - \sum_{j=1}^{N} O_j^{(3)} h_j) \quad (21)$$

$$\Delta c_{l2}(k) = \eta(y_L(k) - y(k)) \cdot \frac{\hat{y}(k) - \hat{y}(k-1)}{u(k-1) - u(k-2)} \frac{2K(O_{l2}^{(2)} - c_{l2})O_{l2}^{(2)}}{\sigma_{l2}^2(\sum_{j=1}^{N} O_j^{(3)})^2} \cdot \sum_{q=1}^{n} O_{q2}^{(2)}(h_{(q-1)n+l}\sum_{j=1}^{N} O_j^{(3)} - \sum_{j=1}^{N} O_j^{(3)} h_j) \quad (22)$$

$$\Delta \sigma_{l1}(k) = \eta(y_L(k) - y(k)) \cdot \frac{\hat{y}(k) - \hat{y}(k-1)}{u(k-1) - u(k-2)} \frac{2K(O_{l1}^{(1)} - c_{l1})^2 O_{l1}^{(2)}}{\sigma_{l1}^3(\sum_{j=1}^{N} O_j^{(3)})^2} \cdot \sum_{q=1}^{n} O_{q2}^{(2)}(h_{(l-1)n+q}\sum_{j=1}^{N} O_j^{(3)} - \sum_{j=1}^{N} O_j^{(3)} h_j) \quad (23)$$

$$\Delta \sigma_{l2}(k) = \eta(y_L(k) - y(k)) \cdot \frac{\hat{y}(k) - \hat{y}(k-1)}{u(k-1) - u(k-2)} \frac{2K(O_{l2}^{(2)} - c_{l2})^2 O_{l1}^{(2)}}{\sigma_{l2}^3(\sum_{j=1}^{l} O_j^{(3)})^2} \cdot \sum_{q=1}^{n} O_{q2}^{(2)}(h_{(q-1)n+l}\sum_{j=1}^{N} O_j^{(3)} - \sum_{j=1}^{N} O_j^{(3)} h_j) \quad (24)$$

where $O_{li}^{(1)}(l = 1, 2, \ldots, n;\ i = 1, 2)$, $O_{li}^{(2)}(l = 1, 2, \ldots, n;\ i = 1, 2)$ and $O_j^{(3)}$ represent the output of the ith node in layer 1, lith node in layer 2 and jth node in layer 3, respectively.

4. EXAMPLE

To demonstrate the effectiveness of the proposed controller design, a distillation process is considered, in which the output variable is the top column composition, y, and the input variable is the reflux flow rate u. This process can be described by the following equations (Eskinat *et al.*, 1991), (Gao *et al.*, 2000):

$$\begin{aligned} y(k) &= 0.757y(k-1) + 0.243g(u(k-1)) \\ g(x) &= 1.04x - 14.11x^2 - 16.72x^3 + 5562.7x^4 \end{aligned} \quad (25)$$

where the input and output variables are both defined as deviations from their respective nominal values.

Independent random signal with uniform distribution between [-0.1 0.1] is used to simulate 100 input-output data points. To proceed with the proposed algorithm, $\mathbf{x}(k)$ is chosen to be $\mathbf{x}(k) = [y(k),\ u(k)]^T$. Then 100 input-output data generated are used to identify the composite model, i.e. a linear ARX model and a NFSM model. The identification result is shown in figure 3.

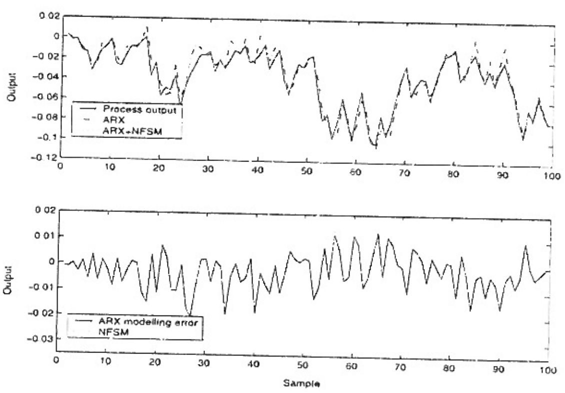

Fig. 3. Identification result

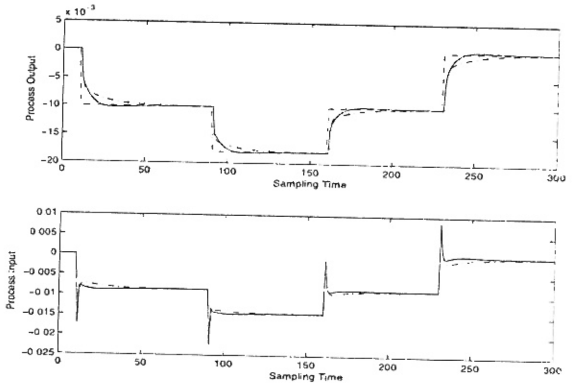

Fig. 4. Response of the proposed design (solid line) and feedback control (dash-dot line)

It is apparent that the ARX model plays a dominant role in the operating range and the linearization errors can be modelled by NFSM accurately. Based on the composite model, the NFSC can be designed next. The structure of NFSC is chosen to be $2 - 14 - 49 - 1$. The initial $\mathbf{h}$ is set as zero, and the initial $\mathbf{c}$ and σ are chosen as $\mathbf{c}_i = [-1,\ -\frac{2}{3},\ -\frac{1}{3},\ 0,\ \frac{1}{3},\ \frac{2}{3},\ 1]$ and $\sigma_{li} = 0.2$ for $i = 1, 2$ and $l = 1, 2, \ldots, 7$ to cover the universe of discourse $[-1,\ 1]$ uniformly. The other parameters used for NFSC are chosen as $K = 10$, $\eta = 0.3$, $\xi_1 = 10$ and $\xi_2 = 0.01$. The feedback controller C is designed based on the IMC-PID design procedure (Rivera *et al.*, 1986).

Figures 4 and 5 compare the set-point responses of the proposed design and feedback controller. It is evident that the proposed control scheme has faster response over the entire operating space. The disturbance rejection capability of these two controller designs is compared in figure 6, where a step unmeasured disturbance of magnitude 0.001 occurs at the one hundred sampling time. Again, the proposed control design has a superior performance over the conventional feedback controller.

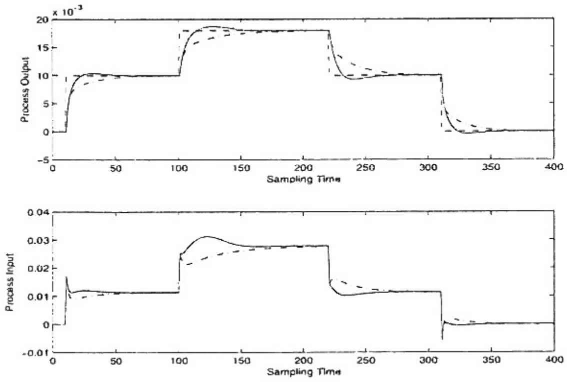

Fig. 5. Response of the proposed design (solid line) and feedback control (dash-dot line)

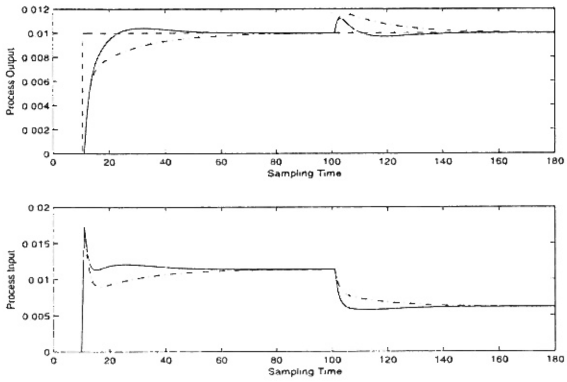

Fig. 6. Response of the proposed design (solid line) and feedback control (dash-dot line)

5. CONCLUSION

An enhanced feedback control scheme is proposed for a class of nonlinear processes with modest nonlinearities.The nonlinear process is first approximated by a composite model consisting a linear ARX model and a NFSM, which captures the linearization error between the process output and predicted output from ARX model. The output of NFSM is viewed as the measured disturbance and NFSC is designed to eliminate this disturbance influence. Simulation results show that the proposed design indeed provides better performance than the feedback control acting alone.

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