Multiple criteria Process Command: a Challenge to Robustness

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***Abstract** – In the area of optimizing industrial process, decision makers are often confronted with multi-objective's problems. In manufacturing, for example, it is often necessary to optimize many conflicting objectives, subject to market constraints.This paper briefly presents a multi-objective optimization procedure - also by introducing a robustness criterion among the criteria directly connected to the end product properties and the technological requirements - which yields an optimal zone containing the solution under the concept of Pareto domination. Pair-wise points are compared, and non dominated points are collected in the Pareto region. Then a ranking is established, and the decision maker selects the first-best solution.*

Keywords: multicriteria optimization, Pareto domain, Pareto domination, indifference thresholds, preference tresholds, veto tresholds, robustness, histogrammization.

I. INTRODUCTION

In the process engineering applications field the most promising optimization principles are based more and more on the multiple criteria approach. Howewer the best solution suggested should satisfy not only compromises established between criteria levels, acceptable by the decision maker but the other *security and stability considerations* related to the process. For this purpose it seems to us essential to introduce a « virtual » maximizing criterion (called robustness criterion) to the criteria originally attached to the process. This *robustness criterion* can be represented by the smallest distance between the possible solutions and the border points of the Pareto set

II. REFLECTIONS ON THE CONCEPT OF PERFORMANCE

The term "robust" was first introduced in statistics by G.E.P. Box in 1953, [2]. An estimator is deemed robust if it is insensitive to small variations in hypotheses for which it was optimized. There are two meanings associated with the term "small": minor variations in all data, or significant variations in a small number of data.

Thus, robustness most often translates into the resistance of the estimate to dissenting data. Mathematically, it is defined as the smallest number of extreme data that modify the value of an estimate reduced to the size of a sample, [4].

The following are reflections on robustness that are the cornerstone of my work. Prior to justifying the reason for addressing robustness in my research, I believe it is important to define what is intended by the concept of *"performance in process engineering"* and *"robustness in performance"*. In my opinion, there is no ambiguity associated with the definition of robustness in stability; however, such is not the case when one considers performance as well, largely because this concept is hard to quantify.

When implemented in an experimental device, a rule of control may indeed lead to acceptable behaviour (for example, a notable attenuation in vibrations), differing, however, from that foreseen in simulation. The closed buckle system is remarkably stable and behaves correctly, but it seems inappropriate to state that the rule is robust in performance. The fact that a compensator can, for all intents and purposes, lead to an acceptable level of performance is insufficient, in my opinion, to prove robustness in performance, [3].

Therefore, in my opinion, it is both the level of measured performance and the concordance between experimentation and simulation that determine the robustness in performance of a rule. It is these criteria, in addition to the wear properties of a product, that are also directly linked to performance. For several years, I have focused on the problem associated with the *multiple criteria approach of biotechnological and chemical engineering processes*. My research has centered on the study and mathematical and chemical design of processes integrating capacities of perception, action and adjacent decisions. Such processes must be able to interact with users in a varying and evolving environment for the robust achievement of different tasks.

III.THE DISTANCE METRICS CHOICE

Since the analysis of results involves criteria space, a two-to-two (i.e. pair-wise) comparison of the vectors of response points is unavoidable.

$$\begin{bmatrix} a_1^{(k)} \\ a_2^{(k)} \\ \vdots \\ a_{n-1}^{(k)} \\ a_n^{(k)} \end{bmatrix} \leftrightarrow \begin{bmatrix} b_1^{(z)} \\ b_2^{(z)} \\ \vdots \\ b_{n-1}^{(z)} \\ b_n^{(z)} \end{bmatrix} \; k \neq z \,; k = 1, \cdots, \Omega - 1; \; z = k+1. \cdots, \Omega$$

where Ω represents the cardinality of a Pareto set, or the number of points to be compared.

Vector factors physically contain different measurements, therefore the Eucledian distance is not adequate because independence between factors cannot be ensured in every circumstance. I surmised that after normalizing the vectors so that the sum of their respective coordinates is equal to 1, one obtains $\sum a_i = \sum b_i = 1.0$, a distance in terms of the angle of the most neutral metric to be used, because this metric eliminates the problem of incomparability between the different physical dimensions. Angle α formed by the two vectors **a** and **b** in hyperspace may be expressed in this cosine trignometric ratio:

$$\cos \alpha = \sum_{i=1}^{m} \frac{a_i}{\sqrt{\sum_{j=1}^{m} a_j^2}} \cdot \frac{b_i}{\sqrt{\sum_{j=1}^{m} b_j^2}},$$ or angle

distance $d_{\alpha}^{(a,b)} \equiv \alpha_{rad} =$

$$\arccos\left(\sum_{i=1}^{m}\frac{a_i}{\sqrt{\sum_{j=1}^{m}a_j^2}}\cdot\frac{b_i}{\sqrt{\sum_{j=1}^{m}b_j^2}}\right)$$ and naturally

$0 \le \alpha \le \pi/2$, the angle distance

expressed as the distance between vector directions in the comparison phase. Although discussion of this basic algebraic vector may seem trivial, it should be noted that the concept of angle distance was addressed at an earlier date by A. Colorni *et al.* (2001), [1].

For argument's sake, a metric is a binary ratio that decribes the distance between two points of a set *E*. This distance is an application of $E \times E \to \mathbb{R}^+$ such that $\forall i, j, k \in E$, such that

1) $d(i,j) = d(j,i)$ symmetry
2) $d(i,j) \ge 0$
3) $d(i,j) = 0 \Leftrightarrow i = j$
4) $d(i,j) \le d(i,k) + d(k,j)$ triangular inequality

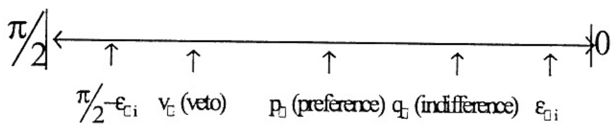
If all these properties are met, one is in the presence of distance metrics. It is clear that $d_{\alpha}^{(a,b)}$ meets the above properties. Therefore, it may "legitimately" be considered suitable for use as a metric. I also normalized vectors of points in Ω. Normalization eliminated any possible ambiguity caused by physical differences in the measurement of vector constituents.

IV. THE PROBLEM OF ANGLE THRESHOLDS

To my knowledge, no technique of choice for this category of thresholds has ever been dealt with in scientific literature. I am taking the liberty of applying purely an esthetic arguments more usually applied to the field of fine arts. Thus, ε_i is the minimum non-zero difference and $\varepsilon_i \ge 0$ is noted between the n^{th} component of criteria vectors during preceding statistical processing. In order to create a security buffer, thresholds q, p and v were set between the interval

$[\pi/2 - \varepsilon_i ; 0 + \varepsilon_i]$, instead of $\pi/2$ and 0. To obtain "an esthetically" robust thresholds, values for q, p and v should be such that the following ratios are observed:

$\frac{v-q}{v-p} = \frac{v-p}{p-q}$ which corresponds to the divine proportion of segment $[\pi/2 - \varepsilon_i ; 0 + \varepsilon_i]$ retained. The schema below illustrates what I have just said concerning the thresholds.



Equality of ratios $\frac{v-q}{v-p}$ and $\frac{v-p}{p-q}$ may be obtained using the divine proportion $\frac{1+\sqrt{5}}{2}$ by producing the famous divine proportions in angle thresholds.

The following table illustrates numerical values of threshold angles meeting robustness criteria

V. A *SINE QUA NON* CONDITION: MASTERY OF THE BORDER POINTS OF THE PARETO SET

In my opinion, the border points of the Pareto set Ω consist of two disjointed subsets of non-dominated points, border

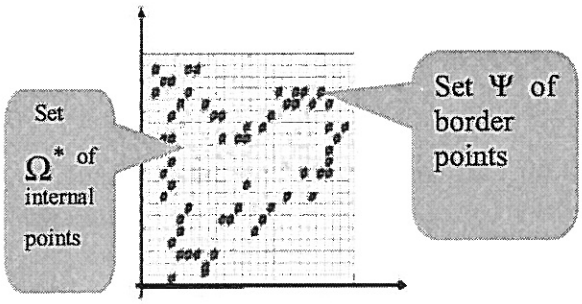


Figure1: Pareto Set Ω

Ψ points on the one hand, and internal Ω^* points on the other hand, where Ψ, $\Omega^* \in \Omega$ with $\Omega^* \cap \Psi = \{\varnothing\}$, et $\Omega = \Psi \cup \Omega^*$ (See Figure 1 above).

Although the Ψ points, like the Ω^* points, represent legitimate solutions that are procedurally achievable, from an industrial point of view Ψ points on the perimeter are less secure because the stability of these points cannot be maintained under the impact of the slightest disturbance.

Therefore, it is in our best interest to privilege a suitable solution from among the internal Ω^* points. To achieve this, a virtual criterion referred to as the robustness criterion must be introduced from among the original criteria associated with the problem or process in which the latter criteria provide a general representation of the wear properties of a product, as well as certain strictly technological and economic indicators. I

Angle Thresholds	***Values in radians and degrees***
Indifference	0.3723534 ≈ 20.423°
Preference	0.6024805 ≈ 31.1°
Veto	0.9748339 ≈ 44.27°
Precision	|0.000000117| ≈ |0.000006688°|

propose that the robustness criterion be interpreted as the minimum distance between border Ψ point and the Ω^* points of the Pareto Ω set. This virtual criterion will then serve as a criterion to maximize in a multi-criteria analysis protocol, where the robustness criterion may further be explained as follows:

$$d_{\alpha}^{(\xi,\omega^*)} \equiv \min\left\{\left\|\xi - \omega^*\right\| \,\middle|\, \xi \in \Psi, \omega^* \in \Omega^*\right\}$$

In order to master the border points, it is vital that one consider the numerical values of the coordinates of each component constituting the vector to which it belongs. To this effect, I developed my own prototype algorithm based on a rather "naïve" principle that consists of the histogramization of Ω elements.

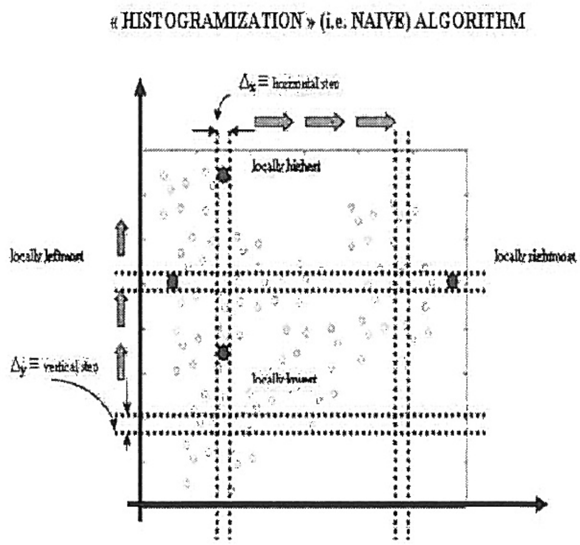


Figure 2: *Horizontal and Vertical Step Histogramization*

VI. NAÏVE ALGORITHM

I had no intention of equalling or even surpassing existing algorithms already catalogued by specialists in the issue, however fascinating they may be. The reason is that they are not applicable in this context, given the fact that most presume a priori knowledge of at least one border point. Although this might appear inconsequential, it is completely unthinkable in process engineering, the field in which I am developing modelling, not to mention the complete inaccessibility of the few codes that have already been published in literature.

Therefore, I developed my own algorithm and corresponding codes. The computer model obtained corresponds precisely to my scientific objective in terms of rapidity, flexibility, precision and ease of use. For a synoptic view of the naïve algorithm, refer to Figure 2. First, I defined the horizontal Δ_x and vertical Δ_Y steps forward where

$\Delta_x \cong \min (x_i - x_j); i \neq j, i = 1, \ldots, \Omega\text{-}1; j = i+1, \ldots, \Omega$, but $\Delta_x > 0$,

and

$\Delta_y \cong \min (y_i - y_j); i \neq j, i = 1, \ldots, \Omega\text{-}1; j = i+1, \ldots, \Omega$, but $\Delta_y > 0$.

As a result of this choice, no point is omitted from the algorithm. Analysis in the horizontal and vertical planes is necessary, since the convexity of the Pareto set is not maintained in all circumstances. More frequently, the border silhouette consists of irregular convex and concave zones.

Suppose that the algorithm is in its λ^{th} step forward on the axis of the abscissa on one of the 2D projections in the data processing run. Before examining the membership of points in the interval under analysis during the λ^{th} step, the membership counter κ of the points must be initialized to 0. One point is singled out in the histogram distribution if:

$$(\lambda\text{-}1)\cdot\Delta_X \leq x_i \leq \lambda\cdot\Delta_X \ ; i = 1,\ldots,\Omega,$$

then $\kappa = \kappa + 1$ et $\xi_{i,\kappa}^{(\lambda)} \equiv x_i$. The seemingly complicated $\xi_{i,\kappa}^{(\lambda)}$ notation is necessary, because one must surmise that a point x_i is ranked as the κ^{th} element of the λ^{th} space.

After the algorithm has run index i on the interval of 1 to Ω, it is evident that **max** $\xi_{i,\kappa}^{(\lambda)}$, as well as **min**$\xi_{i,\kappa}^{(\lambda)}$ can be classed in set Ψ, because they are truly border points. The same procedure is then repeated on the axis of ordinates of each 2D.

After completing the analysis of each 2D, one obtains Pareto points positioned along the border of the Pareto set as follows:

$$\Psi = \bigcap_{\rho=1}^{\binom{\eta}{2}} \left(\bigcup_{i=1}^{\Phi_\rho} \xi_i \right)$$

where

ψ - set of border points;

Φ_ρ - cardinality of ρ^{th} 2D;

ξ_i - i^{th} border point of ρ^{th} 2D, $\xi_i \in \Phi_\rho$;

η - number of variables entering the process;

$\binom{\eta}{2}$- number of 2D, $\rho \in \binom{\eta}{2}$.

VII. THE ROBUSTNESS CRITERION

Here, one seeks the minimum distance between each point and the border points, then the numerical value of the distance found will be added to the set of criteria in order to achieve a kind of virtual criteria. Analytically this can be explained as follows:

$$d_{\alpha}^{(\xi,\omega^*)} \equiv \min\left\{\left\|\xi - \omega^*\right\| \; \middle| \xi \in \Psi, \omega^* \in \Omega^*\right\}.$$

This virtual criterion then becomes a criterion to maximize, therefore **MAX d(α, ω*)**.

This allows compliance not only with the wear property requirements of a product, but also the achievement of stability and security requirements so prized by industry.

VIII. CONCLUDING REMARKS AND FURTHER CONSIDERATIONS

In this paper, a multi-objective decision problem, arising in a process engineering command. The main objective was the determination of the process's optimal operating conditions, in accordance with certain technical, security, stability requirements and with the decision-makers' preference. The first step was the determination of the so-called Pareto region, and the second was the multi-criterion analysis proper. The best trade-off point was then selected from the region. Clearly, this procedure is applicable to the design of the process, but in order to take full advantage of its power, the designer must have some familiarity with the technical environment to which the procedure is applied. Thanks to the applied robustness criterion, the first-best solution is robust under any perturbations in operating input conditions. Our results extend previous decisional procedures, and open new perspectives in the extrapolation of robustness of optimal choice and effective control in process engineering area. Our analysis concerns, first and foremost, a search for a solution to a multi-objective decision problem of the process control area. In some way, it may may be seen as a bridge between decision sciences and the process engineering science. Multi-objective optimization is common in the process industry, and the decision sciences need concrete areas of application. Both stand to benefit.

IX. REFERENCES

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