On the Development of Mamdani PI-Fuzzy Controllers for a Class of Mobile Robots

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Abstract – **The paper deals with the development of Mamdani PI-fuzzy controllers (PI-FCs) meant for solving the tracking problem in a class of nonholonomic mobile robots. A simplified dynamic model, that can well characterize the class of wheeled mobile robots with two degrees of freedom, is first proposed. The control system structure contains two control loops for controlling the forward velocity and the angle between the heading direction and the *x*-axis. The reference trajectory of the robot is based on an obstacle avoidance strategy employing the artificial potential field method. In addition, there is proposed a development method for the PI-FCs employing the Extended Symmetrical Optimum method applied to the basic linear PI controllers. Simulation results validate the proposed PI-FCs as tracking controllers.**

I. INTRODUCTION

The control of nonholonomic mobile robots has received much research interest due to the implications of the nonholonomic constraints on the admissible control inputs of these systems belonging to the general class of nonsmooth or nonholonomic systems [1], [2].

In the general context of constructing mobile robots which are capable of performing various and complex tasks in an autonomous and intelligent way it is important to develop high performance controllers to cope with the three basic navigation problems [3], tracking control (tracking a reference trajectory), path following and point stabilization. The tracking control problem can be further divided in the local and global tracking problems [4].

The majority of controllers developed for nonholonomic mobile robots is based on kinematic or dynamic models [5], [6]. But, the dynamic models do not exploit the dynamics of the actuators, of the measuring devices and of the control equipment as part of the control system. This comes to the first aim of the paper, to propose a simplified dynamic model that can well characterize the class of wheeled mobile robots with two degrees of freedom.

The current approaches to solving tracking control problems include general nonlinear techniques [7] including the popular backstepping [2], [4], the sliding mode approach [5], linear model or passivity based approaches [8], [9], the control Lyapunov function approach [10]. Since the development of the controllers based on these approaches is rather complex, it is necessary to simplify the controller development and the further implementation. This aspect results in the second aim of this paper, to offer a development method for Mamdani PI-fuzzy controllers (PI-FCs) used as tracking controllers. The development method is based on applying the Extended Symmetrical Optimum (ESO) method [11] to the basic linear PI controllers and by adding nonlinear features specific to the fuzzy blocks in order to improve the control system performance.

The control system structure proposed here to solve the tracking control problem contains two control loops for controlling the forward velocity and the angle between the heading direction and the *x*-axis. The reference inputs for these two control loops are obtained by first applying the artificial potential field method used in obstacle avoidance [12] for generating the reference trajectory of the robot. Then, there are performed simple computations that employ the tracking errors for the *x*- and *y*- axes and the maximum accepted values for these errors.

This paper addresses the topics as follows. In the following Section there is proposed the dynamic model used in the tracking control problem for the considered class of mobile robots together with the control system structure. In addition, a formal description of the artificial potential field method is offered. Section III is dedicated to the presentation of the development method for the Mamdani PI-FCs playing the role of tracking controllers. Section IV offers simulation results for a case study to validate the theoretic part. Section V gives the conclusions.

II. DYNAMIC MODEL OF A CLASS OF MOBILE ROBOTS. CONTROL SYSTEM STRUCTURE

The dynamic model of the class of wheeled mobile robots with two degrees of freedom (Fig. 1) can be expressed in terms of (1) by introducing the dynamics to the kinematic model [6] and by extending the dynamic model in [13]:

$$\begin{aligned} &\dot{x} = v \cos \theta \\ &\dot{y} = v \sin \theta \\ &\dot{v} = a_v \\ &\dot{\theta} = \omega \\ &T_{\Sigma 1}\dot{a}_v + a_v = k_{P1}(u_1 + d_1) \\ &T_{\Sigma 2}\dot{\omega} + \omega = k_{P2}(u_2 + d_2) \end{aligned} \quad , \qquad (1)$$

where: (x, y) – the coordinates of the centre of the rear axis of the mobile robot; v and a_v – the forward velocity and acceleration, respectively; θ – the angle between the heading direction and the *x*-axis; ω – the angular velocity; u_1 and u_2 – the control signals, proportional to the torques or to the generalized force variables depending on the actuator types; d_1 and d_2 – disturbance inputs due to the contact with the robot environment; k_{P1} and k_{P2} – gains; $T_{\Sigma 1}$ and $T_{\Sigma 2}$ – small time constants or time constants equivalent to the cumulative effects of the actuator dynamics, the measuring device dynamics, the control equipment dynamics and of the parasitic time constants.

The structure of the model (1) can be illustrated by means of the block diagram of the controlled plant (Fig. 2), that outlines its dynamic subsystems having the transfer functions $H_{P1}(s)$ and $H_{P2}(s)$ of the controlled plant:

$$H_{Pi}(s) = k_{Pi}/[s(1+T_{\Sigma i}s)],\ i=\overline{1,2}, \tag{2}$$

and its kinematic subsystem (KS).

The control system structure employed in solving the tracking control problem for mobile robots with two degrees of freedom is presented in Fig. 3, where: C-1 and C-2 – the forward velocity controller and angle controller, respectively; v_r and θ_r – the reference inputs for the two control loops; RF-1 and RF-2 – the reference filters; $\tilde{v}_r$ and $\tilde{\theta}_r$ – the filtered reference inputs for the two control loops; $e_1=\tilde{v}_r - v$ and $e_2=\tilde{\theta}_r-\theta$ – the control errors; x_r and y_r – the reference positions for x and y, respectively; $e_x = x_r - x$ and $e_y = y_r - y$ – the tracking errors for x and y, respectively; CB-xp and CB-yp – computation blocks providing the estimates $\hat{\dot{x}}_r$ and $\hat{\dot{y}}_r$ of the derivatives $\dot{x}_r$ and $\dot{y}_r$, respectively; $\Delta x_{r,k}$ and $\Delta y_{r,k}$ – the increments of the reference positions x_r and y_r, respectively; k – the index of the current sampling interval.

The proposed control system structure is a cascaded one, with the two inner loops to control v and θ, and the outer loops to provide the reference inputs for the inner loops by means of the blocks CB-xp, CB-yp operating on the basis of the following algorithms:

$$\hat{\dot{x}}_{r,k} = \begin{cases} \Delta x_{r,k}/h & \text{if } |e_{x,k}| \le \varepsilon_x \\ e_{x,k}/h & \text{otherwise} \end{cases}, \tag{3}$$

$$\hat{\dot{y}}_{r,k} = \begin{cases} \Delta y_{r,k}/h & \text{if } |e_{y,k}| \le \varepsilon_y \\ e_{y,k}/h & \text{otherwise} \end{cases}. \tag{4}$$

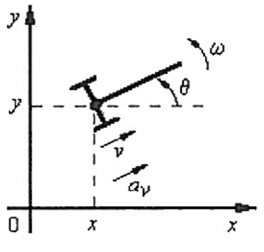

Fig. 1. Mechanical variables of mobile robots with two degrees of freedom

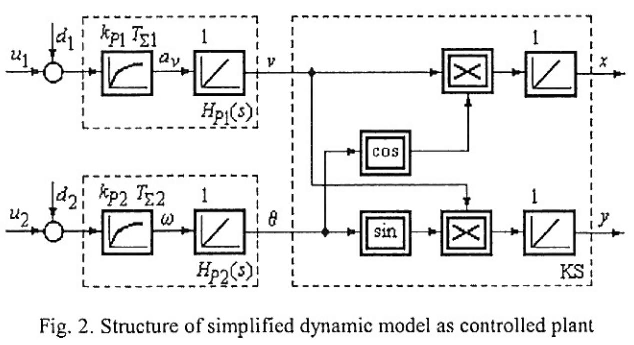

Fig. 2. Structure of simplified dynamic model as controlled plant

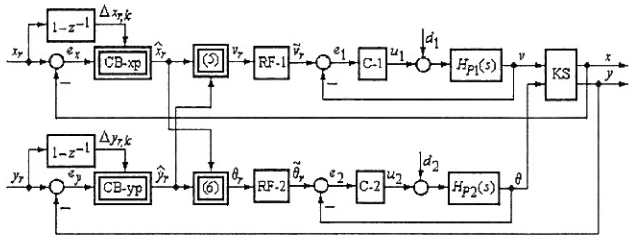

Fig. 3. Control system structure

where: h – the sampling interval; $\varepsilon_x > 0$ and $\varepsilon_y > 0$ – the maximum accepted absolute values of the tracking errors corresponding to the positions x and y. The values of ε_x and ε_y must be specified by the control systems specialist as function of the desired control system performance.

The blocks CB-xp and CB-yp are in fact estimators of derivatives. As it can be seen in (3), (4) and Fig. 3, the feedback in terms of x and y (in the outer control loops) operates only when the absolute values of the tracking errors e_x and e_y exceed the values of ε_x and ε_y, respectively.

For the computation of the reference inputs v_r (the desired value of v) and θ_r (the desired value of θ) fed to the two inner control loops, there can be used the first two equations in (1), modified as follows:

$$v_r = \sqrt{(\hat{\dot{x}}_r)^2 + (\hat{\dot{y}}_r)^2}, \tag{5}$$

$$\theta_r = \tan^{-1}(\hat{\dot{y}}_r / \hat{\dot{x}}_r). \tag{6}$$

The nonlinear blocks (5) and (6) in Fig. 3 operate on the basis of the equations (5) and (6), respectively.

The generation of the reference trajectory (x_r, y_r) for the control system structure in Fig. 3 represents an important task. In the initial phase it is performed off-line, and it is based on drawing a virtual potential field to ensure the obstacle avoidance. This application of the artificial potential field method [12] will be presented as follows.

A local harmonic potential field $\Psi(x, y)$ is constructed that satisfies the Laplace equation (7):

$$\nabla^T \cdot \nabla\Psi(x,y) = \partial^2\Psi(x,y)/\partial x^2 + \partial^2\Psi(x,y)/\partial y^2 = 0. \tag{7}$$

The solution of (7) gives the potential of a singular point of strength q in (0, 0):

$$\Psi(x,y) = -0.5q\cdot\ln(x^2+y^2), \tag{8}$$

and the associated gradient $\rho(x, y) \in R^2$:

$$\rho(x,y) = -\mathrm{grad}\Psi(x,y) = [q/(x^2+y^2)](x,y)^T = (\rho_x, \rho_y) \in R^2, \tag{9}$$

where the upper index T stands for matrix transposition. A fundamental potential field configuration consists of a negative unit singular point in the goal and a positive singular point of strength $0<q<1$ in the obstacle center $q = R/(R+e)$ from the equivalent point placement method, where e is the distance between the goal point and the obstacle center, and R is the radius of the circular obstacle security zone.

By using the artificial potential field method, a collision free trajectory along the vector field lines is guaranteed. The obstacle avoidance aim is to control the orientation of the angle θ to be co-linear to the gradient $\rho(x, y)$. So, the reference input θ_r at (x, y) will be:

$$\theta_r = \tan^{-1}(\rho_y / \rho_x). \tag{10}$$

The desired direction of motion representing the reference input v_r is determined by introducing the additional variable γ, defined as function of the angle error, $e_\theta = \theta_r - \theta$:

$$\gamma = \mathrm{sgn}(v_r),\ \gamma = \begin{cases} 1 & \text{if} \quad -2\pi \le e_\theta < -3\pi/2 \\ -1 & \text{if} \quad -3\pi/2 \le e_\theta < -\pi/2 \\ 1 & \text{if} \quad -\pi/2 \le e_\theta < \pi/2 \\ -1 & \text{if} \quad \pi/2 \le e_\theta < 3\pi/2 \\ 1 & \text{if} \quad 3\pi/2 \le e_\theta < 2\pi \end{cases} . \tag{11}$$

The reference inputs obtained by using (9) ... (11), θ_r and v_r, can be fed directly to the two inner control loops. But, the aim is to solve the tracking control problem and this is the reason why there is necessary to employ the control system structure in Fig. 3 having the reference inputs x_r and y_r and taking the feedback in terms of the actual trajectory of the robot, (x, y).

Finally, the reference trajectory (x_r, y_r) will be obtained by the integration the following equations of motion in terms of using adequate numerical techniques:

$$\begin{aligned} \dot{x}_r &= v_r[\rho_x / \sqrt{(\rho_x)^2 + (\rho_x)^2}]\cos\theta_r \\ \dot{y}_r &= v_r[\rho_y / \sqrt{(\rho_x)^2 + (\rho_x)^2}]\sin\theta_r \end{aligned} . \tag{12}$$

The model of the robot and the control system structure proposed here are quite different from the models and control structures using the reference and actual posture determined by (x, y, θ) [14] because θ is in fact not directly controlled, but it results with acceptable precision since the tracking at the level of x and y is improved.

III. DEVELOPMENT METHOD FOR MAMDANI PI-FUZZY CONTROLLERS

Both Mamdani PI-FCs used as controllers C-1 and C-2 in Fig. 3 have the same structure presented in Fig. 4 and based on adding the dynamics to the basic fuzzy controllers without dynamics $\mathrm{FC_i}$, $i=\overline{1,2}$, by the numerical differentiation of the control error $e_{i,k}$ as the increment of control error, $\Delta e_{i,k} = e_{i,k} - e_{i,k-1}$, and the numerical integration of the increment of control signal, $\Delta u_{i,k} = u_{i,k} - u_{i,k-1}$, $i=\overline{1,2}$.

The development of the Mamdani PI-FCs starts from the development of two basic linear PI controllers. For the considered class of plants with the transfer functions in the forms (2) the use of PI controllers having the transfer function (13):

$$H_{Ci}(s) = \frac{k_{ci}}{s}(1 + sT_{ci}),\ i=\overline{1,2}, \tag{13}$$

with the gains k_{ci} and the integral time constants T_{ci}, tuned in terms of the ESO method [11], can ensure good control system performance when the controllers are tuned in terms of (14):

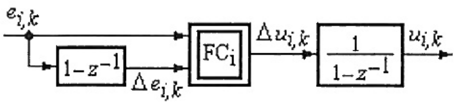

Fig. 4. Structure of Mamdani PI-FCs, $i=\overline{1,2}$

$$k_{ci} = \frac{1}{\sqrt{\beta_i^3 T_{\Sigma i}^2 k_{Pi}}},\ T_{ci} = \beta_i T_{\Sigma i},\ i=\overline{1,2}, \tag{14}$$

where β_i, $i=\overline{1,2}$, represent design parameters, only one for each controller.

By using (2), (13) and (14), the closed-loop transfer functions with respect to the reference inputs and the open-loop transfer functions will obtain their optimal expressions $H_i(s)_{opt}$ and $H_{0i}(s)_{opt}$, respectively:

$$H_i(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i^3} T_\Sigma^3 s^3 + \sqrt{\beta_i^3} T_{\Sigma i}^2 s^2 + \beta_i T_{\Sigma i} s + 1},\ i=\overline{1,2}, \tag{15}$$

$$H_{0i}(s)_{opt} = \frac{1 + \beta_i T_{\Sigma i} s}{\sqrt{\beta_i} s^2 (1 + T_{\Sigma i} s)},\ i=\overline{1,2}. \tag{16}$$

The ESO method represents a generalization of Kessler's Symmetrical Optimum method described in [15], and it performs indeed an optimization by guaranteeing the maximum value of the phase margin for constant controlled plant parameters [11]. Furthermore, the ESO method guarantees a minimum phase margin in the case of variable k_{Pi}.

By the choice of the design parameters β_i in the recommended domain $\beta_i \in (1, 20)$, the control system performance indices {σ_1 – overshoot, $\hat{t}_s = t_s / T_{\Sigma i}$ – settling time, $\hat{t}_1 = t_1 / T_{\Sigma i}$ – first settling time, φ_r – phase margin} can be accordingly modified. A compromise to these indices can be reached by using the diagrams illustrated in Fig. 5. These diagrams have been obtained by processing (15) and (16), and for the sake of simplicity the index i, $i=\overline{1,2}$, has been omitted in the expressions of the control system performance indices.

The control system performance indices can be corrected by suppressing the action of the zero in the open-loop transfer functions. This can be accomplished by adding the reference filters RF-1 and RF-2 with the transfer functions $H_{RF1}(s)$ and $H_{RF2}(s)$, respectively [11]:

$$H_{RFi}(s) = \frac{1}{1 + \beta_i T_{\Sigma i} s},\ i=\overline{1,2}. \tag{17}$$

For the development of the PI-FCs the continuous-time PI controllers (13) are discretized resulting in the discrete-time equations of the PI quasi-continuous digital controllers expressed in their incremental versions (18):

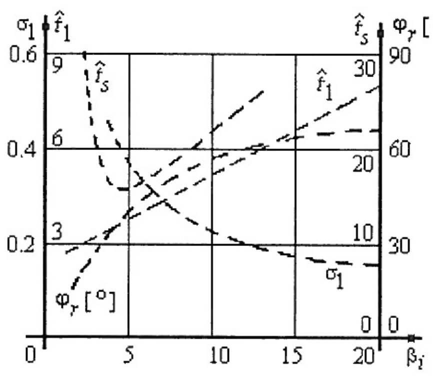

Fig. 5. Control system performance indices versus β_i, $i=\overline{1,2}$

$$\Delta u_{i,k} = K_{Pi}\,\Delta e_{i,k} + K_{Ii}\,e_{i,k} = K_{Pi}(\Delta e_{i,k} + \lambda_i \cdot e_{i,k}),\ i=\overline{1,2}, \tag{18}$$

where the parameters $\{K_{Pi}, K_{Ii}, \lambda_i\}$ depend on $\{k_{ci}, T_{ci}\}$:

$$K_{Pi}=k_{ci}T_{ci}[1-(h/2T_{ci})],\ K_{Ii}=k_{ci}h,\ \lambda_i = K_{Ii}/K_{Pi} = 2h/(2T_{ci}-h),\ i=\overline{1,2}. \tag{19}$$

The fuzzification in the basic fuzzy controllers without dynamics FC_i, $i=\overline{1,2}$, can be solved in the initial phase as follows (Fig. 6):

- for the input variables, $e_{i,k}$ and $\Delta e_{i,k}$, five linguistic terms with regularly distributed triangular type membership functions having an overlap of 1 are chosen;
- for the output variable $\Delta u_{i,k}$, seven linguistic terms with non-regularly distributed singleton type membership functions are chosen.

Other shapes of membership functions can contribute to control system performance enhancement.

The (strictly positive) parameters of the Mamdani PI-FCs are $\{B_{ei}, B_{\Delta ei}, B_{\Delta ui}, m_i, n_i, p_i\}$. The parameters B_{ei}, $B_{\Delta ei}$ and $B_{\Delta ui}$ are in correlation with the shapes of the input and output membership functions. The parameters m_i, n_i and p_i, $m_i < n_i < p_i$, have been added to the standard version of PI-FCs to improve the control system performance by modifying the input-output static map of the blocks FC_i.

The inference engine of the blocks FC_i employs the Mamdani's MAX-MIN compositional rule of inference assisted by complete rule bases expressed as decision tables (Table I), and the centre of gravity method is used for defuzzification.

The development method for the two Mamdani PI-FCs consists of the following development steps:

- express the last two equations in the simplified dynamic model (1) of the mobile robot in terms of the mathematical models of the subsystems of the controlled plant with the transfer functions in (2), $H_{P1}(s)$ and $H_{P2}(s)$, and compute the parameters k_{P1}, k_{P2}, $T_{\Sigma 1}$ and $T_{\Sigma 2}$;

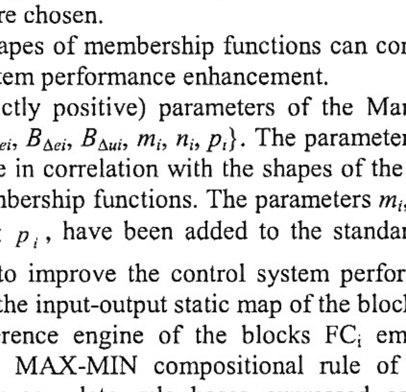

Fig. 6. Accepted initial membership functions of FC_i, $i=\overline{1,2}$

TABLE I

DECISION TABLE OF FC_i, $i=\overline{1,2}$

$\Delta e_{i,k} \backslash e_{i,k}$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

- choose the values of the design parameters β_1 and β_2 as function of the desired control system performance indices by using the diagrams in Fig. 5 and the maximum accepted absolute values ε_x and ε_y of the tracking errors corresponding to the positions x and y, respectively;
- tune the parameters of the basic continuous-time PI controllers $\{k_{c1}, T_{c1}\}$ (for C-1) and $\{k_{c2}, T_{c2}\}$ (for C-2) by (14);
- add to the control system structure the reference filters RF-1 and RF-2 (for example, the versions (17));
- choose a sufficiently small sampling period, h, accepted by quasi-continuous digital control and take into account the presence of zero-order hold blocks;
- discretize the two continuous-time PI controllers and compute the parameters of the two quasi-continuous digital PI controllers, $\{K_{P1}, K_{I1}, \lambda_1\}$ and $\{K_{P2}, K_{I2}, \lambda_2\}$, by (19);
- choose the values of the parameters m_i, n_i, p_i and B_{ei} of the PI-FCs and apply (20), resulted from the modal equivalence principle [16], to obtain the values of the parameters $B_{\Delta ei}$ and $B_{\Delta ui}$:

$$B_{\Delta ei} = \lambda_i B_{ei},\ B_{\Delta ui} = K_{Ii} B_{ei},\ i=\overline{1,2}. \tag{20}$$

By applying this method for tuning the parameters of the Mamdani PI-FCs, the parameters of the basic linear PI controllers (13), k_{c1}, T_{c1}, k_{c2} and T_{c2}, are taken into consideration in the values of the parameters $B_{\Delta ei}$ and $B_{\Delta ui}$.

Besides the choice of the parameters m_i, n_i, p_i and B_{ei} in accordance with the user's experience in controlling the plant, the choice of the inference method and of the defuzzification method represents the user's option as well and can improve the overall control system performance.

IV. CASE STUDY. DIGITAL SIMULATION RESULTS

The simulation results are obtained by accepting that the simplified dynamic model (1) characterizes the wheeled mobile robot with two degrees of freedom. The considered case study corresponds to the following values of the parameters $k_{P1} = k_{P2} = 1$ and $T_{\Sigma 1} = T_{\Sigma 2} = 1$ s.

There are considered two experiments with different placements of three obstacles. In the experiment 1, the obstacles are placed in the points having the following coordinates: (3, 3), (9, 3), (6, 7), with the potentials 0.3 for all obstacles, and the initial position of the robot is in the point (10, 4). In the experiment 2, the obstacles are placed in the points (1, 4), (10, 8) and (2, 10) having the potentials 0.3, 0.3 and 0.5, and initial position of the robot is (5, 2). For both experiments the goal, representing the desired final position, is placed in the point (6, 11) with the potential −1.

For the considered obstacles and robot positions the application of the artificial potential method (Section II) leads to the vector field and to the reference trajectory (x_r, y_r) presented in Fig. 7 and Fig. 8 for both experiments.

The control system structure is that presented in Fig. 3. The Mamdani PI-FCs playing the roles of C-1 and C-2 are developed and tuned in terms of the method presented in the previous Section. For the accepted case study, in the conditions of $\beta_1=\beta_2=4$ and $h=0.1$ s, the parameters of the PI quasi-continuous digital controllers will obtain the values $K_{P1}=K_{P2}=0.4938$, $K_{I1}=K_{I2}=0.0125$, $\lambda_1=\lambda_2=0.0253$. By choosing $B_{e1}=0.3$, $B_{e2}=0.45$, $m_1=m_2=0.8$, $n_1=n_2=1.2$ and

$p_1=p_2=1.4$, the values of the other parameters of the Mamdani PI-FCs will be tuned as $B_{\Delta e1}=0.0076$, $B_{\Delta u1}=0.0037$, $B_{\Delta e2}=0.0114$, $B_{\Delta u2}=0.0056$.

In both experiments there have been fed the disturbance inputs d_1 and d_2:

$$d_i = \delta_i\sigma(t-110) - \delta_i\sigma(t-220) - \delta_i\sigma(t-440) + \\ + \delta_i\sigma(t-550) + \delta_i\sigma(t-770) - \delta_i\sigma(t-880), \\ t \in [0, t_f],\ i = \overline{1,2}, \tag{21}$$

where: σ – the unit step signal; $\delta_1=0.006$, $\delta_2=0.09$, $t_f=1100$ s in the experiment 1; $\delta_1=0.003$, $\delta_2=0.07$, $t_f=1000$ s in the experiment 2. These variations are acceptable to model the contact of the robot with the environment.

In the conditions of these disturbance inputs and of the reference inputs computed off-line by using (9) ... (11), Fig. 7 and Fig. 8, the synthesis of the digital simulation results for the control system with Mamdani PI-FCs is presented as follows:

- the reference trajectory (dotted line) and the actual trajectory (continuous line) in Fig. 9 and Fig. 10 in the case of the experiments 1 and 2, respectively;
- the variations of the control signal u_1 (defined in Fig. 3) in Fig. 11 and Fig. 12 in the case of the experiments 1 and 2, respectively;
- the variations of the control signal u_2 (defined in Fig. 3) in Fig. 13 and Fig. 14 in the case of the experiments 1 and 2, respectively.

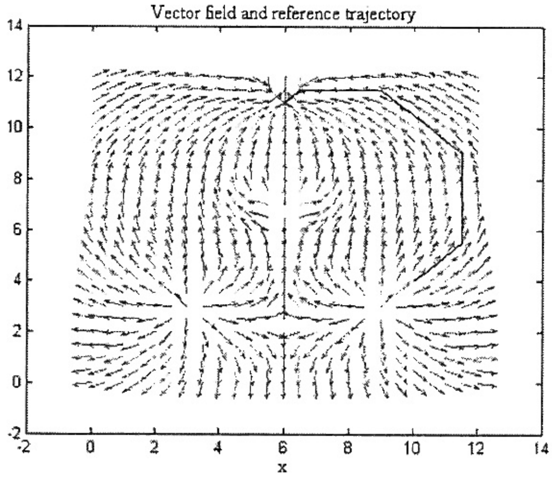

Fig. 7. Vector field and reference trajectory in the experiment 1

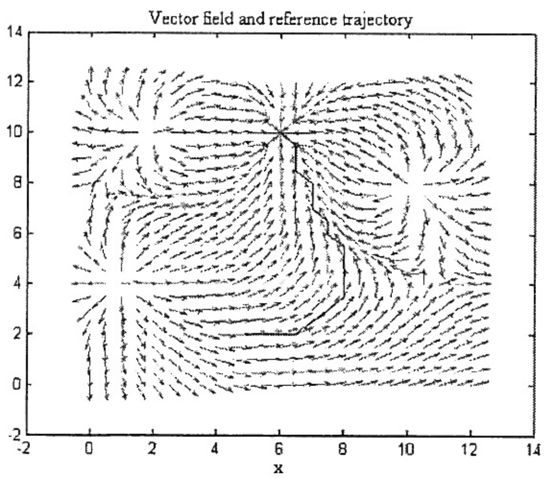

Fig. 8. Vector field and reference trajectory in the experiment 2

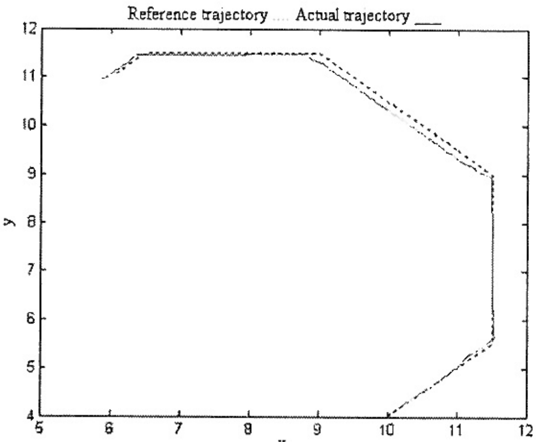

Fig. 9. Reference and actual trajectory in the experiment 1

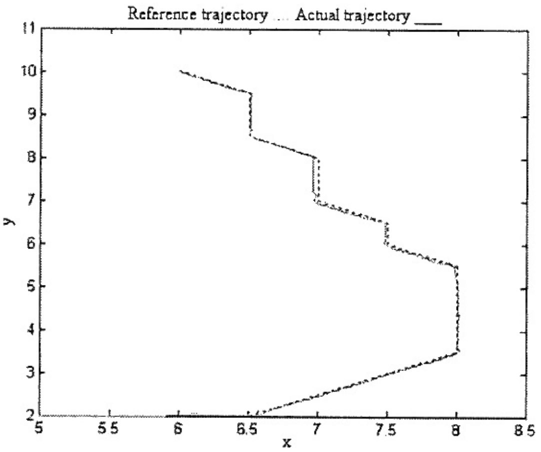

Fig. 10. Reference and actual trajectory in the experiment 2

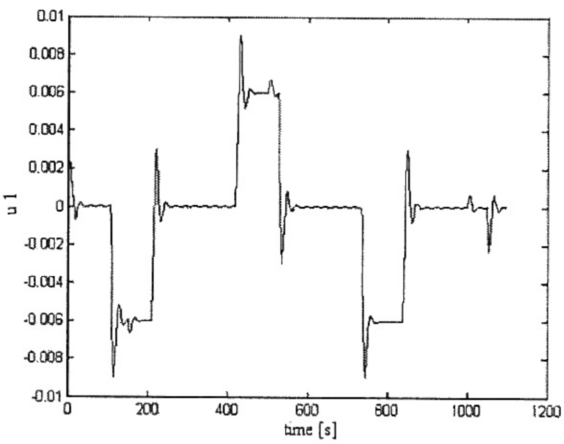

Fig. 11. Control signal u_1 versus time in the experiment 1

V. CONCLUSIONS

The paper proposes a new development method for Mamdani PI-fuzzy controllers, to solve the tracking problem for wheeled mobile robots with two degrees of freedom. The controllers can be implemented as tracking controllers in a cascaded control system structure using the off-line generation of the reference inputs by means of the artificial potential field method for obstacle avoidance.

In addition, the paper proposes a simplified dynamic model of the mobile robots with two degrees of freedom. The proposed model is sufficiently general to test the Mamdani PI-FCs developed in the paper as tracking controllers.

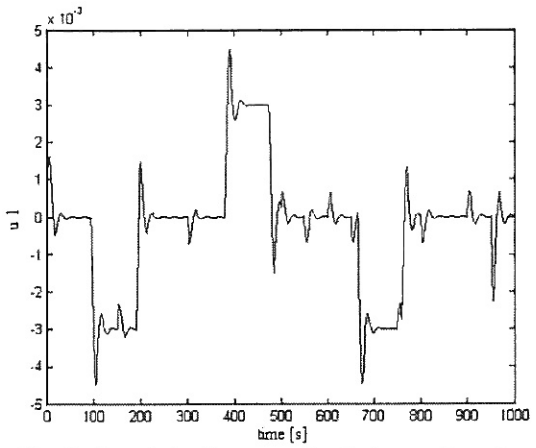

Fig. 12. Control signal u_1 versus time in the experiment 2

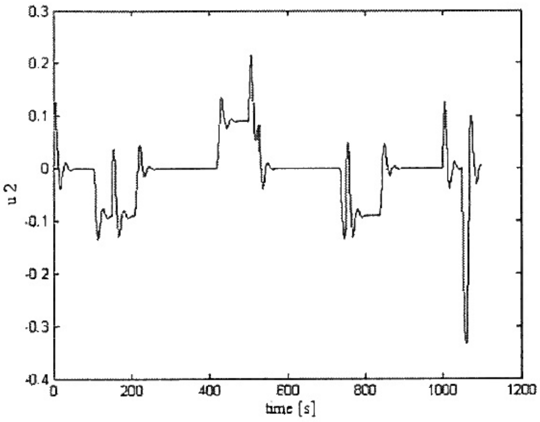

Fig. 13. Control signal u_2 versus time in the experiment 1

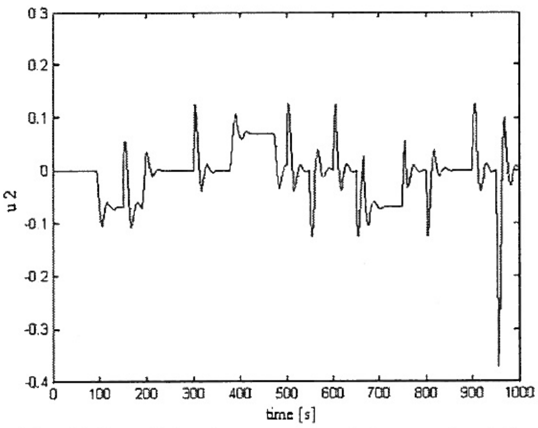

Fig. 14. Control signal u_2 versus time in the experiment 2

The development method for the Mamdani PI-FCs has been validated with good results on the simplified dynamic model by performing two digitally simulated experiments.

The presence of the double integrator on both control channels (of x and y) ensures a favorable behavior with respect to a ramp variation of the reference input and the rejection of the effects of constant disturbances as well. The control system behavior can be improved by the development of optimal PI-FCs.

There is proved that fuzzy controllers can cope with tracking control involving relatively small initial tracking errors, not arbitrary ones as in the global tracking problem. However, the obtained results draw the conclusion that the Mamdani PI-FCs can be applied in simpler or more complex control problems such as path following and point stabilization. There can be developed also Takagi-Sugeno PI-FCs without major difficulties [17], [18].

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VII. REFERENCES

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