V-PATH AND U-CIRCUIT GENERATION IN ORDINARY PETRI NET

Gi-Bum Lee
Research Institute of Industrial Science and Technology
Pohang, 790-600 KOREA
gblee@rist.re.kr

Han Zandong
Tsinghua University
Beijing 100084, P.R. CHINA
hanzd@mail.tsinghua.edu.cn

Moon-Gab Joo
Pukyong National University
Busan, 608-737 KOREA
gabi@pknu.ac.kr

Jin S. Lee
Pohang University of Science and Technology
Pohang, 790-784 KOREA
jsoo@postech.ac.kr

Abstract – **This paper presents a method of generating a V-path and a U-circuit structure. A Petri Net consists of three kinds of structures: 1) made only of the V-path; 2) composed only of the U-circuit, and; 3) consists of both the V-path and the U-circuit. The V-path describes the one-way path of a token, whereas the U-circuit has the circulation structure of a token. All Petri Nets can be represented as the combination of the V-path and U-circuit.**

I. INTRODUCTION

Petri Nets are widely recognized as a powerful modeling and analysis tool for discrete event systems particularly those characterized by synchronization, concurrency, and parallelism [1, 2, 3]. They also possess the structural characterization of fundamental features, i.e., causality, choice, and concurrency.

A Petri Net (PN) can be analyzed using various methods such as reachability graph, reduction, and fundamental equation. First, the reachability graph [4, 5] is essential in analyzing these properties. But the more Petri Net structure increases, the more reachability graph becomes hugely. Second, the reduction method [6, 7] is useful in analyzing a large system. This method is powerful, but it is difficult to detect and correct design mistakes. The third approach is to solve the fundamental equation [8, 9]. The solution of the matrix equation provides a firing count vector [10] describing the relationship between the initial marking and destination marking. Still its major problem is the lack of information on firing sequences and the existence of spurious solutions.

In this paper, the general solution of $\mathbf{AX}=\mathbf{M}_d-\mathbf{M}_0$ is described in the form of one minimal particular solution and an arbitrary linear combination of the T-invariant solutions to analyze a Petri Net including the initial marking and destination marking. The V-path route is created from minimal particular solutions, whereas the U-circuit structure is generated from minimal T-invariant solutions. All Petri Nets can be represented as the combination of the V-path and the U-circuit.

II. PRELIMINARY

This section presents some definitions and notations necessary for the following sections.

Definition 2.1. Let PN=(P, T, I, O, M_0) be a marked Petri Net. $P=\{p_1, p_2, \ldots, p_n\}$ is a finite set of places, and $T=\{t_1, t_2, \ldots, t_m\}$, a finite set of transitions. I denotes the input function, O, the output function, and M_0, the initial marking.

Definition 2.2. A PN is an ordinary Petri net iff $I(p, t)\rightarrow\{0, 1\}$ and $O(t, p)\rightarrow\{0, 1\}$ for any given $p \in P$ and $t \in T$. ■

$\mathbf{AX}=\mathbf{M}_d-\mathbf{M}_0$ is the fundamental equation where $\mathbf{M}_d$ is the destination marking. $\mathbf{M}_0$ represents the initial marking, whereas $\mathbf{A}=\mathbf{O}-\mathbf{I}$ is the incidence matrix. **O** and **I** are the output and input function matrices, respectively. Let $\mathbf{X}=[x_1\ x_2\ \ldots\ x_m]^T$ be a column vector. The transition set $T(\mathbf{X})$ is represented as the support of **X** if it is composed of transitions associated with the positive elements of **X**, i.e. $T(\mathbf{X})=\{t_i|x_i>0\}$. p° is the set of output transitions of p, $^\circ p$ is the set of input transitions of p, t° is the set of output places of t, and $^\circ t$ is the set of input places of t.

Definition 2.3. A vector **X** that is a solution of $\mathbf{AX}=0$ is known as a T-invariant [1]. A solution **X** is positive if every element of **X** is a non-negative value.

Definition 2.4. A positive T-invariant solution **U** of $\mathbf{AU}=\mathbf{0}$ is minimal if it satisfies the following condition:
For any other T-invariant $\mathbf{U}_i$, at least one element of $\mathbf{U}-\mathbf{U}_i$ is a negative value. The set of minimal T-invariant solutions is $U=\{\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_s\}$.

Definition 2.5. A positive particular solution **V** of $\mathbf{AV}=\mathbf{M}_d-\mathbf{M}_0$ is minimal if it satisfies the following condition:
For any T-invariant **U** of PN, there must be at least one negative element in the solution of **V-U**, i.e. $\{\mathbf{U}|\mathbf{V}-\mathbf{U}\geq 0, \mathbf{U}$ is a T-invariant$\}=\Phi$. The set of minimal particular solutions is $V=\{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_q\}$.

The general solution of $\mathbf{AX}=\mathbf{M}_d-\mathbf{M}_0$ is represented as shown in (1). It must be expressed in the form of one minimal particular solution and an arbitrary linear combination of the T-invariant solutions:

$$\mathbf{X} = \mathbf{V}_i + \sum_{j=1}^{r} k_j U_j \quad (1)$$

where $\mathbf{V}_i \in V$, k_j is a non-negative integer.

III. V-PATH AND U-CIRCUIT GENERATION

The set of minimal particular solutions is $V=\{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_q\}$. The set of minimal T-invariant solutions is $U=\{\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_s\}$. As a subset of V, $B=\{\mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_n\}$ is generated as follows:

For every $\mathbf{V}_i$, if $T(\mathbf{V}_i)\subset T(\mathbf{U}_j)$, then $\mathbf{V}_i\notin B$. If $T(\mathbf{V}_i)\not\subset T(\mathbf{U}_j)$, then $\mathbf{V}_i\in B$. For each pair of $(\mathbf{B}_i, \mathbf{U}_j)$ where $i=1, 2, \ldots, |B|$, $j=1, 2, \ldots, s$, and $|B|$ is the cardinality of set B. If ${}^{\circ}T(\mathbf{B}_i)\cap T(\mathbf{U}_j)^{\circ}\neq\Phi$ and $T(\mathbf{U}_j)\not\subset T(\mathbf{B}_i)$ are satisfied, then $\mathbf{D}_i=\mathbf{B}_i-\max(\mathbf{B}_i)\cdot\mathbf{U}_j$, where $\max(\mathbf{B}_i)$ is the maximum value of the elements in $\mathbf{B}_i$.

Let $\mathbf{D}_i(r)$ be the r-th element of $\mathbf{D}_i$.

$$\mathbf{W}_i(r)=f(\mathbf{D}_i(r)), \text{ where } f(x)=\begin{cases}\mathbf{D}_i(r), & \text{if } \mathbf{D}_i(r)>0\\ 0, & \text{if } \mathbf{D}_i(r)\leq 0\end{cases}, \quad r=1, 2, \ldots, m. \qquad (2)$$

Definition 3.1. V-path

The transitions of an i-th V-path consist of $T(\mathbf{W}_i)$, where $\mathbf{W}_i$ is the same m-vector as $\mathbf{D}_i$. The V-path places consist of the input and output places of $T(\mathbf{W}_i)$ except the output places of $T(\mathbf{U}_j)$. The input places of the V-path are $P_{vi}=\{p|p\in{}^{\circ}T(\mathbf{W}_i)\}-\{p|p\in T(\mathbf{U}_j)^{\circ}\}$. In contrast, the output places of the V-path are $P_{vo}=\{p|p\in T(\mathbf{W}_i)^{\circ}\}-\{p|p\in T(\mathbf{U}_j)^{\circ}\}$.

Definition 3.2. U-circuit

The transitions of a j-th U-circuit consist of the transitions of $T(\mathbf{U}_j)$. The U-circuit places consist of the output places of $T(\mathbf{U}_j)$. Let $P_{ui}=\{p|p\in{}^{\circ}T(\mathbf{U}_j)\}$ be the input place of a U-circuit. Let $P_{uo}=\{p|p\in T(\mathbf{U}_j)^{\circ}\}$ be the output place of a U-circuit. Therefore $P_{ui}=P_{uo}$.

To analyze systematically the correlation between $T(\mathbf{W}_i)$ and $T(\mathbf{U}_j)$, a PN is divided into V-path and U-circuit structures and reformed. The PN structure of the V-path suggests the one-way route of a token as shown in Fig. 1. On the other hand, the PN of the U-circuit describes the circulation structure of a token. The relationship between a V-path and a U-circuit is linked with arcs as connected U-circuit places to V-path transitions or V-path transitions to U-circuit places. There are two connection methods: one-way connection and bi-directional connection.

i) One-way connection

- Connects the V-path transition to the U-circuit place.

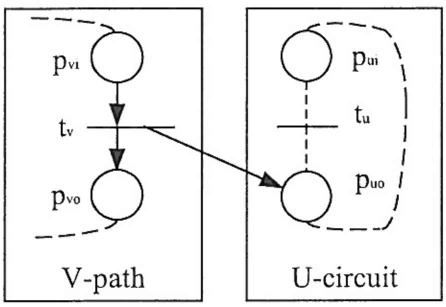

Fig. 1. One-way connection (I).

- Connects the U-circuit place to the V-path transition.

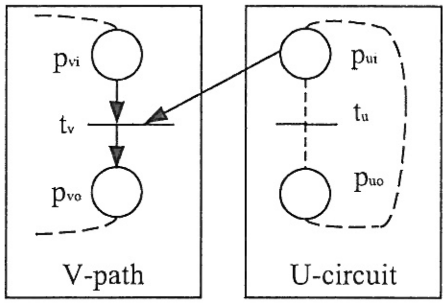

Fig. 2. One-way connection (II).

ii) Bi-directional connection

- Connects the U-circuit places with the V-path places.

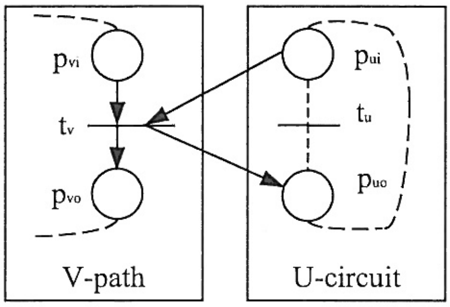

Fig. 3. Bi-direction connection.

The ● and o symbols represent the current markings and destination markings, respectively. A Petri Net with the initial marking and destination marking can be divided into the V-path and the U-circuit:

Example 1. Partition of the V-path and the U-circuit.

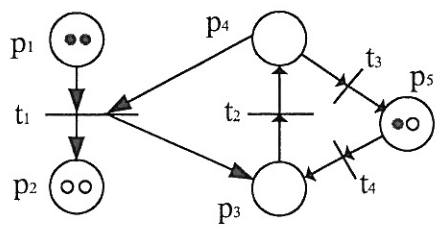

Fig. 4. Petri Net containing a V-path and a U-circuit.

The fundamental equation is $\mathbf{AX}=\mathbf{M}_d-\mathbf{M}_0$.

Fig. 2.4 gives rise to the following:

$$\begin{bmatrix}-1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ -1 & 1 & -1 & 0\\ 1 & -1 & 0 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}\begin{bmatrix}t_1\\ t_2\\ t_3\\ t_4\end{bmatrix}=\begin{bmatrix}-2\\ 2\\ 0\\ 0\\ 0\end{bmatrix}.\ \mathbf{V}=\begin{bmatrix}2\\ 2\\ 0\\ 0\end{bmatrix} \text{ and } \mathbf{U}=\begin{bmatrix}0\\ 1\\ 1\\ 1\end{bmatrix}. \quad (3)$$

Since $T(\mathbf{V})\not\subset T(\mathbf{U})$, $\mathbf{B}=\mathbf{V}$.

For a pair of $(\mathbf{B}, \mathbf{U})$, since ${}^{\circ}T(\mathbf{B})\cap T(\mathbf{U})^{\circ}\neq\Phi$ and $T(\mathbf{U})\not\subset T(\mathbf{B})$,

$$D=\begin{bmatrix}2\\2\\0\\0\end{bmatrix}-2\cdot\begin{bmatrix}0\\1\\1\\1\end{bmatrix}=\begin{bmatrix}2\\0\\-2\\-2\end{bmatrix},\ W=\begin{bmatrix}2\\0\\0\\0\end{bmatrix}. \quad (4)$$

Therefore, the V-path transition is t_1 and the U-circuit transitions are t_2, t_3, and t_4.

The abovementioned example has shown that the Petri Net has a V-path and a U-circuit. Nonetheless, not all Petri Nets have both the V-path and the U-circuit. A Petri Net can have three kinds of structures: 1) made only of the V-path; 2) composed only of the U-circuit; 3) represented as the combination of the V-path and the U-circuit.

IV. CONCLUSION

This paper shows that a Petri Net could be represented as a combination of the V-path and the U-circuit. The V-path transition and the U-circuit transition are calculated from the fundamental equation of a Petri Net. The V-path shows the one-way route of a token, whereas the U-circuit describes the circulation structure of a token. In the future, this division method is expected to be utilized in analyzing token flow.

V. REFERENCES

[1] David. *Petri Net and Grafcet,* Prentice Hall International Ltd., 1992.

[2] MengChu Zhou. *Petri Nets in Flexible and Agile Automation,* Boston: Kluwer Academic Publishers Group, 1995.

[3] Tadao Murata, "Petri Nets: Properties, Analysis and Applications," in *Proceedings of the 1989 IEEE, 77,* pp. 541-580.

[4] Ernst W. Mayr, "An Algorithm for the General Petri Net Reachability Problem," *SIAM J. COMPUT.*, vol. *13,* no. 3, 1984, pp. 441-449.

[5] Gi Bum Lee, Han ZanDong, and Jin S. Lee, "Generalized State Equation of Petri Nets with Priority," *International Journal of Intelligent Systems*, vol. 18, Nov. 2003, pp. 1145-1153.

[6] Kwang-Hyung Lee and Joel Favrel, "Hierarchical Reduction Method for Analysis and Decompostion of Petri Nets," *IEEE Trans. On Systems, Man, and Cybernetics,* SMC vol.15, no. 2, 1985, pp. 272-280.

[7] Peterson. *Petri Net Theory and the Modeling of Systems,* London: Prentice Hall international (UK) Ltd., 1981.

[8] Gi Bum Lee and Jin S. Lee, "The state equation of Petri Net for the LD program," *IEEE International Conference on Systems, Man & Cybernetics,* 2000, pp. 3051-3056.

[9] Iko Miyazawa, Haruki Tanaka, and Takashi Sekiguchi, "Classification of Solutions of Matrix Equation Related to Parallel Structure of a Petri Net," *IEEE Conference on Emerging Technologies and Factory Automation,* 1996, pp. 446-452.

[10] Jorg Desel and Javier Esparza, *Free choice Petri nets,* Cambridge University Press, 1995.