Stabilization with motion planning of a four rotor mini-rotorcraft for terrain missions

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***Abstract*— We present in this paper the stabilization with motion planning of six independent configuration of a mini Unmanned Areal Vehicle (UAV) with four rotors, called *X4-flyer*. Naturally, the yaw motion can be stabilized without difficulty and independently of other motions. While the remaining other dynamics are linearly approximated around a small roll and pitch angles. We show that the system presents a flat output which can be useful to solve the motion generation problem. The stabilizing feedback control with motion planning are established using the concept of flatness. The resulting controller involves the time derivative of the lift (collective) input which can be obtained using the flat output of the system.**

I. Introduction

Terrain missions control of Unmanned Areal Vehicles (UAV) is still an open and difficult problem for scientific research and in engineering design. In our laboratory *Lsc*, a mini-UAV, is under construction subject of some industrial constraints. The areal flying engine couldn't exceed $2kg$ in mass, and $50cm$ in diameter with $30mn$ as time of autonomy in flight. With these characteristics, we can presume that our system belongs to the family of mini-UAV. The idea of our group was fixed on a four-rotor flyer with protected rotor blades. It is an autonomous hovering system, capable of vertical take-off, landing and quasi-stationary (hover or near hover) flight conditions. Compared to helicopters, named quad-rotor [8], [10], [1], the four-rotor rotorcraft have some advantages [9], [4]: given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend to cancel in trimmed flight. An X4-flyer operates as an omnidirectional UAV. Vertical motion is controlled by collectively increasing or decreasing the power for all four dc-motors. Lateral motion, in x-direction or in y-direction, is achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations.

A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotor vertical take-off and landing (VTOL), named as X4-flyer vehicle, is given in [4]. Dynamic motor effects are incorporating [4] and a bound of the perturbation error was obtained for the coupled system. The X4-flyer stabilization problem is also studied and tested by Castillo [2] where the nested saturation algorithm is used. The idea is to grantee a bound of the roll and pitch angles with a fixed bound of control inputs. Motivated by the need to stabilize aircrafts that are able to take-off vertically as helicopters, the control problem was solved for the Planar Vertical Take-Off and Landing (PVTOL): I/O linearization procedure in [7], theory of flat systems in [3] [5] and [6].

In this paper, flatness and motion planning are combined to solve the stabilizing control problem of the X4-flyer. we show that the system is flat with $\xi = (x, y, z)$ as a flat-output. By virtue of the system being flat, we can write all state and input trajectories satisfying the differential equation in terms of the flat output and its time derivatives. The idea will be considered here to the point to point control problem with a predefined path following.

II. Configuration descriptions and modelling

The X4-flyer is a system consisting of four individual electrical fans attached to a rigid cross frame. It is an omnidirectional (vertical take-off and landing) VTOL vehicle ideally suited to stationary and quasi-stationary flight conditions. We consider a local reference airframe $\Re_G = \{G, E_1^g, E_2^g, E_3^g\}$ attached to the center of mass G of the vehicle. The center of mass is located at the intersection of the two rigid bars, each of them supports two motors. Equipment (controller cartes, sensors, etc.) onboard are placed not far from G. The inertial frame is denoted by $\Re_o = \{O, E_x, E_y, E_z\}$ such that the vertical direction E_z is upwards. Let the vector $\xi = (x, y, z)$ denote the position of the center of mass of the airframe in the frame $\Re_o$. While the rotation of the rigid body is determined by a rotation $R : \Re_G \to \Re_o$, where $R \in SO(3)$ is an orthogonal rotation matrix. This matrix is defined by the three Euler angles ψ(yaw), θ(pitch), ϕ(roll) which are regrouped in $\eta = (\psi, \theta, \phi)$. The studied X4-flyer is given in figures (1,2). Dynamic modelling of an aircraft with four rotors is given with the lagrangian method in [2], while in [4] and [1] the model is presented in compact form. Here after, we give an idea about the dynamic model obtained with the Newton-Euler method.

A. Translation motion

We consider the translation motion of $\Re_G$ with respect to (*wrt*) $\Re_o$. The position of the center of mass *wrt* $\Re_o$ is defined by $\overrightarrow{OG} = (x\ y\ z)^t$, its time derivative gives the velocity *wrt* to $\Re_o$ such that $\frac{d\overrightarrow{OG}}{dt} = (\dot{x}\ \dot{y}\ \dot{z})^t$, while the second time derivative permits to get the acceleration: $\frac{d^2\overrightarrow{OG}}{dt^2} = (\ddot{x}\ \ddot{y}\ \ddot{z})^t$ denoted by

$$\frac{d^2\overrightarrow{OG}}{dt^2} = \overrightarrow{\gamma_G}\,|_{\Re_o} \tag{1}$$

Applying the first Newton equation of mechanics, we obtain the following compact expression of the translational motion

$$m\overrightarrow{\gamma_G}\,|_{\Re_O} = -mg\overrightarrow{e_z} + R(\psi,\theta,\phi)\overrightarrow{u} \tag{2}$$

where m is the total mass of the vehicle. The vector $\overrightarrow{u}$ combines the principal non conservative forces applied to the X4-flyer airframe including forces generated by the motors (figure 2) and drag terms. Drag forces and gyroscopic due to motors effects will be not considered in this work. $\overrightarrow{e_z}$ is the unit vector of E_z. The lift(collective) force $\overrightarrow{u}$ is the sum of the four forces, such that

$$\overrightarrow{u} = \sum_{i=1}^{4}\overrightarrow{f_i} \tag{3}$$

with $\overrightarrow{f_i} = k_i\omega_i^2\overrightarrow{e_3}$ and $\overrightarrow{e}_3$ is the unit vector along E_3^g. $k_i > 0$ is a given constant (we assume $k_i = k$) and ω_i is the angular speed of motor i. The form of the rotation matrix used in (2) is as follow

$$R_{\psi\theta\phi} = \begin{pmatrix} c_\theta c_\psi & s_\psi s_\theta & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\theta s_\psi s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\theta s_\psi c_\phi - c_\psi s_\phi & c_\theta c_\phi \end{pmatrix} \tag{4}$$

One substitute (3) into (2), we obtain

$$m\overrightarrow{\gamma_G}\,|_{\Re_O} = -mg\overrightarrow{e_z} + uR_{\psi\theta\phi}\overrightarrow{e_3} \tag{5}$$

and where the scalar $u = \sum_{i=1}^{4} k_i\omega_i^2$.

B. Rotational motion

The rotational motion of the X4-flyer will be defined *wrt* to the local frame, but expressed in the inertial frame. According to classical mechanics, and knowing the inertia matrix I_G of the X4-flyer at the center of mass and its local velocity of rotation Ω, the kinetic moment $\overrightarrow{\sigma_G}$ is defined by

$$\overrightarrow{\sigma_G} = I_G\overrightarrow{\Omega} \tag{6}$$

or the rotation vector is related to the $\eta(\psi,\theta,\phi)$ vector through

$$\overrightarrow{\Omega} = J(\eta)\dot{\eta}\overrightarrow{e} \tag{7}$$

where in the local frame $\overrightarrow{\Omega} = \Omega\overrightarrow{e}$, $\overrightarrow{e} = (\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3})$ and

$$J(\eta) = \begin{pmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{pmatrix}$$

Then, from (6-7) we get

$$\overrightarrow{\sigma_G} = I_G J(\eta)\dot{\eta}\overrightarrow{e} \tag{9}$$

In the following let $\Pi_G(\eta) \equiv I_G J(\eta)$. Using the derivative of (9), the dynamic moment

$$\overrightarrow{\delta_G} = \dot{\Pi}_G(\eta)\dot{\eta}\overrightarrow{e} + \Pi_G(\eta)\ddot{\eta}\overrightarrow{e} \tag{10}$$

The rotational motion is subject of the following relation

$$\dot{\Pi}_G(\eta)\dot{\eta}\overrightarrow{e} + \Pi_G(\eta)\ddot{\eta}\overrightarrow{e} = \sum\overrightarrow{M}_{ext} \tag{11}$$

where the external moments *wrt* G ($l_i = l, i = 1, ..., 4$ distance from G to the motor i which are considered identical)

$$\sum\overrightarrow{M}_{ext} = \tau_\psi\overrightarrow{e_3} + \tau_\theta\overrightarrow{e_1} + \tau_\phi\overrightarrow{e_2} \tag{12}$$

and where

$$\begin{aligned} \tau_\psi &= k(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \\ \tau_\theta &= l(\omega_2^2 - \omega_4^2) \\ \tau_\phi &= l(\omega_3^2 - \omega_1^2) \end{aligned} \tag{13}$$

The equality from (11) is ensured, meaning that

$$\ddot{\eta} = \Pi_G(\eta)^{-1}(\tau - \dot{\Pi}_G(\eta)\dot{\eta}) \tag{14}$$

with $\tau = (\tau_\psi\ \tau_\theta\ \tau_\phi)^t$.
At first, the model above can be I/O linearized by tacking the decoupling feedback

$$\tau = \Pi_G(\eta)\tilde{\tau} + \dot{\Pi}_G(\eta)\dot{\eta} \tag{15}$$

and the decoupled dynamic model of rotation

$$\ddot{\eta} = \tilde{\tau} \tag{16}$$

with $\tilde{\tau} = (\tilde{\tau}_\psi\ \ \tilde{\tau}_\theta\ \ \tilde{\tau}_\phi)^t$ which will be the subject of investigations due to the flatness propriety of the system.

Using the translational and rotational motions (2)(16), the equation of the dynamic of the X4-flyer can be written in the following explicit form

$$\begin{aligned} m\ddot{x} &= -us_\theta \\ m\ddot{y} &= uc_\theta s_\phi \\ m\ddot{z} &= uc_\theta c_\phi - mg \\ \ddot{\psi} &= \tilde{\tau}_\psi \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\phi} &= \tilde{\tau}_\phi \end{aligned} \tag{17}$$

which is a nonlinear differential equation with drift.
The four inputs u, $\tilde{\tau}_\psi$, $\tilde{\tau}_\theta$ and $\tilde{\tau}_\phi$ will be calculated in order to stabilize the system around any desired equilibrium which is defined by $(x_d, y_d, z_d, 0, 0, \psi_d)$. It is clear that for each equilibrium the control inputs should verify: $u = mg$, $\tilde{\tau}_\psi = \tilde{\tau}_\theta = \tilde{\tau}_\phi = 0$.
Note in (17) that the appropriate choice of $\tilde{\tau}_\psi$ permits to stabilize ψ at any desired value $\psi_d \in R$. As well as for its first and second time derivatives. So in the following section we will omit the dynamic of ψ.

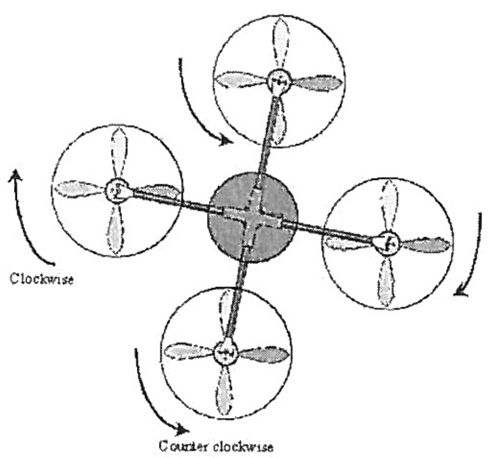

Fig. 1. The Understudy LSC' four rotor rotorcraft.

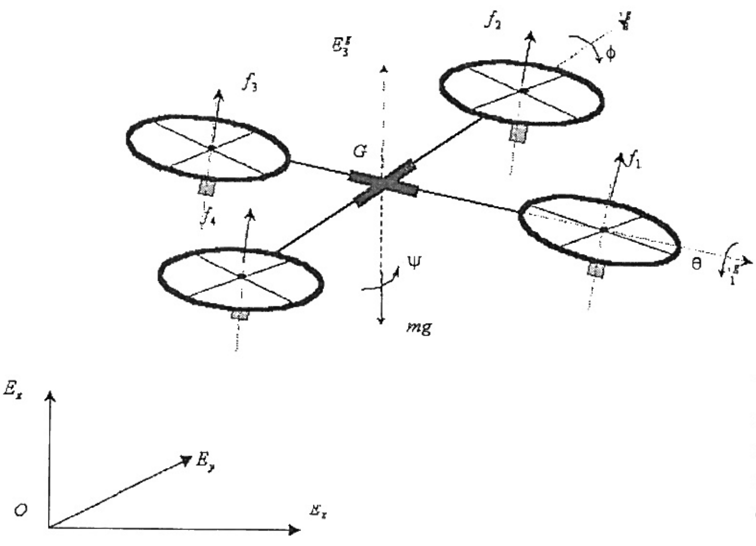

Fig. 2. The four rotor frames.

III. Motion Planning and Flatness

A system is flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration [6]. Flatness was first defined by Fliess *et al.* [3]. It was viewed and redefined by Martin *et al.* in motion planing. More precisely, if the system has states $x \in R^n$, and inputs $u \in R^m$, with

$$\dot{x} = f(x,u) \tag{18}$$

where f is a smooth vector field, then the system is flat if we can find outputs $y \in R^m$ of the form

$$y = h(x, u, \dot{u}, ..., u^{(r)})$$

such that

$$x = \varphi(y, \dot{y}, ..., y^{(q)})$$
$$u = \alpha(y, \dot{y}, ..., y^{(q)})$$

When a system is flat it is an indication that the nonlinear structure of the system is well characterized and one can exploit that structure in designing control algorithms for motion planing, trajectory generation, and stabilization.

Proposition 1: The X4-flyer described by the dynamic (17) is flat with $\xi = (x, y, z)$ is its flat output.

Proof: First, we define the state by $X = (x, y, z, \theta, \phi, \dot{x}, \dot{y}, \dot{z}, \dot{\theta}, \dot{\phi})$, we denote $\dot{X}$ its time derivative, and the input vector is regrouped in $U = (u, \tilde{\tau}_\theta, \tilde{\tau}_\phi)$, then the system can be written as

$$\dot{X} = f(X, U) \tag{19}$$

To prove that the defined state and the control vector are function of the flat output and its derivative, for any given trajectory $(x(t), y(t), z(t))$ smooth enough, we get $(u > 0)$

$$\begin{aligned} u =& m\left(\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2\right)^{\frac{1}{2}} \\ \phi =& arctg\left(\frac{\ddot{y}}{\ddot{z}+g}\right) \\ \theta =& -\, arctg\left(\frac{c_\phi \ddot{x}}{\ddot{z}+g}\right) \end{aligned} \tag{20}$$

Indeed, u, θ, ϕ, $\dot{u}$, $\dot{\theta}$ and $\dot{\phi}$ are function of $\ddot{\xi}, \xi^{(3)}$. So, it is straightforward to verify that $X = \varphi(\xi, \dot{\xi}, \xi^{(2)}, \xi^{(3)})$. Moreover, we can derive $\theta(t)$, $\phi(t)$ and prove the ξ-dependence of $\tilde{\tau}_\theta = \alpha_{\tilde{\tau}_\theta}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$ and $\tilde{\tau}_\phi = \alpha_{\tilde{\tau}_\phi}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$. ■

It follows from the fact that the system is flat that the feasible trajectories of the system are completely characterized by the motion of the center of mass of the X4-flyer. By converting the input constraints on the system to constraints on the curvature and higher derivatives of the motion of G, it is possible to compute efficient techniques for trajectory generation.

IV. Stabilization of the Equilibrium $(x_d, y_d, z_d, 0, 0, \psi_d)$

In this section we will develop a control strategy for stabilizing any equilibrium of the form $(x_d, y_d, z_d, 0, 0, \psi_d)$ of the X4-flyer, integrating motion planning. The flatness property of the system will serves for the trajectory planning between the given initial flat output (ξ_i, t_i) and the final one (ξ_f, t_f) where t_i and t_f are the initial and final time, respectively. As we have demonstrated in section III, we can write all trajectories $(X(t), U(t))$ satisfying the differential equation type (19) in terms of the flat output and its derivatives. In the following, we see that time derivatives at fourth order of the flat output will be needed. In the simple stabilization control problem, *i.e.* without motion planning, time derivatives of the reference flat output are equal to zero. In our cases, these derivations appear. Thus, our investigation can be viewed like cases of the tracking problem.

At first, we assume that $(\theta, \phi) \in (0,0)$, then (17) can be written as

$$\begin{aligned} m\ddot{x} =& -u\theta \\ m\ddot{y} =& u\phi \\ m\ddot{z} =& u - mg \\ \ddot{\psi} =& \tilde{\tau}_{\psi} \\ \ddot{\theta} =& \tilde{\tau}_{\theta} \\ \ddot{\phi} =& \tilde{\tau}_{\phi} \end{aligned} \tag{21}$$

A. Altitude z-stabilization and ψ-control

The control of the vertical position (altitude) can be obtained with the application of the following control input

$$u = mg + m\ddot{z}_d - mk_1^z(\dot{z} - \dot{z}_d) - mk_2^z(z - z_d) \tag{22}$$

where k_1^z, k_2^z are the coefficients of Hurwitz and z_d is the desired altitude.
The yaw attitude can be stabilized to a desired value with the following tracking feedback control

$$\tilde{\tau}_{\psi} = \ddot{\psi}_d - k_1^{\psi}(\dot{\psi} - \dot{\psi}_d) - k_2^{\psi}(\psi - \psi_d) \tag{23}$$

where k_1^{ψ}, k_2^{ψ} are stable coefficients.
Indeed, introducing (15) into (14), we obtain :

$$\begin{aligned} \ddot{x} =& -(g + f(z, z_d))\theta \\ \ddot{y} =& (g + f(z, z_d))\phi \\ \ddot{z} =& f(z, z_d) \\ \ddot{\psi} =& \ddot{\psi}_d - k_1^{\psi}(\dot{\psi} - \dot{\psi}_d) - k_2^{\psi}(\psi - \psi_d) \\ \ddot{\theta} =& \tilde{\tau}_{\theta} \\ \ddot{\phi} =& \tilde{\tau}_{\phi} \end{aligned} \tag{24}$$

where the function $f(z, z_d) = \ddot{z}_d - k_1^z(\dot{z} - \dot{z}_d) - k_2^z(z - z_d)$ is assumed to be regular *wrt* to their arguments.
The following investigation concerns the system

$$\begin{aligned} \ddot{x} =& -(g + f(z, z_d))\theta \\ \ddot{y} =& (g + f(z, z_d))\phi \\ \ddot{z} =& f(z, z_d) \\ \ddot{\theta} =& \tilde{\tau}_{\theta} \\ \ddot{\phi} =& \tilde{\tau}_{\phi} \end{aligned} \tag{25}$$

which can be subdivided on two independent cascade dynamics, given by

$$\begin{aligned} \ddot{x} =& -(g + f(z, z_d))\theta \\ \ddot{\theta} =& \tilde{\tau}_{\theta} \end{aligned} \tag{26}$$

and

$$\begin{aligned} \ddot{y} =& (g + f(z, z_d))\phi \\ \ddot{\phi} =& \tilde{\tau}_{\phi} \end{aligned} \tag{27}$$

Designing the control $\tilde{\tau}_{\theta}$ in the dynamic (26) permits to stabilize (bound) the pitch angle which will be viewed like a control input for the x-motion. As soon as for (27), where $\tilde{\tau}_{\phi}$ will be determined and ϕ is the input to stabilize the y-motion.

B. x-stabilization and θ-control

As the output x is flat, then its dynamic is effectuated in order to make appear the control $\tilde{\tau}_{\theta}$. We recall

$$\ddot{x} = -(g + f(z, z_d))\theta \tag{28}$$

When one derive twice this expression, we get

$$x^{(4)} = -\ddot{f}(z, z_d)\theta - 2\dot{f}(z, z_d)\dot{\theta} - (g + f(z, z_d))\tilde{\tau}_{\theta} \tag{29}$$

Proposition 2: By the fact that $g + f(z, z_d) = \frac{1}{m}u$, which is by hypothesis positif definite as $u > 0$ (see equation (3)), the asymptotic stability of x and θ is asserted by

$$\tilde{\tau}_{\theta} = -\frac{1}{g + f(z, z_d)}(\nu_x + \ddot{f}(z, z_d)\theta + 2\dot{f}(z, z_d)\dot{\theta}) \tag{30}$$

with

$$\begin{aligned} \nu_x =& x_d^{(4)} - k_1^x(x^{(3)} - x_d^{(3)}) - k_2^x(\ddot{x} - \ddot{x}_d) \\ &- k_3^x(\dot{x} - \dot{x}_d) - k_4^x(x - x_d) \end{aligned} \tag{31}$$

k_1^x, k_2^x, k_3^x, k_4^x are positives and stable coefficients.

Proof: Incorporate (30) into (29), it leads to a decoupled x-motion

$$x^{(4)} = \nu_x \tag{32}$$

further, tacking ν_x as given in (31), x and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (17), θ reaches its equilibrium ($\theta(t_f) = 0$). ∎

C. y-stabilization and ϕ-control

As we have detailed above, ϕ denotes the roll angle. This attitude has the same behavior like for θ. Roll allure is necessary to the x4-flyer to correct motion in the y-direction. Thus, these variables are related by the cascade system

$$\begin{aligned} \ddot{y} =& (g + f(z, z_d))\phi \\ \ddot{\phi} =& \tilde{\tau}_{\phi} \end{aligned} \tag{33}$$

We will proceed as before, four time derivatives of the flat output y are necessary. Then

$$y^{(4)} = \ddot{f}(z, z_d)\phi + 2\dot{f}(z, z_d)\dot{\phi} + (g + f(z, z_d))\tilde{\tau}_{\phi} \tag{34}$$

Proposition 3: By the fact that $g + f(z, z_d) = \frac{1}{m}u$, which is by hypothesis positif definite as $u > 0$, the asymptotic stability of y and ϕ is asserted by

$$\tilde{\tau}_{\phi} = -\frac{1}{g + f(z, z_d)}(\nu_y + \ddot{f}(z, z_d)\phi + 2\dot{f}(z, z_d)\dot{\phi}) \tag{35}$$

with

$$\begin{aligned} \nu_y =& y_d^{(4)} - k_1^y(y^{(3)} - y_d^{(3)}) - k_2^y(\ddot{y} - \ddot{y}_d) \\ &- k_3^y(\dot{y} - \dot{y}_d) - k_4^y(y - y_d) \end{aligned} \tag{36}$$

k_1^y, k_2^y, k_3^y, k_4^y are positives and stable coefficients.

Proof: Incorporate (35) into (34), it leads to a decoupled y-motion

$$y^{(4)} = \nu_y \tag{37}$$

further, tacking ν_y as given in (36), y and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (17), ϕ reaches its equilibrium ($\phi(t_f) = 0$).

Remark 1: The proposed $\tilde{\tau}_\theta$ and $\tilde{\tau}_\phi$ stabilizing controller involve the first and second time derivatives of $f(z, z_d)$. We can easily calculate it from (22) and (17). Therefore, $\dot{f}(z, z_d) = z_d^{(3)} + ((k_1^z)^2 - k_2^z)\dot{e}_z + k_1^z k_2^z e_z$ and $\ddot{f}(z, z_d) = z_d^{(4)} - ((k_1^z)^3 - 2k_1^z k_2^z)\dot{e}_z - ((k_1^z)^2 k_2^z - (k_2^z)^3)e_z$.

V. Simulations

Recall that the objectives consist to stabilize the x4-flyer at a desired configuration. What we need to compare is the stabilizing problem with and without motion planning. Motion generation is described here by an important and limited acceleration in ascent following by an important deceleration which permits to reach at $t - f$ the desired point. The generated motion could satisfy $\xi_d(t_i) = \dot{\xi}_d(t_i) = 0$ and $\xi_d(t_f) = \xi_d, \dot{\xi}_d(t_f) = 0$. The final time t_f couldn't be reduced enough to limit an excessive reference acceleration. Without motion planning ξ_d can't be more then $1m$, otherwise the system diverges. Tests have been effectuated as follow: for $\xi_d = \xi_d(t_f) = 1m(t_f = 4s)$ (with and without motion planning) and $\xi_d(t_f) = 10m(t_f = 8s)$ (only with motion planning). All control parameters are $k_1^z = 8$, $k_2^z = 16$, $k_1^x = k_1^y = 20$, $k_2^x = k_2^y = 150$, $k_3^x = k_3^y = 500$ and $k_4^x = k_4^y = 625$. The masse is $m = 2kg$.

A. Results and comments

Any equilibrium of the X4-flyer is defined by $(x_d, y_d, z_d, 0, 0, \psi_d)$. Let $\xi_d(x_d, y_d, z_d) = 10m$. To reach this configuration, the imposed motion planning is given in figure 3. The last sub-figure shows that $f(z, z_d) > -g$, then the well validity of the controllers. Moreover, $\dot{z}_d$ and $\ddot{z}_d$ behavior are sketched in figure 3. The allure of inputs (figure 4) shows that $u > 0$ and $u = mg$ when the X4-flyer reaches the equilibrium, in addition a good errors behavior are demonstrated in figure 5. At the equilibrium, the attitude $\phi(t_f)$ and $\theta(t_f)$ are equal to zero. Without motion planning the system becomes instable and incapable to reach the final configuration. In order to compare the flight with/without an imposed motion, let us examine figures (5,6,7,8). Without motion planning, the amplitude of controllers are important (8), chattering dominate the behavior of these inputs when the system leaves its initial configuration. While with a predefined path a minimum of energy is asserted which is requested for flying vehicles.

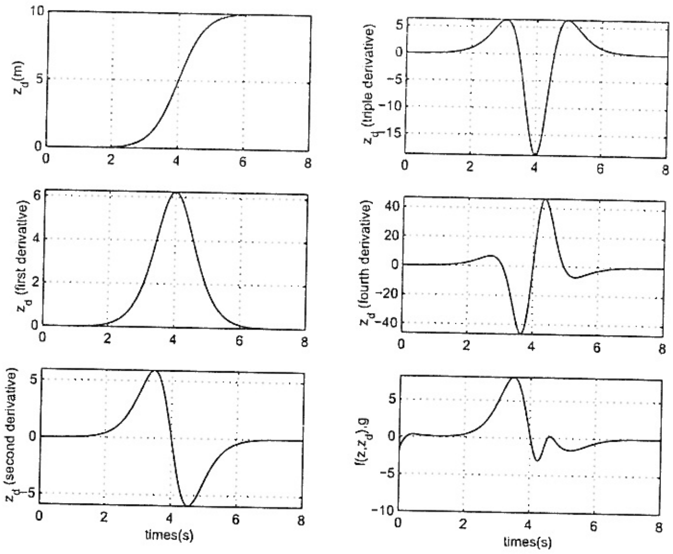

Fig. 3. Motion planning to reach ($x_d = y_d = z_d = 10m$).

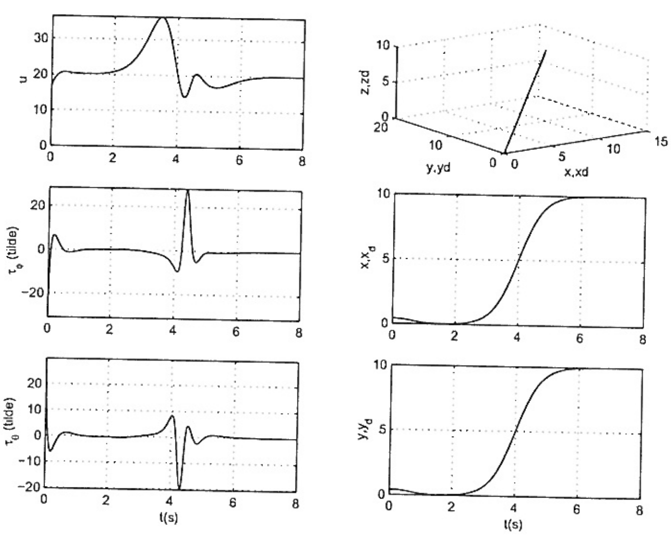

Fig. 4. Necessary inputs to stabilize ($x_d = y_d = z_d = 10m$).

VI. Conclusions

Modelling and controlling aerial vehicles (blimps, mini rotorcraft) are the principal preoccupation of the LSC, LIM-groups. In this topic and in order to stabilize a four rotor rotorcraft, the dynamic modelling involving three inputs was derived. The system presents a flat output which was efficiency exploited in designing the control algorithm for motion planning and stabilization. By carefully choosing the necessary final time for the trajectory realization, we can assert a limit in acceleration, consequently economy in energy. The problem of energy is primordial and justified as the autonomy of batteries don't exceed thirteen minutes.

Acknowledgments

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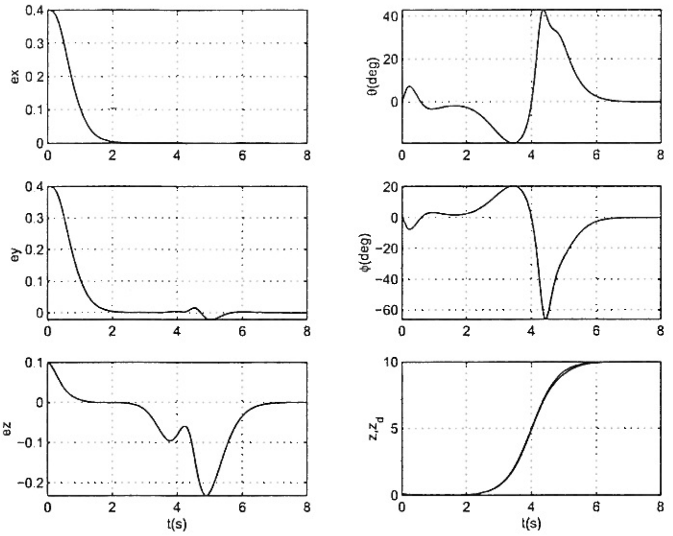

Fig. 5. Stabilization errors with motion planning.

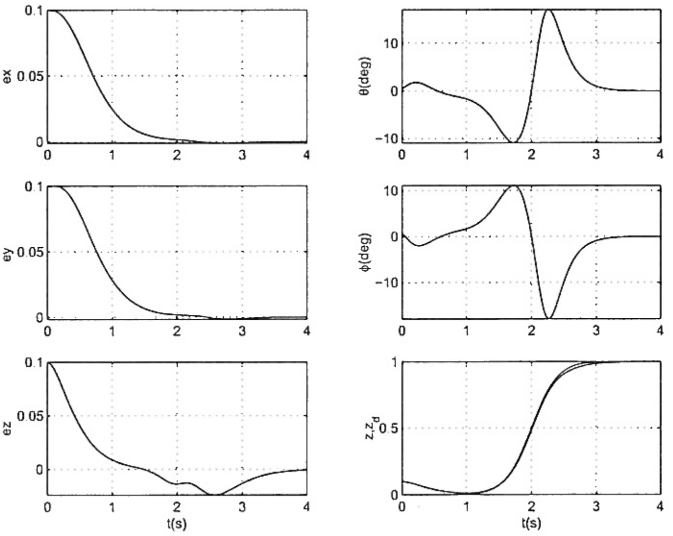

Fig. 6. Errors behavior to reach ($x_d = y_d = z_d = 1m$).

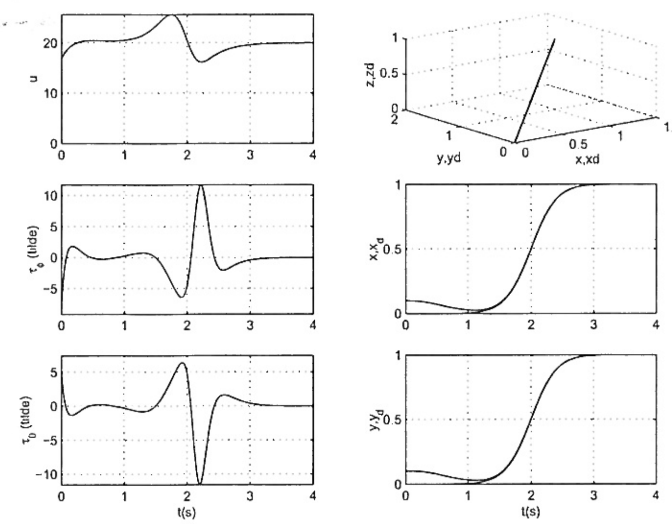

Fig. 7. Inputs to stabilize ($x_d = y_d = z_d = 1m$) with motion planning.

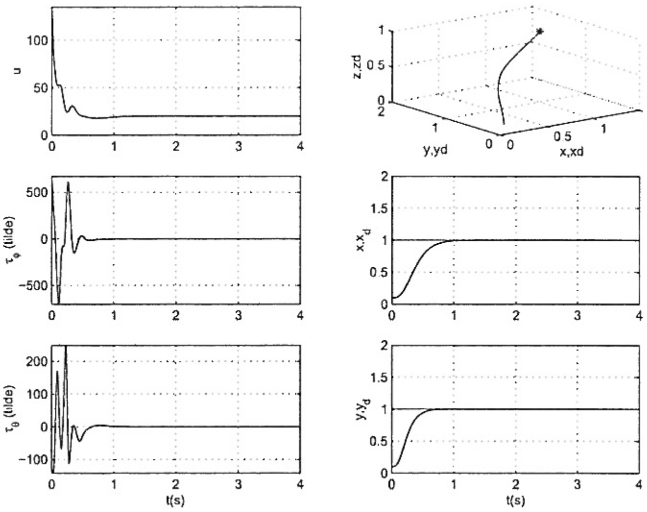

Fig. 8. Necessary inputs to stabilize ($x_d = y_d = z_d = 1m$) without motion planning.

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