The Air Hockey playing Robot: Algorithm for the puck trajectory prediction problem

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Abstract— **In this paper we present an algorithm based on Kalman filter approach for the puck trajectory prediction problem. The problem rises from the development project of an air hockey playing robotic system that highlights growing interests in entertainment robots, or robotic players, also as an educational test-bed for tracking problems, Artificial Intelligence algorithms, signals filtering techniques, control skills and visual systems. Simulation results are shown and compared with a least-square algorithm.**

I. Introduction

This work is part of researches focused on the educational development of an air hockey playing robotic system [18] and highlights growing interests in entertainment robots or robotic players, also as a test-bed for tracking problems, Artificial Intelligence algorithms, signals filtering techniques, control skills and visual systems ([7], [8], [9], [10], [12]). The goal of the air hockey playing robot project is to control a puck that slides on a table, through impacts with a paddle, which is maneuvered by a robot, a problem proposed by [1], [2], [3], [4]. While the work of Mark Spong and his students was to develop a three-degree-of-freedom air hockey playing robot, in our project we are interested to use a general-purpose five-degree-of-freedom robot and its controller, the CataLyst-5 manipulator produced by the CRS Robotics Corporation and its C500C controller. This robot is ideal for laboratory automation systems specifically for medical, pharmacological and biochemical applications and for advanced manufacturing, and it is not specifically developed as a robotic player; for this reason, we had to adapt some game rules and equipments to be able to use this robot (e.g. we established the table dimensions according to the robot work space). Moreover, the architecture of the C500C controller prevents us to implement some advanced visual control strategy [19], [20], [21].
In this paper our aim is to provide a correct and accurate estimate of the puck trajectory, given sensed puck location [1], and to predict the puck position at the intercept line. The intercept line is an imaginary line near the goal protected by the robot and where the robot can intercept and control the puck through impacts with the paddle. The system must provide the puck position estimate as soon as possible to make the robot ready to intercept the puck. We decided to model *model uncertainties* and *measurement errors* as a white noise. Under this assumption the prediction of the puck intercept position has been obtained using a Kalman filter approach [13], [15], [16], [14].

A least-square technique has been proposed in [1] for state prediction but this method requires, in order obtaining acceptable results, a sophisticate *model of the plant*; it is very difficult to obtain a model that takes into account the complete dynamics of the system. In [5], [6] a predictor based on neural networks has been proposed, but the network provides an accurate estimate of the final puck intercept state if an accurate data training set of trajectories is used, all important state parameters are included as inputs and the network has an adequate number of internal nodes. Simulation results are reported and compared with those obtained using the least-square algorithm proposed in [1].

The paper is organized as follows. In Section II we present the equipment to reproduce the human game of air hockey and in Section III we derive the sliding puck model. Section IV introduces the algorithm proposed to predict the puck intercept line, based on Kalman filter approach [13]. Computer simulations are presented in Section V. Some conclusive remarks end the paper.

II. The Equipment

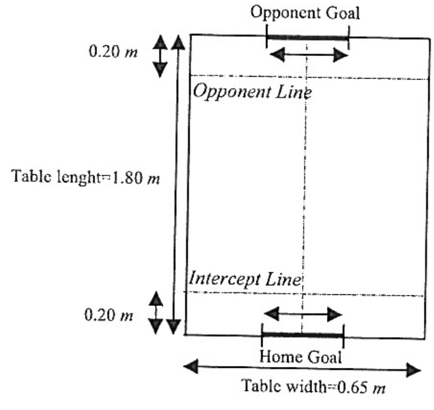

Fig. 1. Air table

In order to reproduce the human game of air hockey, the following equipment is required: a black cylindrical puck, a mallet, an air table, a vision system and a robotic player.
The air table is a bounded, rectangular, low-friction surface that the puck slides on. The table dimensions are shown in Figure 1.
The vision system consists of a DALSTAR CA-D6-0512 CCD camera (with 516×532 image plane) mounted above the table so that its optical axis is perpendicular to the plane of the table, which reduces the vision problem to two dimensions. The camera provides a measure of the puck position over a sampling period $\Delta := t_{k+1} - t_k$.
The DALSTAR CA-D6-0512 camera provides premium performance and premium image quality; the camera's four outputs provide truly impressive frame rates. The camera has a frame rate of 262 *fps* and for this reason it is ideal for motion analysis, and in particular for this application.
The robot used such as air hockey player is a five-degree-of-freedom robot, the CataLyst-5 manipulator produced by the CRS Robotics Corporation. The CataLyst-5 is an articulated robot that is ideal for applications that require complex and flexible movements without sacrificing speed or reliability.
Even if experiments are performed in simulation rather than on the real system, the available robot and vision systems have been presented as necessary for parametres evaluation in (22)-(24).

III. System Modelling

Under the assumption that model uncertainties and measurement errors are white noises, that is stochastically independent random noise terms, normally distributed with expectation 0 and covariance matrix Q and R respectively, the Kalman filter, which is a typical method for filtering of motion trajectory, can predict the puck intercept state properly.
The use of Kalman filtering techniques requires a stochastic state-space representation of the puck motion on the air table and of the measure process that is derived in this section.
A number of physical models can be used for the puck motion. The simplest model considers the puck to be an extended particle, all collisions to be elastic and force tangent to impact surfaces negligible (thus the puck's spin is ignored) [see Fig.2]. For more exact models it is necessary to analyze dynamic effects of spin, friction, inelastic collisions and fluid dynamics (for the airflow). It is also important to model the exact dynamics of the table that requires a complete understanding of the topology of the table along with a correct knowledge of the material properties of the components involved. The dynamics of the air hockey system may change over time because, for example, the air supply for the table may change in pressure and the puck or the mallet may scuff the surface. Because of the difficulty of physically modelling all aspects of the system and the environment, the model used for the prediction is the most simplistic, which assumes linear translation of the puck and ignores spin and friction. Localization of the puck on the table requires the knowledge of the two coordinates x and y, so the analysis of the puck motion on the table refers to

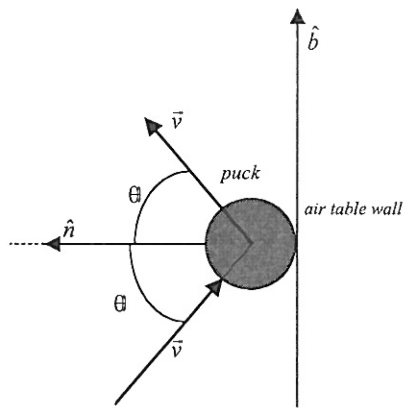

Fig. 2. Puck/wall impact

the inertial coordinate system (x, y). Denote with s_k the state vector at time t_k:

$$s_k = [\ \dot{x}_k\ \ x_k\ \ \dot{y}_k\ \ y_k\ ]' \tag{1}$$

where x_k and y_k are the puck's center of gravity positions at time t_k along x-axis and y-axis, respectively, $\dot{x}_k$ and $\dot{y}_k$ are their respective components of linear velocity. In the case of no friction, the puck velocity remains constant during motion. The model of the puck motion from t_k to t_{k+1} without impacts between the puck and table's walls can be written in the following compact form:

$$s_{k+1} = F s_k + w_k \tag{2}$$

The term w_k is the noise term representing model uncertainties. In particular, it can be assumed to have uncertainties that affect the puck velocity model, in term of its modulus (which can decrease during an impact) and of its direction (which can change on account of collisions with table's walls). The state evolution matrix F has the following form:

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \Delta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \Delta & 1 \end{bmatrix} \tag{3}$$

where Δ is the sampling period. To modelling the puck motion on the surface in a correct way, it is necessary to analyze impacts between the puck and the wall of the air table. The simplifying assumption that all collisions are elastic and force tangent to impact surfaces negligible, makes the change in sign of the normal component of the puck velocity be the only effect of puck/wall impacts. If there is a puck/wall impact between t_k and t_{k+1}, the model of the puck motion can be expressed in the following form

$$s_{k+1} = F s_k + w_k \tag{4}$$

$$s_{k+1} = U s_{k+1} + V(s_{k+1}) \tag{5}$$

where

$$U = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

and V is a vector which depends on s_{k+1} in the following way

$$V = [\, 0 \;\; 2(D-r) \;\; 0 \;\; 0 \,]' \quad (7)$$

if $x_{k+1} \geq (D-r)$, that is the impact occurs with the right-hand wall of the table, and

$$V = [\, 0 \;\; 2r \;\; 0 \;\; 0 \,]' \quad (8)$$

if $x_{k+1} \leq r$, that is the impact occurs with the left-hand wall of the table; r is the puck radius and D the table width.
Denote with z_k the measures obtained from the camera at the sampling time t_k, which are only measures of the puck position on the table.
The measure equation is given by

$$z_k = H' s_k + v_k \quad (9)$$

where H' is the following matrix

$$H' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

and v_k is the noise term representing the measurement error of sensed location.

IV. Prediction of the puck intercept state

Once a stochastic state-space representation of the puck motion and of the measure process has been derived, which is expressed in (2)-(9) in the case of no impacts, and in (4)-(5)-(9) in the case of impacts, the Kalman filtering prediction equations may be written in the following form:

$$K_k = F\Sigma_{k/k-1}H \left[H'\Sigma_{k/k-1}H + R\right]^{-1} \quad (11)$$

$$\Sigma_{k+1/k} = F\Sigma_{k/k-1}F' - F\Sigma_{k/k-1}H \cdot \left(H'\Sigma_{k/k-1}H + R\right)^{-1} H'\Sigma_{k/k-1}F' + Q \quad (12)$$

$$\hat{z}_{k+1/k} = H'\hat{s}_{k/k-1} \quad (13)$$

$$\hat{s}_{k+1/k} = \left[F - K_k H'\right]\hat{s}_{k/k-1} + K_k z_k \quad (14)$$

If $\hat{x}_{k+1/k} \geq (D-r)$ or $\hat{x}_{k+1/k} \leq r$ then

$$\hat{s}_{k+1/k} = U\hat{s}_{k+1/k} + V\left(\hat{s}_{k+1/k}\right) \quad (15)$$

The term $\hat{s}$ is the filter state estimate, Σ the filter estimate of the state covariance, Q is the covariance matrix of the system noise, R is the covariance matrix of the measurement noise and K the filter gain.
The notation $k+1/k$ indicates the value of the variable predicted for time step $k+1$ given k observations. The quantity $\hat{z}$ is the predicted observation, the position of the predicted state in observation space.
The equation (15) shows impact effects according to description presented in (5) and contains the nonlinear dynamics of the system. The filter state estimate, expressed in the equation (14), is modified if necessary according to equation (15).
The covariance matrix R is assumed to have the following diagonal form:

$$R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \quad (16)$$

this means that no correlation is assumed between measurement errors and also that none of the sensed positions is exact.
The covariance matrix of system noise, Q, can be expressed as:

$$Q = \begin{bmatrix} \sigma_w^2 & 0 & 0 & 0 \\ 0 & \sigma_w^2 & 0 & 0 \\ 0 & 0 & \sigma_w^2 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix} \quad (17)$$

because of the assumption that uncertainties affect the puck velocity model.
The action of the Kalman filter is to improve the prediction $\hat{s}_{k+1/k}$ by blending the predicted observation $\hat{z}_{k+1/k}$ with the real observation z_{k+1} in such a way that the total mean square error on the estimate (the trace of the matrix Σ) is minimized.
In this application, the Kalman filter uses the observations at time step k to provide an accurate estimate of the puck position at the intercept line, so the filter predicts the value of the puck state for time step $k+N_k$ given k observations.
The equations representing the predicted intercept state are the following ones:

$$\hat{s}_{k+N_k/k} = F^{N_k-1}\hat{s}_{k+1/k} \quad (18)$$

and

$$\begin{aligned} \hat{\dot{x}}_{k+N_k/k} &= \hat{\dot{x}}_{k+N_k/k} \\ \hat{x}_{k+N_k/k} &= r + (\hat{x}_{k+N_k/k} \; mod \; (D-r)) \end{aligned} \quad (19)$$

if $\lceil \hat{x}_{k+N_k/k}/(D-r) \rceil$ is even, and

$$\begin{aligned} \hat{\dot{x}}_{k+N_k/k} &= -\hat{\dot{x}}_{k+N_k/k} \\ \hat{x}_{k+N_k/k} &= (D-r) - \left(\hat{x}_{k+N_k/k} \; mod \; (D-r)\right) \end{aligned} \quad (20)$$

if $\lceil \hat{x}_{k+N_k/k}/(D-r) \rceil$ is odd.
The predicted intercept state of equation (18) is corrected according to equation (19) or (20). These equations spring from nonlinear dynamics of the puck. This expedient made the Jacobian of the dynamics not necessary to be included in the update equation for the covariance matrix (12).
The number of steps N_k must be estimated with accuracy at each time step k once the puck velocity is estimated using position measurements [11]. The simplest velocity estimation method is the Euler approximation that takes the difference between the last two sampled positions divided by the sampling period. When the positions are precisely sampled, this Euler approximation gives the simplest and most efficient velocity estimation. Thus

$$N_k = \lceil \hat{t}_k/\Delta \rceil \quad \text{where} \quad \hat{t}_k = \left(z_{y_k} - \bar{y}_{il}\right)/\hat{\dot{y}}_k \quad (21)$$

where z_{y_k} is the measurement value of the puck location along y-axis at time t_k and $\bar{y}_{il}$ is the value of the y-coordinate at the intercept line.
The Kalman filter can be implemented once estimates of R

and Q are available and initial conditions are assigned. In general, complete information about the covariance matrices is not available but it is well know how poor knowledge of noise statistics may seriously degrade the Kalman filter performance. Therefore, the proposed algorithm is:

- Inizialize $\hat{s}_{0/-1}$ and $\Sigma_{0/-1}$
- Repeat
1) Compute K_k from (11) and $\Sigma_{k+1/k}$ from (12)
2) Compute $\hat{s}_{k+1/k}$ from (14) and (15)
3) Compute N_k from (21)
4) Compute $\hat{s}_{k+N_k/k}$ from (18) according to (19)-(20)
- until *the puck crosses the intercept line.*

Evaluation of R and Q

The maximum errors along x and y axes have been evaluated in [18] according to the specifications of the DALSTAR CA-D6-0512 camera, to its resolution of 516×532 *pixels* and to the frame rate 262 *fps*, and taking into account the dimensions of the table. The following estimation of the noise covariance matrices has been derived in [18] considering the errors along x and y axes larger than that estimated, to take into account not only camera resolution but also quantization noise:

$$R = (0.005)^2 I_{2\times 2} \tag{22}$$

The matrix Q takes into account the effects of the puck dynamics model error and has been evaluated in [18] as:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (0.003)^2 \times \Delta \tag{23}$$

The sampling period Δ has been chosen larger than that estimated according to the visual sample rate to consider image feature extraction software [18].

$$\Delta = \frac{1}{frameRate} = \frac{1}{200}\ sec = 0.005\ sec \tag{24}$$

The initial state covariance has been estimated in [18] as follows:

$$\Sigma_{0/-1} = (0.005)^2 I_{4\times 4} \tag{25}$$

V. Simulation Results

In order to evaluate the performance of the proposed algorithm, several simulations have been carried out but for sake of simplicity, only four of them has been reported and the results have been compared with those obtained with the algorithm proposed in [1], which is a least-square approach in two dimensions.

For each simulation the puck trajectory and the resulting intercept position prediction errors are shown.

The model used in the simulator takes into account both the lost of energy due to the friction between the puck and the table's walls and inelastic puck/wall impacts. The lost of energy is modelled by considering an attenuation of the magnitude of the puck linear speed in the form

$$|\tilde{v}| = |v| \sqrt{1-\varepsilon} \tag{26}$$

where ϵ is a random variable chosen from a uniform distribution on the interval $[0, 5 * 10^{-3}]$ and the inelastic puck/wall impacts are modelled by considering the reflection angle in the form

$$\tilde{\theta} = \theta + d\theta \tag{27}$$

where θ is the angle of incidence and $d\theta$ is a random variable chosen from a uniform distribution on the interval $[-0.025, 0.025]$ degrees.

Otherwise, the model used in the Kalman filtering prediction algorithm is a simpler dynamic model, as expressed in (2)-(9) and in (4)-(5)-(9).

In Figs. 3, 4, 5, 6 are shown puck's no bounce, one-bounce, four-bounce and too many bounces trajectories, respectively, for the simulator and the model used in the Kalman filter. The incoming parameters used in the simulations are depicted in Table I, where x_{opp} is the measured puck location on the opponent line and $|v_{opp}|$ and θ_{opp} are respectively the estimated magnitude and angle of the puck velocity on the opponent line.

TABLE I

Incoming Parameters in the Simulations

Trajectory	x_{opp} $[m]$	$\lvert v_{opp}\rvert$ $[m/s]$	θ_{opp} $[deg]$
no bounce	0.2	1.3	−85
one-bounce	0.5	1.4	−110
four-bounce	0.6	2	−150
too many bounces	0.4	1.2	−170

In Figs. 7, 8, 9, 10 are shown the errors on the prediction of the puck position at the intercept line, obtained with the proposed Kalman filter (solid line) and the least-square algorithm (dashed line) for no bounce, one-bounce, four-bounce and too many bounces trajectories depicted in Fig. 3, 4, 5, 6 respectively.

In Table II we resume our simulations in prediction error terms and we use as performance index the Normalized Integral Square Error, defined as

$$ISE = \frac{1}{T}\int_0^T e^2(t)dt$$

where $e(t)$ is the puck location prediction error. As evident from Table II the proposed Kalman algorithm is better than the least-square algorithm, and in particular it produces better predictions when a puck/wall impact occurs (see Figs. 7, 8, 9, 10). This improvement obtained using the Kalman filter approach is referable to the fact that the filter takes into account the change in sign of the puck velocity along the x-axis after a puck/wall impact. Otherwise, the least-square algorithm needs many measurements in order to evaluate the new correct trajectory.

TABLE II

INTEGRAL SQUARE ERROR (ISE) OF THE PUCK INTERCEPT POSITION PREDICTION

Trajectory	*Kalman Filter*	*LSE*
no bounce	$5.77 * 10^{-6}$	$1.6 * 10^{-3}$
one-bounce	$5.76 * 10^{-5}$	$3.3 * 10^{-3}$
four-bounce	$1.87 * 10^{-4}$	$1.34 * 10^{-2}$
too many bounces	$2.2 * 10^{-3}$	$1.35 * 10^{-2}$

VI. CONCLUSION

In this paper we have presented the main virtues of the Kalman filter approach by the use of an interesting paradigm in the prediction of the puck intercept position. This problem rises from an educational project of a robotic air hockey player. A simplified puck dynamics model has been used and uncertainties are modelled as a white noise. Simulation results show that this approach is better than the approach proposed in [1] in prediction error terms (ISE performance index).

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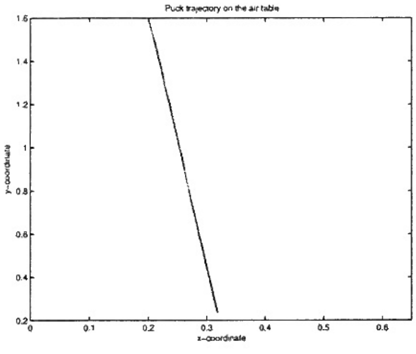

Fig. 3. No bounce trajectory for the simulator (solid line) and the model in the filter (dashed line)(traslational velocity is 1.3 m/sec)

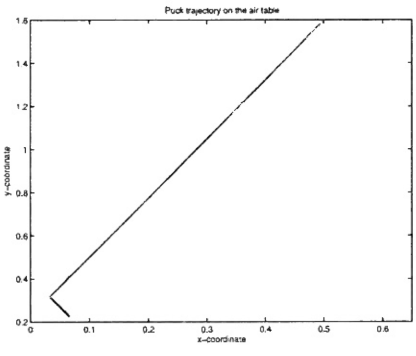

Fig. 4. One-bounce trajectory for the simulator (solid line) and the model in the filter (dashed line) (traslational velocity is 1.4 m/sec)

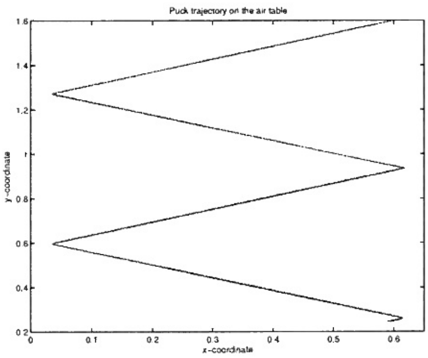

Fig. 5. Four-bounce trajectory for the simulator (solid line) and the model in the filter (dashed line) (traslational velocity is 2 m/sec)

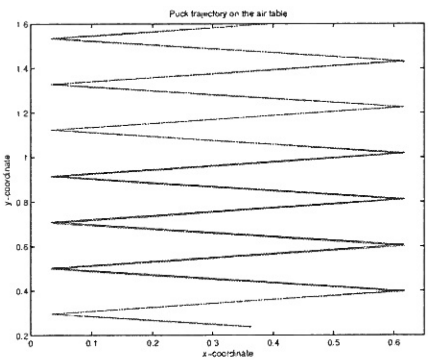

Fig. 6. Too many bounces trajectory for the simulator (solid line) and the model in the filter (dashed line) (traslational velocity is 1.2 m/sec)

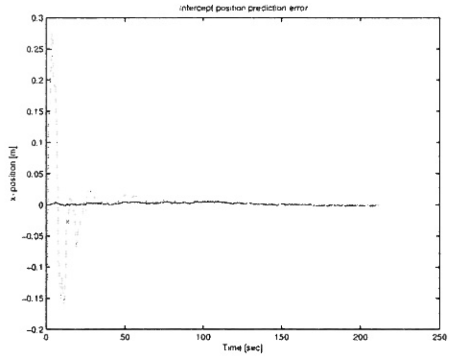

Fig. 7. Intercept position prediction error for no bounce trajectory (LSE-dashed line and Kalman Filter-solid line)

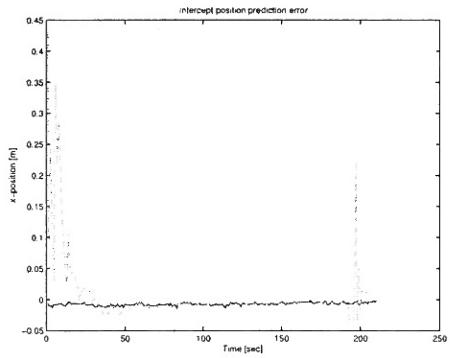

Fig. 8. Intercept position prediction error for one-bounce trajectory (LSE-dashed line and Kalman Filter-solid line)

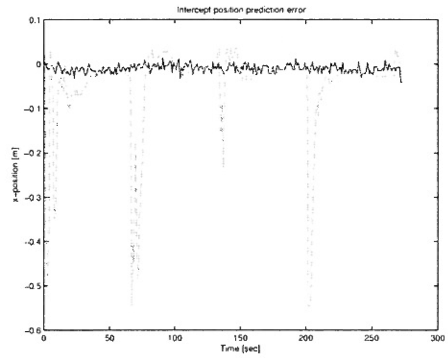

Fig. 9. Intercept position prediction error for four-bounce trajectory (LSE-dashed line and Kalman Filter-solid line)

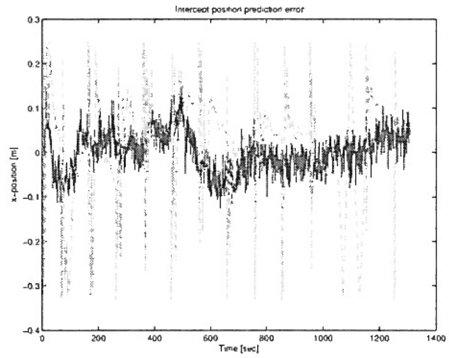

Fig. 10. Intercept position prediction error for too many bounces trajectory (LSE-dashed line and Kalman Filter-solid line)