Quality Improvement of Observer Based Robust Fuzzy Control of Nonlinear Systems

Michal Polanský
Faculty of Electrical Engineering and Communication
Brno University of Technology
Božetěchova 2, 612 66 Brno
Czech Republic
polansky@feec.vutbr.cz

Abstract – **The aim of this paper is to introduce an original fuzzy control method for nonlinear systems. Its development was motivated by the problem, that robust controller is often slower than an optimal one. This is a drawback of robust control methods that should be eliminated by proposed method. It increases the performance of the control process by interpolation of optimal and robust controller. Takagi-Sugeno fuzzy control methods have been adopted for design of robust and optimal controller and also for their interpolation. Inverted pendulum system has been chosen to analyze its robustness and performance.**

I. INTRODUCTION

Most of the systems in industry have nonlinear character. There are still usually difficulties to control them and universal control design methodology does not exist. Real system models aren't always precise and have severe uncertainties that are difficult to describe. In the past some design techniques were developed for modeling and control of nonlinear uncertain systems. Very interesting approach were done in the fuzzy modeling and control, especially with Takagi-Sugeno (T-S) fuzzy modeling and related Parallel Distributed Compensation (PDC) control algorithm. This method uses local linear models in the consequent of fuzzy rules. Decision variables are designed to divide the state space of the system into areas, where the linear local models describe precisely the nonlinear system. In the overlapped parts of these areas are local models interpolated according to fuzzy membership functions. Takagi-Sugeno model based control of nonlinear systems is quite popular now for its simple and effective design based usually on Linear Matrix Inequalities (LMIs). Modern techniques, such as the one published in [1] can find a solution of control design problem by optimization of quality criteria function as is quadratic error minimization or H_2 control design.

The robustness of the controller is also very important property if model parameters are uncertain or change in time or if the initial state variables are not known. Robustness in fuzzy model based control has been extensively studied in the past, such as the stability robustness against modeling errors or Hinf control. Some robust control techniques for systems described by Takagi-Sugeno fuzzy model were published in [2] and [3]. A drawback of robust controllers is that they are usually slower than the controller, which is designed for an exact model of the system and work on it.

Another important problem is that state variables of the system are often unavailable. For T-S systems, where the decision variables are often derived from state variables, it is necessary to design an observer based control. Authors of [4], [5] and [6] studied fuzzy observer designs for T-S fuzzy control systems, and they proved that a state feedback controller and an observer always yields a stabilizing output feedback control provided that the stabilizing property of the control and asymptotic convergence of the observer are guaranteed by the Lyapunov method. The paper [2] shows a methodology of observer and controller design and the conditions in the LMI format. However, inaccuracy found in this method yields to problems in controller and observer design. Hence the method is also presented here in the right form.

The optimal PDC controller can be found by other methods. The sub-optimal controller design method [1] for control of nonlinear systems gives the possibility to choose matrixes determining the importance of speed and energy demand during control process.

The inverted pendulum control system is adopted to analyze performance and robustness of both methods. The observer and control system capability to handle with different initial conditions were tested and evaluated also for both designed controllers.

II. TAKAGI-SUGENO FUZZY MODELLING OF SYSTEMS WITH PARAMETRIC UNCERTAINTIES

The standard Takagi-Sugeno fuzzy model consists of the set of fuzzy rules:

Rule i:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i and … and $z_n(t)$ is M_n^i

$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad y(t) = C_i x(t) \tag{1}$$

where $z^T(t) = [z_1(t),...,z_n(t)]$ are premise variables,

$y^T(t) = [y_1(t),...,y_l(t)]$ is the output vector,

$x^T(t) = [x_1(t),...,x_n(t)]$ is a state vector and

$u^T(t) = [u_1(t),...,u_m(t)]$ is control input vector.

$i = 1,2,\ldots,q$ denotes the area's number. q is the number of areas and thus also of fuzzy rules. M_j^i is a fuzzy set ($M_j^i(z(t))$ is the grade of membership of premise variable $z_j(t)$ in the area number i). m is the number of inputs and l is the number of outputs of T-S fuzzy system. Matrixes $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{l\times n}$ are system, input and output matrixes respectively in the area number i.

Parametric uncertainties in T-S fuzzy model

Parametric uncertainties in the system can be added to the model in the following way:

Rule i:
IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i and … and $z_n(t)$ is M_n^i
THEN $\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)$, (2)
$y(t) = C_i x(t)$

It is clear, that uncertainty in the matrix C_i can be involved into matrixes ΔA_i and ΔB_i.
The defuzzyfied output can be then represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t))[A_i x(t) + B_i u(t)] + \sum_{i=1}^{q} \mu_i(z(t))[\Delta A_i x(t) + \Delta B_i u(t)] \quad (3)$$

$$y(t) = \sum_{i=1}^{q} \mu_i(z(t))C_i x(t)$$

where $\mu_i(z(t)) = \dfrac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}$

and $w_i(z(t)) = \prod_{j=1}^{n} M_j^i(z_j(t))$.

If *z(t)* is in specified range, then holds that

$$\sum_{i=1}^{q} \mu_i(z(t)) = 1$$

This representation of defuzzyfied model can be easily implemented into Matlab model.

Assumption 1: To incorporate the parameter uncertainties into design method, it is assumed, that they are norm-bounded, in the form:

$$(\Delta A_i, \Delta B_i) = D_i F_i(t)[E_{1i}, E_{2i}] \quad (4)$$

$$F_i^T(t)F_i(y) \le I \quad (5)$$

where D_i, E_{1i} and E_{2i} are known real constant matrices of appropriate dimension, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and I is the identity matrix of appropriate dimension.

III. FUZZY STATE OBSERVER AND ROBUST PDC FUZZY CONTROL ALGORITHM

The fuzzy rules for the PDC controller are similar to that for T-S model:

Controller rule i:
IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i and … and $z_n(t)$ is M_n^i (6)
THEN $u(t) = -K_i x(t)$,

where $K_i \in R^{m \times n}$ is a constant feedback gain to be determined.
After defuzzyfication we get:

$$u(t) = -\sum_{i=1}^{q} \mu_i(z(t))K_i x(t) \quad (7)$$

If we do not measure all states of the system, then we need a fuzzy state observer to reconstruct the state variables and eliminate parametric uncertainties. It will be defined in the following form:

Observer rule i:
IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i and … and $z_n(t)$ is M_n^i
THEN $\dot{\hat{x}}(t) = A_i x(t) + B_i u(t) + G_i[y(t) - \hat{y}(t)]$, (8)
$\hat{y}(t) = C_i \hat{x}(t)$

where $G_i \in R^{n \times l}$ is a constant observer gain to be determined.
After defuzzyfication we get a fuzzy state observer represented as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{q} \mu_i(z(t))\{A_i x(t) + B_i u(t) + G_i[y(t) - \hat{y}(t)]\}, \quad (9)$$

$$\hat{y}(t) = \sum_{i=1}^{q} \mu_i(z(t))C_i \hat{x}(t)$$

We can define observation error as

$$e(t) = x(t) - \hat{x}(t) \quad (10)$$

Our objective is to design a fuzzy output feedback controller based on the observer and Takagi-Sugeno fuzzy model, which will provide robust stabilization of the system in the form

$$u(t) = -\sum_{i-1}^{q} \mu_i(z(t))K_i \hat{x}(t) \quad (11)$$

We can incorporate the control signal as follows.

$$\dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))\{(B_i + \Delta B_i)K_j e(t) + [A_i + \Delta A_i - (B_i + \Delta B_i)K_j]x(t)\} \quad (12)$$

$$\dot{\hat{x}}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))[G_i C_j e(t) + (A_i - B_i K_j)x(t)] \quad (13)$$

$$\dot{e}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))[(\Delta A_i - \Delta B_i K_j)x(t) + (A_i + \Delta B_i K_j - G_i C_j)e(t)] \quad (14)$$

The stability conditions for the system with parametric uncertainties are summarized in the following theorem:

Theorem 1: If there exist symmetric and positive definite matrixes P_1 and P_2, some matrixes $K_i = M_i Q^{-1}$ and $G_i = P_2^{-1} N_i$, and scalars ε_{ij}, *(i, j = 1,…,q)*, such that the following LMIs are satisfied, then the T–S fuzzy system is asymptotically stabilizable via the T–S fuzzy-model-based output-feedback controller:

$$\begin{bmatrix} \Phi_{ii} & * & * \\ E_{1i}Q - E_{2i}M_i & -(\varepsilon_{ii}^{-1}+1)^{-1}I & * \\ D_i^T & 0 & -(\varepsilon_{ii}^{-1}+1)^{-1}I \end{bmatrix} < 0 \quad (15)$$

$$\begin{bmatrix} T_{ii} & * & * \\ E_{1i} - E_{2i}K_i & -(\varepsilon_{ii}^{-1}+1)^{-1}I & * \\ D_i^T P_2 & 0 & -(\varepsilon_{ii}^{-1}+1)^{-1}I \end{bmatrix} < 0 \quad (16)$$

$(1 \le i \le q)$

$$\begin{bmatrix} \Psi_{ij} & * & * & * & * \\ E_{1i}Q - E_{2i}M_j & -\xi_{ij}^{-1}I & * & * & * \\ E_{1j}Q - E_{2j}M_i & 0 & -\xi_{ij}^{-1}I & * & * \\ D_i^T & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^T & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix} < 0 \quad (17)$$

$$\begin{bmatrix} \Xi_{ij} & * & * & * & * \\ -E_{2i}K_j & -\xi_{ij}^{-1}I & * & * & * \\ -E_{2j}K_i & 0 & -\xi_{ij}^{-1}I & * & * \\ D_i^T & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^T & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix} < 0 \quad (18)$$

$(1 \le i < j \le q)$

where the control objective is to minimize *trace(Q)* and

$\xi = [(\frac{\varepsilon_{ij}}{2})^{-1} + \frac{1}{2}]$.

$\Phi_{ii} = QA_i^T + A_iQ - M_i^T B_i^T - B_iM_i + I$,

$\Psi_{ij} = QA_i^T + A_iQ - M_j^T B_i^T - B_jM_i +$

$\quad + QA_j^T + A_jQ - M_i^T B_j^T - B_iM_j + I,$

$T_{ii} = A_i^T P_2 + P_2A_i - C_i^T N_i^T - N_iC_i + K_i^T B_i^T B_iK_i,$

$\Xi_{ij} = A_i^T P_2 + P_2A_i - N_j^T B_i^T - B_jN_i + A_j^T P_2 + P_2A_j -$

$\quad - N_i^T B_j^T - B_iN_j + \dfrac{K_i^T B_j^T B_jK_i + K_j^T B_i^T B_iK_j}{2}.$

IV. SUB-OPTIMAL FUZZY PDC CONTROL ALGORITHM

The optimization control objective is to minimize

$$J = \sum_{k=0}^{\infty} \left(y^T(t)Wy(t) + u^T(t)Ru(t) \right) \quad (19)$$

where $W = W^T > 0$ a $R = R^T > 0$.

The objective J depends also on initial conditions x(0). Following LMIs lead to the sub-optimal PDC controller.

$$\begin{bmatrix} 1 & x^T(0) \\ x(0) & Q \end{bmatrix} \ge 0, \quad (20)$$

$$\begin{bmatrix} D & QC_i^T W^{\frac{1}{2}} & M_i^T R^{\frac{1}{2}} \\ W^{1/2}C_iQ & \gamma I & 0 \\ R^{1/2}M_i & 0 & \gamma I \end{bmatrix} > 0, \quad (21)$$

$$\begin{bmatrix} T & QC_i^T W^{\frac{1}{2}} & QC_j^T W^{\frac{1}{2}} & M_i^T R^{\frac{1}{2}} & M_j^T R^{\frac{1}{2}} \\ W^{1/2}C_iQ & \gamma I & 0 & 0 & 0 \\ W^{1/2}C_jQ & 0 & \gamma I & 0 & 0 \\ R^{1/2}M_i & 0 & 0 & \gamma I & 0 \\ R^{1/2}M_j & 0 & 0 & 0 & \gamma I \end{bmatrix} > 0, \quad (22)$$

where the objective is to minimize γ and

$D = -A_iQ - QA_i^T - B_iM_i - M_i^T B_i^T$

$T = -A_iQ - A_jQ - QA_i^T - QA_j^T - B_iY_j - B_jY_i - Y_i^T B_j^T - Y_j^T B_i^T$

The sub-optimal controller gain can be then computed as

$$K_i = -M_iQ^{-1} \quad (23)$$

IV. ENHANCED ROBUST PDC CONTROL

As it is shown above, both robust and optimal PDC controllers are described by fuzzy rules with the same antecedent. Mathematical description is the same and they can be interpolated. Each controller computes its output and they are weighted according to model validity appreciation. The fuzzy rules for interpolation are:

Rule 1: IF $z_{V1}(t)$ is M_{V1}^1 and $z_{V2}(t)$ is M_{V2}^1 and ...
and $z_{Vn}(t)$ is M_{Vn}^1 (24)
THEN $u(t) = -K_{Oi}x(t)$,

Rule 2: IF $z_{V1}(t)$ is M_{V1}^2 and $z_{V2}(t)$ is M_{V2}^2 and ...
and $z_{Vn}(t)$ is M_{Vn}^2 (25)
THEN $u(t) = -K_{Ri}x(t)$,

where $K_{Oi} \in R^{m \times n}$ and $K_{Ri} \in R^{m \times n}$ are a constant feedback gain of sub-optimal and robust controller respectively.

After defuzzyfication the final control output is computed as

$$u(t) = -\mu_O(z_V)\sum_{i=1}^{m}\mu_i(z)K_{Oi}x(t) - \\ -\mu_R(z_V)\sum_{i=1}^{m}\mu_i(z)K_{Ri}x(t) \quad (26)$$

Weights μ_i are same, as in (3) and μ_R and μ_O are normalized by

$$\mu_O(z_V) = \frac{w_O(z_V)}{w_R(z_V) + w_O(z_V)}, \quad \mu_R(z_V) = \frac{w_R(z_V)}{w_R(z_V) + w_O(z_V)}$$

where $\mu_R(t)$ and $\mu_O(t)$ are fuzzy membership functions that come from model validity analysis.

The scheme of the control system can be seen on Fig. 1.

IV. INVERSE PENDULUM FUZZY CONTROL

To demonstrate the design methods, the inverted pendulum system is adopted. Simulation has been done in Matlab Simulink. The system is modeled by the following T-S model:

Plant rule 1:
IF $x_1(t)$ is about 0
THEN $\dot{x}(t) = (A_1 + \Delta A_1)x(t) + B_1u(t)$, (27)
$y(t) = C_1x(t)$

Plant rule 2:
IF $x_1(t)$ is about $\pm\frac{\pi}{3}$
THEN $\dot{x}(t) = (A_2 + \Delta A_2)x(t) + B_2u(t)$, (28)
$y(t) = C_2x(t)$

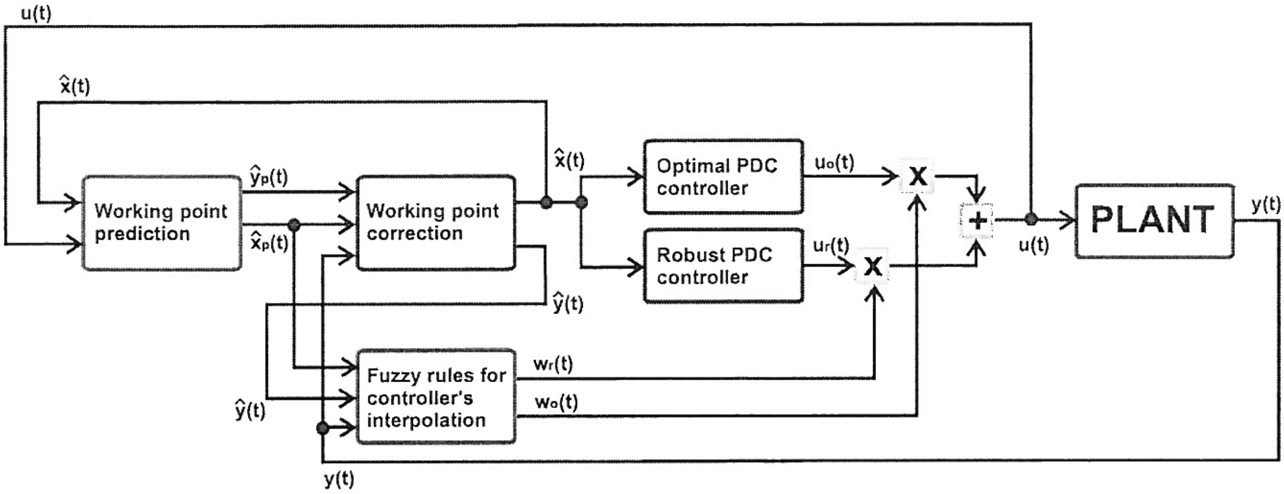

Figure 1: Enhanced Robust PDC Control schematic

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 29.2529 & -0.3149 & 0 & 44.1811 \\ 0 & 0 & 0 & 1 \\ -1.2637 & -0.0041 & 0 & -16.7096 \end{bmatrix} \quad (29)$$

$$B_1 = \begin{bmatrix} 0 \\ -1.9280 \\ 0 \\ 0.7292 \end{bmatrix} \quad (30)$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 22.0587 & -0.2872 & 0 & 20.1425 \\ 0 & 0 & 0 & 1 \\ -0.4765 & -0.0019 & 0 & -15.2361 \end{bmatrix} \quad (31)$$

$$B_2 = \begin{bmatrix} 0 \\ -0.8790 \\ 0 \\ 0.6649 \end{bmatrix} \quad (32)$$

Membership functions for plant areas are

$$\mu_1(x_1(t)) = \left\{1 - \frac{1}{1+e^{-7[x_1(t)-\pi/6]}}\right\}\frac{1}{1+e^{-7[x_1(t)+\pi/6]}} \quad (33)$$

$$\mu_2(x_1(t)) = 1 - \mu_1(x_1(t)) \quad (34)$$

Robust controller

It was designed to be robust for parametric uncertainties within the range of 30% of the nominal values of matrixes A_1 and A_2. Based on assumption 1 were defined

$D_1 = D_2 = 0.3I$,

$E_{11} = E_{12} = 15I$,

$E_{21} = E_{22} = 0$

Feedback gain K_{Ri} and observer gain G_{Ri} matrices can be obtained by performing Theorem 1:

K_{R1} = [-69.1254 -11.2047 -7.8689 -34.0224] (35)

K_{R2} = [-154.1245 -30.2409 -9.8612 -37.0122] (36)

$$G_1 = \begin{bmatrix} 69.042 & 34.682 \\ 1198.1 & 1609.3 \\ 0.671 & 45.766 \\ 11.991 & 234.79 \end{bmatrix}, G_2 = \begin{bmatrix} 68.149 & 14.392 \\ 1108.1 & 609.31 \\ 0.4509 & 49.166 \\ 10.566 & 284.09 \end{bmatrix} \quad (37)$$

Sub-optimal controller

The gain for sub-optimal controller has been computed by solving LMIs (20-22) for $x(0)=[\frac{\pi}{3} 0 0 0]$:

K_{O1} = [-259.2652 -53.0929 -70.1098 -74.7717] (38)

K_{O2} = [-188.1872 -38.2351 -46.7570 -58.5087]

4 SIMULATION RESULTS

Both robust and sub-optimal control system was tested under various conditions. Its A_i matrix was multiplied by the coefficient *mulA* to show the robustness against parametric uncertainties.

$$A_{ipert} = mulA \cdot A_i$$

Initial conditions of the system $x(0)$ and of the model $\bar{x}(0)$ were set different to prove, how the observer adjusts the state space variables.

At first the simulation results for ***robust control system*** is presented. They are on the following figures.

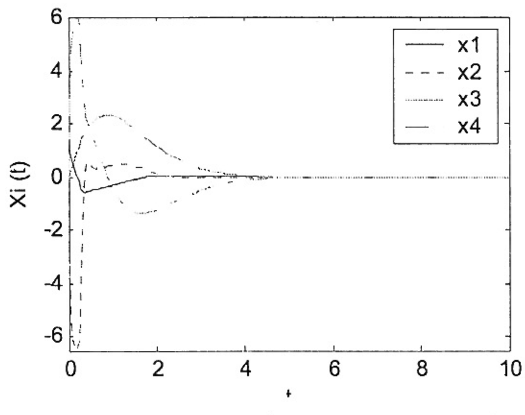

Figure 2: Response of the system without perturbations and with the same initial conditions $mulA = 1$, $x(0) = \bar{x}(0) = [\frac{\pi}{3}\ 0\ 0\ 0]$

Next figures show simulation results, when the system matrix was perturbed by 30% error in both sides.

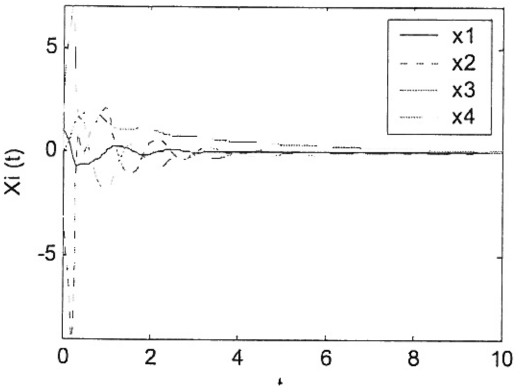

Figure 3. $mulA = 0.7,\ x(0) = \hat{x}(0) = [\frac{\pi}{3}\ 0\ 0\ 0]$

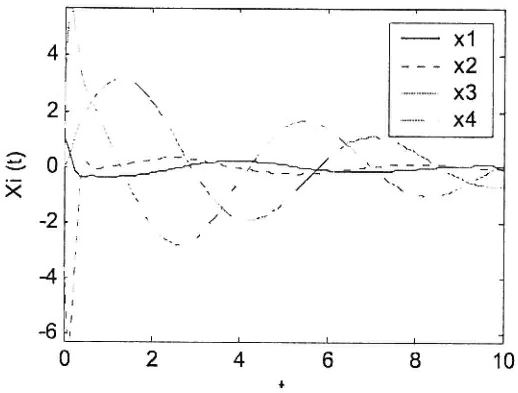

Figure 4. $mulA = 1.3,\ x(0) = \hat{x}(0) = [\frac{\pi}{3}\ 0\ 0\ 0]$

These results are worse, but still stabilizing the system. They prove our assumption on experimental base. The mathematical proof can be found in [1].

The following figures 5. and 6. show, how can the observer handle with the difference between initial conditions of the system and of the model. Initial conditions of the model are set to zero as if the initial conditions of the system were unknown. Uncertainties in angle and position were tested.

The area of initial condition of state variables x_1 (angle) and x_3 (position of the cart) for which the control process is stable is on the Figure 7. This result has been acquired by simulation in Matlab.

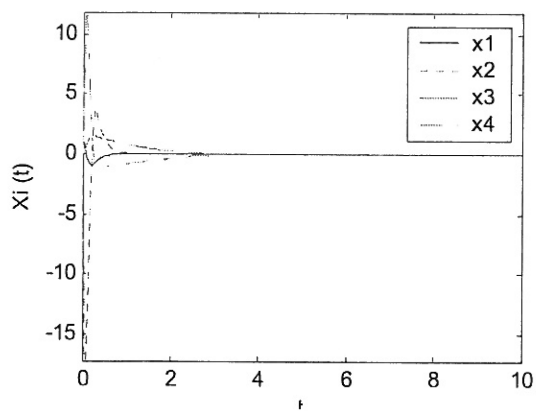

Figure 5. $mulA = 1,\ x(0)=[\frac{\pi}{6}000], \hat{x}(0)=[0000]$

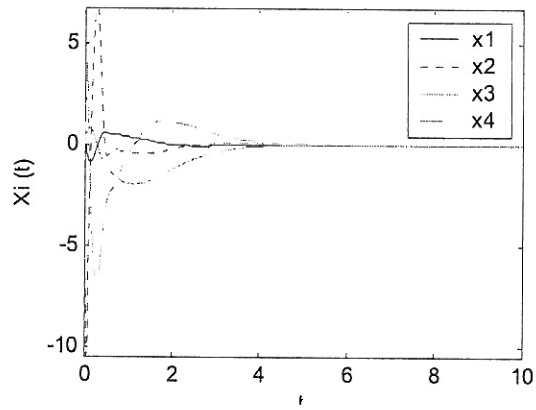

Figure 6. $mulA = 1, x(0) = [0\ 0\ 0.5\ 0], \hat{x}(0) = [0\ 0\ 0\ 0]$

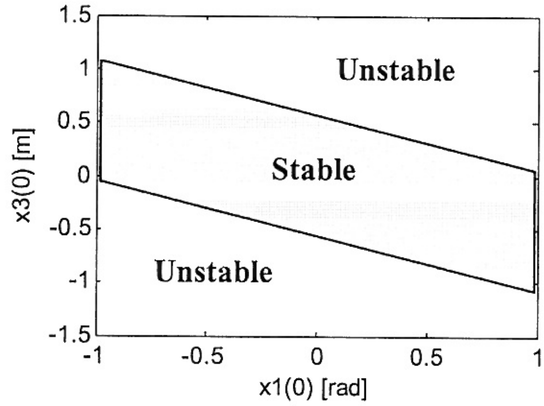

Figure 7. Initial conditions for stable control process for robust controller

For <u>sub-optimal control system</u> the next results have been acquired.

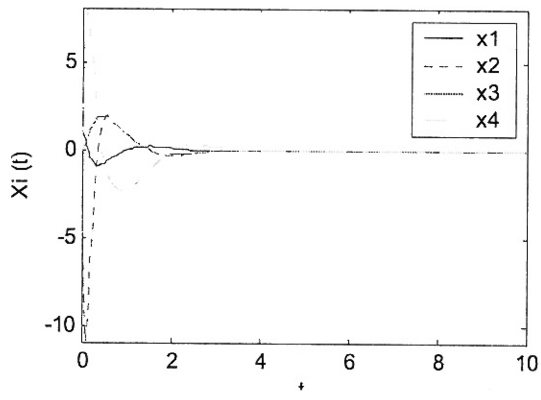

Figure 8: Response of the system without perturbations and with the same initial conditions $mulA = 1,\ x(0) = \hat{x}(0) = [\frac{\pi}{3}\ 0\ 0\ 0]$

The response of sub-optimal control system is here much better than of the robust controller. The system is still stable for *mulA* = 1.3, as is shown in the Figure 9, but it is unstable for *mulA* < 0.76. The required robustness is not maintained.

The stabilizing area of initial conditions with mulA=1 is on the Figure 10. It is smaller than the area for the robust controller. The optimal controller is then less robust against the error in initial conditions estimation.

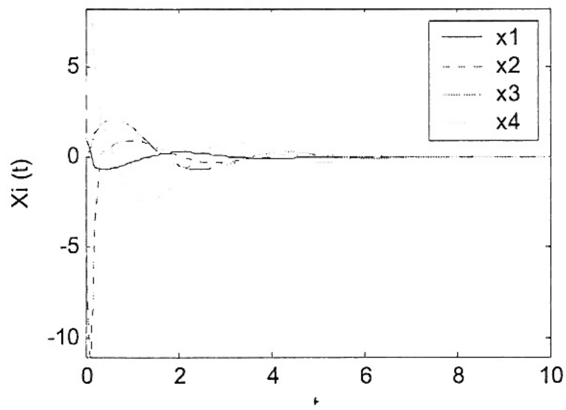

Figure 9. $mulA$=1.3, $x(0) = \hat{x}(0) = [\frac{\pi}{3}\ 0\ 0\ 0]$

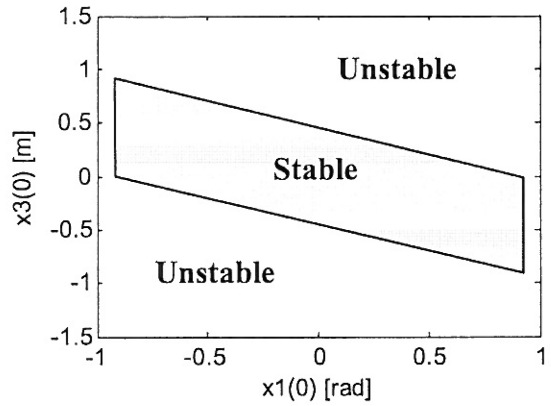

Figure 10. Initial conditions for stable control process with sub-optimal controller

5 CONCLUSION

The original method of observer based fuzzy control algorithm for systems with parameter's uncertainties have been presented. This method interpolates the robust and sub-optimal controller to increase the performance of robust control. The robust and sub-optimal controllers were designed for inverted pendulum benchmark problem and experimental examinations of control robustness against parametric uncertainties were done. Also the behavior of the control system was studied when the initial conditions of the system and its model were different. Some important conclusions implies from this design and experiments.

- The first proposed method leads to stabilizing robust controller and observer for parameters in decided boundaries.
- The second method leads to the controller that is faster, but does not maintain required robustness.
- The initial conditions of the system and the model have to be taken in mind since the high difference among them leads to unstable control.
- The area of initial condition for stable control process is presented.

These methods are based on Takagi-Sugeno fuzzy modeling and control that is continuously developing area, which has a great potential for practical realization.

Identification of the system for construction of T-S model can be done with help of program toolbox FMID for Matlab. It constructs the model from response of the system on some input signal. We are thus able to model a system with a good accuracy, even if we do not have much information about it. By T-S model we can represent almost any linear or nonlinear system.

Linear Matrix Inequalities (LMIs) are used for fast control algorithm design. They can be effectively solved for example in the LMI Control Toolbox for Matlab.

Future development of the method for quality improvement of robust controller will be concerned on finding the appropriate method for online model verification and for robust and sub-optimal controller interpolation.

This method brings a new view into this area, that can be used, when both, robustness and high performance of the control is required.

ACKNOWLEDGMENTS

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