A Fuzzy Approach To The Tracking Control Systems

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Abstract – **The target tracker of a guidance missile is disturbed by the noise sources. This condition has led the fuzzy controller is an alternative to the conventional deterministic PD control to reject the noise effects. Firstly, a mathematical model of the tracker is derived, then two different control algorithms are applied to this model in a simulation environment. The encouraged results for the future studies are obtained from the fuzzy proportional derivative control.**

I. INTRODUCTION

Since World War II, guided missiles have played an increasingly important role in warfare. Earlier papers [1], [2] reviewed the development of the inertial guidance systems that made possible the accurate delivery of long range ballistic (and other) missiles for which the target is a known set of earth coordinates. These systems are not suitable for guidance of missiles against targets such as maneuvering aircraft, which requires an ability to sense the target location in real time and respond to rapid changes. To accomplish this, modern air defense missiles use homing guidance, in which a tracker servo provides the target data [3]. Because of the continually improving quality of target information as the missile closes in, homing guidance provides an accuracy that is unmatched by any other form of missile guidance.

In the homing systems, the tracker is in the missile and the relative movement of target and missile is relevant. A target tracker attempts to align its electrical null axis (boresight) with a line joining the tracker and target called the 'Line of Sight' (LOS) [4]. As known, 'the tail pursuit' is one of the main methods in the homing guidance systems. Essentially, two devices are used to provide the target data to achieve the interception in the tail pursuit applications. The "seeker" or radar receiver may be used alone in short range air - to - air missiles for which target maneuvers and LOS variations are limited within the operation time interval. But, for the medium range applications more system capability and a "tracker" is required because the targets may adopt some escape policy.

The tracking error may take large values beyond the seeker limits, finally all the measurable variables may have to be used to predict the target behavior. Therefore, a tracker mechanism and related measurement systems should be introduced into the model and control system. The noise which occurs at the tracker system input perturbs this system. The main source of noise in radar receivers is 'thermal noise' . It gives rise to an electrical noise voltage having 'white noise', characteristic, its spectrum is independent of frequency from d.c to a frequency far in excess of any servo tracker bandwith. There are many other sources of noise associated with receivers including environmental background noise but in practice it is observed that if receiver noise is significant , it is largely due to thermal noise.

Since this noise affects the input of the tracking system, the angular position λ and angular velocity $\dot{\lambda}$ of the tracker with respect to the target can not be considered as a precise variable. In control systems, fuzzy logic is considered as an alternative for conventional control theory in the control of plants where variables have fuzzy characteristics or deterministic mathematical models are difficult or impossible [5],[6]. In DC motor drives and motion control areas, FLC is proposed to control different systems like Induction and DC motors [7],[8], Robots and AC servos [9]. In these applications, it has been reported that the FLC is more robust to plant parameter value changes than a classical control algorithm and has better noise rejection capabilities.

The purpose of this paper is to investigate the performance of FLC application when a noise occurs at the system input and to compare it with the classical PD control algorithm performance. Simulation results display the system performances under these two control approaches.

II. MATHEMATICAL MODEL

Tracker is a servomechanism which has the input ε_T and tracking the target to have $\varepsilon_T = 0$. The angular position of the tracker with respect to the missile frame is λ_m and its angular velocity with respect to the missile frame is $\dot{\lambda}_m$. These variables can directly be measured from the servomechanism. The configuration of the tracker is given on Fig. 1.

From Fig. 1., the equations

$$\lambda = \sigma - \alpha \quad (1)$$

$$\dot{\lambda} = \dot{\sigma} - \dot{\alpha} \quad (2)$$

can be written.

Inputs of the tracker controller, from Figure 1. are

$$\varepsilon_T = \lambda - \lambda_m \quad (3)$$

$$\dot{\varepsilon}_T = \dot{\lambda} - \dot{\lambda}_m \quad (4)$$

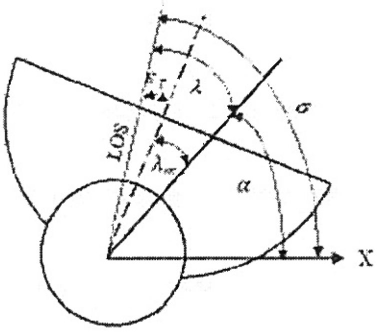


Fig. 1. Tracker Configuration

The mathematical model of the tracking servomotor is used in the simulation study as the controlled plant. The equation of the motion of the tracker drive motor in the simplest form is:

$$J\ddot{\lambda}_m + B\dot{\lambda}_m = k.u_T \qquad (5)$$

where J is the total effective inertia, k is the servomotor gain and B is the total effective viscous friction coefficient. The model of tracking servo can be obtained from equation (5). When the state variables are assigned as:

$$X_1 = \lambda_m \qquad X_2 = \dot{\lambda}_m$$

The following state model is obtained for numerical integration:

$$\left.\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -\frac{B}{J}X_2 + \frac{k}{J}u_T \end{aligned}\right\} \qquad (6)$$

The transfer function from u_T to λ_m is

$$G(s) = \frac{K_T}{s(\tau_T s + 1)} \qquad (7)$$

where $\tau_T = \frac{J}{B}$ is the tracker time constant and $K_T = \frac{k}{B}$ its static gain. The tracker controller will generate u_T according to the proposed control schemes in this work.

III. PD CONTROL

Equations (3) and (4) are used to get a classical PD control algorithm which can be written as:

$$u_T(t) = K_p(\varepsilon_T(t) + \tau_d\dot{\varepsilon}_T(t)) \qquad (8)$$

where K_p is the proportional gain coefficient and τ_d is the differential control coefficient.

The transfer funtion for PD control is written as

$$K(s) = K_p(\tau_d.s + 1) \qquad (9)$$

Then the transfer function from ε_T to λ will be written as

$$\frac{E_T(s)}{\Lambda(s)} = \frac{1}{1 + \dfrac{K_p(\tau_d s + 1)K_T}{s(\tau_T s + 1)}} \qquad (10)$$

where $E_T(s)$ is defined as the laplace transform of $\varepsilon_T(t)$ and $\Lambda(s)$ is the laplace transform of $\lambda(t)$.

When the equation (10) is arranged

$$\frac{E_T(s)}{\Lambda(s)} = \frac{s(\tau_T s + 1)}{\tau_T s^2 + s(1 + K_p\tau_d K_T) + K_p K_T} \qquad (11)$$

is obtained. This equation may be rewritten in the following form.

$$\frac{E_T(s)}{\Lambda(s)} = \left(\frac{1}{K_p.K_T}\right).\frac{w_n^2.s(\tau_T s + 1)}{s^2 + 2\zeta w_n s + w_n^2} \qquad (12)$$

where w_n and ζ are respectively the natural frequency and the damping ratio of the controlled system and are defined in function of system parameters as:

$$2\zeta w_n = \frac{1 + K_p\tau_d K_T}{\tau_T} \qquad (13)$$

$$w_n^2 = \frac{K_p K_T}{\tau_T} \qquad (14)$$

Because the second order characteristic equation of the controlled system, system dynamic behavior can be specified by desired values of w_n and ζ. Controller coefficients K_p and τ_d can be found using second order system step response requirements and corresponding desired w_n and ζ values. Since the controlled system G(s) is of type 1, PD control is enough to obtain zero steady-state error for step variation of λ. The behavior of this tracker system under noisy inputs can be investigated by simulation of the system, adding a noise *w(t)* to a mean value $\bar{\lambda}(t)$ of $\lambda(t)$. Then we will have, for simulation purposes, $\lambda(t) = \bar{\lambda}(t) + w(t)$ and consequently according to equation (3) and (4)

$$\varepsilon(t) = (\bar{\lambda}(t) + w(t)) - \lambda_m(t) \qquad (15)$$

$$\dot{\varepsilon}(t) = (\dot{\bar{\lambda}}(t) + \dot{w}(t)) - \dot{\lambda}_m(t) \qquad (16)$$

$\lambda(t)$ is the LOS angle of the target with respect to the missile axis, *w(t)* is the noise used to test the system performance in simulation environment, $\varepsilon(t)$ and $\dot{\varepsilon}(t)$ are used as the tracker controller inputs.

IV. FUZZY CONTROL

The noise which occurs at the system input lead to measurements with fuzzy characteristics, then the defendable reasons for a fuzzy control application exist. In this work, the 'white noise' with zero mean value is taken as disturbance input. The graphical representation of the 'noise' with respect to time as a random input is shown on Fig.2 . Fuzzy control structure and principles is shortly introduced in this section. Fuzzy logic controller has four modules: fuzzification, rule base, inference mechanism and defuzzification modules. Fuzzification converts the inputs of the controller to a convenient form to be used in the rule base and the inference mechanism modules. The fuzzification module creates the membership functions of the input values. In the rule base module, the inputs of the controller are oriented by a linguistic method. The orientation is realized with IF....THEN rules which are evaluated in the inference mechanism module. These rules are modified to the crisp form in the defuzzification module and finally they are transmitted to the controlled plant.

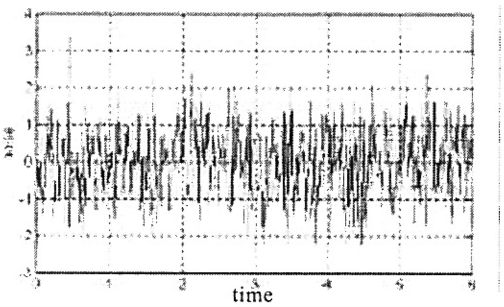
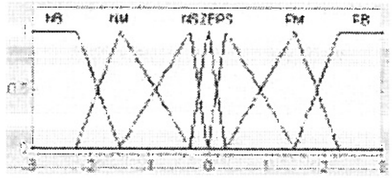


Fig. 2. Noise Distribution

The inputs of the FLC are ε_T and $\dot{\varepsilon}_T$ and they are described by equation (15) and (16). 'Fuzzification' constructs both input and output membership functions. The input functions are shown on Fig. 3, output function is displayed on Fig. 4. IF...THEN rules are derived from the membership functions are shown in the Table 1. In this table, PS means "positive small", NB "negative big" etc. In accordance with membership definition functions presented above. The rule base table should be interpreted as IF ε_T belongs to NS membership distribution and $\dot{\varepsilon}(t)$ belongs to PB membership distribution, THEN u_T will be derived from PM "positive medium" distribution. These rules will be interpreted by the inference mechanism. They are derived in a similar way as presented for dc motor control [10].

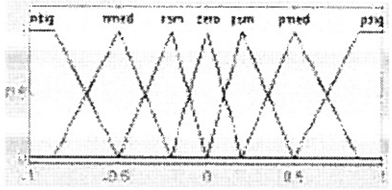
The inference mechanism has two basic tasks [11]: 1) Determining the extent to which each rule is relevant to the current situation as characterized by the inputs $\varepsilon_T, \dot{\varepsilon}_T$. This task is called "matching"; 2) Drawing conclusions using the current inputs and the information in the rule-base, thisk task is called "inference step".

$f(\varepsilon_T)$



$\varepsilon_T / \varepsilon_{T\max}$

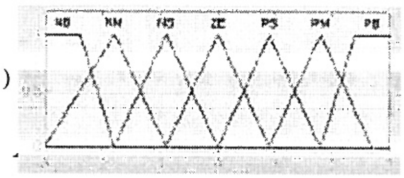
$f(\dot{\varepsilon}_T)$



$\dot{\varepsilon}_T / \dot{\varepsilon}_{T\max}$

Fig. 3. Fuzzification of the Inputs

$f(u_T)$



$u_T / u_{T\max}$

Fig. 4. Fuzzification of the Output

Table 1. Rule Base

u_T		$\dot{\varepsilon}_T$						
		NB	NM	NS	ZE	PS	PM	PB
ε_T	NB	NB	NB	NB	NB	NM	NS	ZE
	NM	NB	NB	NB	NM	NS	ZE	PS
	NS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	PS	NM	NS	ZE	PS	PM	PB	PB
	PM	NS	ZE	PS	PM	PB	PB	PB
	PB	ZE	PS	PM	PB	PB	PB	PB

The defuzzification phase is needed to send the rules which are evaluated in the inference phase as an unique control signal to the controlled plant (tracking servomotor). The most common method among the others is the 'Center of Gravity' (COG) method. This method determines the center of area below the clipped or scaled fuzzy sets. This requires the computation of overlapping area. COG method leads to the equation (16) given below.

$$u^{crisp} = \frac{\sum_i b_i \int \mu_i}{\sum_i \int \mu_i} \qquad (16)$$

μ_i is the the value of each membership funtion for any time in the simulation program, and b_i symbolizes their centers. Scaling coefficients are considered to tune the input and output membership functions according to the desired system behavior in the fuzzy-PD controller. How to get the scaling factors for Fuzzy PD controller is explained in reference [12]. Crisp control action is calculated in this work using the center of gravity (COG) defuzzification procedure providing better continuity [13] .

V. SIMULATION RESULTS AND CONCLUSION

Simulation results for PD and FLC tracker controller are given in a comparative form on Fig. 5. Control coefficients in PD application are obtained from equation (13) and (14) for a natural freqency of $w_n = 6Hz$ and a damping ratio of $\zeta = 0.7$. The coefficients which correspond to these parameters are K_p =80, τ_d=0.2. Simulation results for tracking system show that the fuzzy controller provides the better noise rejection as expected. Step response of the system for fuzzy PD application is more damped than PD control and control variable variations have smaller amplitudes due to the adopted defuzzification method. System settling time to reference input has an acceptable value (approximately 3.3s). The results obtained from this study has led to the further developments in the tracking control area.

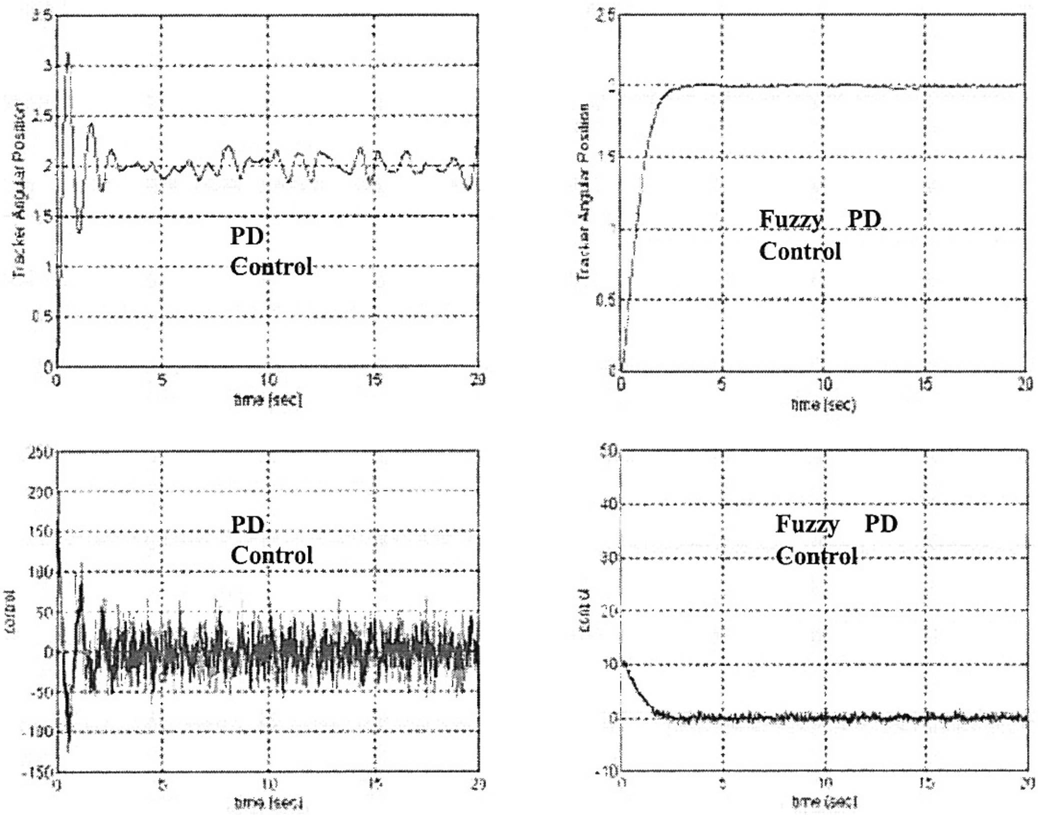


Fig. 5. The Comparative Form of the System Behaviors

VI. NOMENCLATURE

λ_m : angular position of the tracker

λ: angle between LOS and missile speed vector

K_p : proportional gain

K_v : differential gain

τ_d : differential time constant

τ_T : dc motor time constant

ε_T : tracker error

$\dot{\varepsilon}_T$: change of tracker error

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